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Taking into account the rate of convergence in CLT under Risk evaluation on financial markets

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Abstract: This paper examines "fat tails puzzle" in the financial markets. Ignoring the rate of convergence in Central Limit Theorem (CLT) provides the "fat tail" uncertainty. In this paper, we provide a review of the empirical results obtained "fat tails puzzle" using innovative method of Yuri Gabovich based on the rate of convergence in CLT to the normal distribution, which is called G-bounds. Constructed G-bounds evaluate risk in the financial markets more carefully than models based on Gaussian distributions. This statement was tested on the 24 financial markets exploring their stock indexes. Besides, this has tested Weak-Form Market Efficiency for investigated markets. As a result, we found out the negative correlation between the weak effectiveness of the stock market and the thickness of the left tail of the profitability density function. Therefore, the closer the risk of losses on the stock market to the corresponding risk of loss for a normal distribution, the higher the probability that the market is weak effective. For non-effective markets, the probability of large losses is much higher than for a weak effective.

Subjects: Financial Mathematics; Quantitative Finance; Econometrics

Keywords: fat tails; non-gaussianity; risk evaluation; G-bounds; CLT; The convergence to the normal distribution; weak-form market efficiency (WFE)

JEL classifications: C32; C52; C58; G14

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PUBLIC INTEREST STATEMENT

For economic variables, 1 case of 10,000, or 1 day of the 40 years, explains most of the excesses of the financial markets—as a measure of fat tails. "The difference between the return on the market portfolio, a hypothetical portfolio of all securities, and the riskless rate of return, which is usually defined as the return on a 90 day Treasury bill". An example is the Black Monday in 1987, a day when the Dow Jones index fell by 22.6%. L. Bachelier, R. Merton, F. Black, M. Scholes, and many other well-known scientists have spent many years in the construction of their methodologies, based on a normal distribution. They became so popular that the problem of fat tails became ignored. This statement is one of the most pressing issues of modern financial mathematics, as well as the existence of fat tails is ignored in many models of risk assessment.

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1. Introduction

In economic theory and in practice, often used models have with a normal distribution. But empirical studies show that the practical use of the normal distribution does not take into account a "fat tail puzzle" of fat tails of distributions.

As an alternative to the normal distribution of financial mathematics, there have been developed a variety of approaches, including: model with stable distributions (Fama, 1963), Clarke model, mixed distribution model, General Levy processes, the models with variable and stochastic volatility, microstructural model, and various models of non-normal distributions (Jondeau, Poon, & Rockinger, 2007).

Formally, the random variable *X* has a fat tail, if:

$$\Pr\left[X > x\right] \sim x^{-\alpha}, \text{ when } x \to \infty \text{ and } \alpha > 0.$$

If X has distribution density function $F_x(x)$:

$$\mathrm{Fx}(x) \sim x^{-(1+\alpha)}, \text{ when } x \to \infty \text{ and } \alpha > 0.$$

In particular, fat tails arise when one uses a normal distribution ignoring the rate of convergence in the Central Limit Theorem (hereinafter referred to as the CLT).

Distribution of return on financial markets is not normal. This statement is one of the most urgent problems of modern financial mathematics, as the existence of fat tails is ignored in many models of risk assessment. An example is one of the main risk assessment models' portfolios Value at Risk (VaR), created by the JPMorgan bank.

2. Objectives and hypotheses

This research examines the question of using G-bounds method for risk evaluation and testing WFE hypothesis.

Our research intends to reach the next objectives:

(1) To test G-bound method in the different financial markets.
(2) To test the relationship between market efficiency and deviation from the normal distribution.

The following hypotheses have been tested:

H0: G-bound evaluates risks on the financial markets more accurately than normal distribution and CLT.

H1: Markets of researched countries are effective in the weak form.

H2: There is negative correlation between market efficiency and risk of large losses on the investigated stock market.

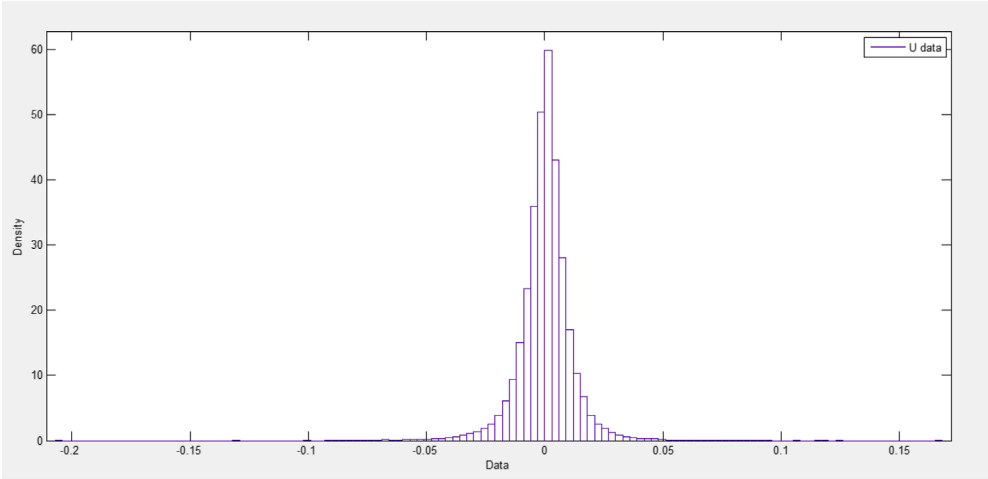
3. Data, methodology, and results

As an example of non-Gaussianity phenomena in the stock market, a dynamics of the price of S and P500 from 03.01.1928 to 31.05.2015 has been considered (Thomson Reuters database, XXXX) (Figure 1).

The Kolmogorov–Smirnov test and the Jarque–Bera test reject normality of this distribution at 5% significance level, as well as in case of all the other stock indices for the countries under investigation.

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Figure 1. Dynamics of a price change S & P500 index.



For comparison of the results, evaluation of fat tail distribution by Value at Risk (VaR) model based on the historical data of the index has been performed (Figure 2).

As seen in Table 1, sigma distribution of left tail S & P500 index profitability does not correspond to analogous for the normal distribution.

One of the main lacks of VaR model is the fact, that regardless on the calculus method, VaR approach always uses historical market data. In case of abrupt changes in the market as sudden and abrupt change in volatility as well as changes in the assets correlation, VaR will consider the changes only after some period of time and before that moment evaluation would be incorrect. Also it does not take into account characteristics of market liquidity in VaR models. Various methods of risk evaluation exist in the framework of this model, which lead to the formation of model risks.

VaR can work correctly in cases of stable markets and cease to adequately show the risk amount at the market's excesses, which carries a great risk related to the fat-tailed distributions.

All mentioned data demonstrate that existing mathematical models of risk evaluation using normal distribution are far from the true risk values (Yahoo Finance, XXXX). Hence, there is need to switch from the idea of using the normal distribution in the standard form to risk evaluation which takes into account convergence rate. This approach allows building risk assessment close to that obtained from historical data.

Figure 2. Sigma quintiles for a normal distribution.

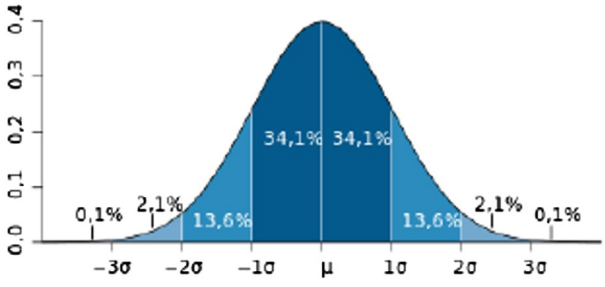


Table 1. VaR distribution by sigma for the logarithms of daily profitability S and P 500

	Number of observation	21,643	
	Average	0.00028621	
	St. dev.	0.011871591	
1	1 sigma	−0.011585381	19,609
2	2 sigma	−0.023456972	21,093
3	3 sigma	−0.035328563	21,449
3,5	3,5 sigma	−0.041264359	21,507
4	4 sigma	−0.047200154	21,557
5	5 sigma	−0.059071745	21,601
6	6 sigma	−0.070943336	21,643

3.1. G-bounds evaluation

In this article, we focus on left tails of distribution of stock index profitability, which are criteria of loses in stock markets.

Consider equity prices $S_j > 0$ over n time intervals: S_0, S_1, ... S_n, and define X_j as logarithms of stock returns: $X_1 = \ln(S_1/S_0)$, ..., $X_n = \ln(S_n/S_{n-1})$. Logarithm of the stock return over the whole period $Y_n = \ln(S_n/S_0)$ can be represented as a sum of X_j:

$$Y_n = X_1 + \ldots + X_n$$

In order to demonstrate that $F(t)$ is a characteristic suitable for $F_n(t)$, expand the $F_n(t)$ as follows:

$$F_n(t) = \left[F_n(t) - \Phi(t)\right] + \Phi(t)$$

This simple observation explains the phenomenon of fat tails and it is very simple and makes doubtful the traditional use of $F(t)$ in characteristics for the $F_n(t)$ on the whole space (n, t).

In this paper, we will consider two types ($k = 1, 2$) of generalized assessments (G-Bounds) $G_{k,n}(t)$, which are functions of n and t. $G_{k,n}(t)$ are designed to provide an upper bound for the $F_n(t)$.

$$F_n(t) \le G_{k,n}(t)$$

Consider $Y_n = \sum_{j=1}^{n} X_j$, composed of independent and identically distributed random variables with finite third absolute moments $M_3 = E|X_j|^3$.

We assume that the random variables X_1 degenerate with a standard deviation $\sigma > 0$.

$$\text{Defines constants} \rho_3 = \frac{E|X_1|^3}{\sigma^2}.$$

Building $G_{k,n}(t)$, bounds follow from the well-known results on the rate of convergence to the normal distribution (Berry, 1941) and inequalities for sums of independent random variables.

We can use Berry–Esseen's-type inequality:

For c.d.f. $F_n(t)$ of Y_n, there exists such normal $\Phi_n(t)$ and non-dependent on n constant C that for all t (Esseen, 1942):

$$sup_t|F_n(t) - \Phi(t)| \le \frac{C\rho}{\sqrt{n}}$$

According to this: $F_n(t) = \left[F_n(t) - \Phi(t)\right] + \Phi(t) \le \frac{C\rho}{\sqrt{n}} + \Phi(t)$

Using the Nagaev–Nikulin-type inequalities (Nikulin, 2010), for sums of independent random variables, we get:

$$|F_n(t) - \Phi(t)| \le \frac{C(t)\rho}{\sqrt{n}(1 + |t|^3)}$$

The following two types of G-bounds have been constructed analytically using combinations of the above-mentioned inequalities in the following way:

G1-bounds $G_{1,n}(t)$ - combination of Berry-Esseen-type estimates with Chebyshev-type inequality;

G2-bounds $G_{2,n}(t)$ - combination of G1-bounds with Nagaev-Nikulin-type inequality.

Table 2 contains detailed information about results of constructing G-bounds.

Description of Table 2 values: $H(t)$—historical frequencies of observed one-year losses in excess of $t = K^*\sigma$; $\Phi(t)$—left tail of a standard normal distribution; $\Psi(\Phi, t)$—thickness ratio measured with respect to the normal distribution; ΔKS—Berry–Essen-type estimate of convergence to a normal distribution; KS—tail estimate based on the sum of ΔKS and $\Phi(t)$; CH(t)—tail estimate based on Chebyshev's inequality; G1(t)—G1-bound, constructed based on Berry–Esseen and Chebyshev inequalities; $\Psi(G1, t)$—thickness ratio measured with respect to G1-bound; NC(t)—Nikulin's estimates of constant C(t); NN(t)—Nagaev–Nikulin's bound; G2(t)—G2-bound, constructed based on the G1-bound and Nagaev-type inequalities; $\Psi(G2, t)$—thickness ratio measured with respect to the G2-bound.

As seen from the Table 2, thickness of left tail related to standard normal distribution is considerably less than the thickness corresponding to G-bounds.

Based on these results, the hypothesis about the analytical construction of G-bounds corresponding to the frequency of losses in the stock markets is fully confirmed.

Historical data demonstrate significant losses at the level of −6σ, while the corresponding value for the standard normal distribution is much smaller:

$$\frac{F_{258}(-6\sigma)}{\Phi(-6\sigma)} \sim 10^7$$

Investors encounter losses −6σ 10 million times greater than the predicted loss with the help of the Central Limit Theorem. The stated hypothesis is fully confirmed and shows the importance of further study and implementation of G-bounds as a measure of risk close to the market.

For all 24 countries, Y. Gabovich's conclusion (Gabovich, 2013) is confirmed by the historical evaluation of distribution tails of log profitability of all 24 tested countries; they did not go beyond the G-bounds.

3.2. Analysis of WFE hypotheses

The question about evaluation of the effectiveness in the stock market is one of the fundamental questions of financial theory. In 1970, the American scientist J. Fama formulated the hypothesis on the information efficiency of the stock market (Fama, 1970). According to this hypothesis, there are three groups of efficient markets, characterized by the amount of information available to investors.

Table 2. G-bounds evaluation for daytime MICEX index for the period from inception to 18 May 2015

0	1*σ	2*σ	3*σ	4*σ	5*σ	6*σ	7*σ	8*σ	9*σ	10*σ
H(t)	0.001822	0.001822	0.001822	0.001822	0.001822	0.001822	0.001822	0.001822	0.001822	0.001822
Ψ(Φ,t)	0.01148	0.079908	1.401468	57.47345	6348.113	1845905	1.42E+09	2.93E+12	1.61E+16	2.39E+20
Φ(t)	0.1587	0.0228	0.0013	3.17E−05	2.87E−07	9.87E−10	1.28E−12	6.22E−16	1.13E−19	7.62E−24
ΔKS	0.013949	0.013949	0.013949	0.013949	0.013949	0.013949	0.013949	0.013949	0.013949	0.013949
CH(t)	0.5	0.2	0.1	0.0588	0.0385	0.027	0.02	0.0154	0.0122	0.0099
KS	0.172649	0.036749	0.015249	0.01398	0.013949	0.013949	0.013949	0.013949	0.013949	0.013949
G1(t)	0.172649	0.036749	0.015249	0.01398	0.013949	0.013949	0.013949	0.013949	0.013949	0.013949
Ψ(G1,t)	0.010553	0.049578	0.119481	0.13032	0.130614	0.130617	0.130617	0.130617	0.130617	0.130617
NC(t)	29.117	29.117	29.117	22.1853	16.024	11.8046	9.059	7.2512	6.0329	5.737
NN(t)	16.42365	3.717361	1.227884	0.411264	0.151953	0.065031	0.03155	0.016741	0.009658	0.007083
G2(t)	0.172649	0.036749	0.015249	0.01398	0.013949	0.013949	0.013949	0.013949	0.009658	0.007083
Ψ(G2,t)	0.010553	0.049578	0.119481	0.13032	0.130614	0.130617	0.130617	0.130617	0.18864	0.257236

The developed algorithm analyzes the weak form of efficiency and consists of the following steps:

(1) Analysis of the test results of Runs
(2) Analysis of the test results on the random walk
(3) If the results of the random walk test and the Runs test does not reject by the 5% significance level the hypothesis, that the distribution of logarithmic daily profitability are said to be randomly distributed and obeys the law of the random walk, the country will be defined as weak form effective. Otherwise, the country was regarded as not effective.

3.2.1. Runs test

Runs test results are interpreted as follows: if the value of Z statistic is greater than 1.96 and/or less than −1.96, the null hypothesis that the sampling units are distributed randomly is rejected. Runs test is used to check the following null hypothesis:

H0: Changes in asset prices are random and independent.

H1: asset price changes are not random and independent.

Based on the runs test results, two groups of countries were identified:

(1) H0 is not rejected at the 5% level for the following countries: Argentina, Australia, Belgium, Brazil, Britain, Germany, Denmark, Ireland, Israel, Spain, Turkey, France, Switzerland, Japan, and the USA.
(2) Hypothesis H0 is rejected at the 5% level for the following countries: Austria, Hong Kong, India, Indonesia, Canada, Malaysia, Mexico, the Netherlands, and Russia.

The criterion of a random distribution of log returns of stock indexes is a fundamental issue in the study hypothesis of weak-form market efficiency (Table 3).

3.2.2. Random walk test

The test results of random walks, Lo and Mackinlay (1988), are more common than the results of a similar test, the Dickey–Fuller (Dickey & Fuller, 1979), since it uses the fact that if the value of the asset is described by a random walk, the price increments are not correlated, and the dispersion of these increments increases linearly in the given time interval.

The H0 hypothesis states that:

Y_t is the time series obtained from the following equation:

$Y_t = u + Y_{t-1} + \epsilon_t$, where u is some drift parameter.

For the key properties of a random walk, we check if $E(\varepsilon_t) = 0$ for all t and $E(\varepsilon_t, \varepsilon_{t-1}) = 0$ (Table 4).

According to the test results, the following two groups are obtained:

H0 is not rejected for the following countries: Australia, Argentina, Austria, Brazil, France, Germany, Hong Kong, Denmark, Israel, Spain, Canada, Malaysia, the Netherlands, Turkey, France, Switzerland, Japan, and the USA.

H0 is rejected for the following countries: Belgium, India, Indonesia, Ireland, Mexico, and Russia.

After analyzing the results of random walk test and runs test, we note that they do not match. It is difficult to confirm the hypothesis of the weak efficiency of the stock market of any country.

Table 3. Runs test results for logarithms of daily profitability of indexes

Runs test	Number of runs	No. of "+" symbol	No. of "–" symbol	Z-stats
Australia	3,785	2,904	2,788	−0.2987
Austria	3,621	2,832	2,742	−3.0128
Argentina	3,039	2,336	2,241	−0.4266
Belgium	4,015	3,116	2,984	−1.564
Brazil	3,635	2,799	2,677	−0.4968
United Kingdom	5,337	4,126	3,980	−1.7606
Germany	4,122	3,179	3,014	−0.1959
Hong Kong	4,621	3,638	3,416	−2.3016
Denmark	3,965	3,064	2,978	−1.9173
Israel	3,805	3,621	555	1.40
India	2,833	2,270	2,149	−4.026
Indonesia	2,798	2,224	2,123	−3.5916
Ireland	2,982	2,248	2,272	−1.0996
Spain	2,794	2,616	2,551	1.0533
Canada	5,864	4,638	4,532	−6.1714
Malaysia	3,375	2,716	2,581	−5.0894
Mexico	3,738	2,999	2,891	−5.8257
Netherlands	3,771	2,966	2,792	−2.1099
Russia	2,850	2,152	2,237	−2.7151
Turkey	3,797	2,904	2,777	0.278
France	4,257	3,255	3,131	−0.0049
Switzerland	4,107	3,148	3,038	−0.5076
Japan	5,114	3,932	3,791	−0.9311
USA	13,905	10,556	11,364	−1.882

Table 4. Results of the random walk test for daily price indexes

Variance ratio test	Prob.	Stats	Variance ratio test	Prob.	Stats
Australia	0.2042	−1.27	Ireland	0.0095	2.60
Austria	0.1284	1.52	Spain	0.0504	−1.96
Argentina	0.5072	0.66	Canada	0.6369	0.47
Belgium	0.0026	3.01	Malaysia	0.5597	0.58
Brazil	0.5405	−0.61	Mexico	0.0085	2.63
United Kingdom	0.5602	−0.58	Netherlands	0.3749	0.89
Germany	0.9836	−0.02	Russia	0.0047	−2.83
Hong Kong	0.5919	0.54	Turkey	0.7646	−0.30
Denmark	0.0551	1.92	France	0.2588	−1.13
Israel	0.6936	−0.39	Switzerland	0.1115	1.59
India	9.78E−04	3.30	Japan	0.7743	−0.29
Indonesia	0.0266	2.22	USA	0.3604	0.91

Applying the above algorithm for testing the weak-form of efficiency, we consider that the following countries were classified as effective in the weak form: Argentina, Australia, Brazil, Britain, Germany, Denmark, Israel, Spain, the Netherlands, Turkey, France, Switzerland, Japan, and the USA.

3.3. Correlation between weak-form efficiency and thickness of the distribution left tail

For analyzing the relationship between thickness of the left tail of the distribution of stock indices profitability and WFE, the logit model was built.

A binary variable stands for the results of a vector which was found while testing WFM:

- 1 if the market satisfies the WFE hypothesis
- 0 if the market does not satisfy the WFE hypothesis.

The distribution of the random component in the model is supposed to be as follows:

$$F(z) = \frac{1}{1 + e^{-z}}$$

Z—is the matrix with dimension of 24 × 10, which presents the data of the thickness of the left tail of the distribution and is broken by Sigma.

The thickness of the characteristics with respect to Φ (*t*) is defined as the ratio of observed historically left tail H(*t*) to Φ(*t*)—corresponding to the left tails of the standard normal distribution:

$$\Psi(\Phi, t) = \frac{H(t)}{\Phi(t)}$$

A deviation from the average thickness of the left tail distribution of the logarithmic daily returns of stock indexes was considered. While constructing a logit regression, the logit regression between the above relationships (1) and market weak-form efficiency by Fama are analyzed.

Table 5. Logit test model results

		Coef.	Std. err.	*z*	*P* > |*z*|	[95% Conf. interval]	
−1*σ	*X*	2066309	3,027,078	0.68	0.495	−3,866,654	7,999,272
	Const	−0.9540378	1.929778	−0.49	0.621	−4.736332	2.828257
−2*σ	*X*	101,228.70	1,194,578.00	0.08	0.93	−2,240,101.00	2,442,559.00
	Const	0.22	1.44	0.15	0.88	−2.60	3.04
−3*σ	*X*	−226,253.40	276,671.00	−0.82	0.41	−768,518.60	316,011.70
	Const	1.68	1.71	0.98	0.33	−1.68	5.03
−4*σ	*X*	−14,209.27	12,722.60	−1.12	0.26	−39,145.11	10,726.58
	Const	1.60	1.23	1.30	0.19	−0.81	4.02
−5*σ	*X*	−440.3902	223.7824	−1.97	0.049	−878.9956	−1.784768
	Const	2.06	0.99	2.08	0.04	0.12	4.01
−6*σ	*X*	−3.02	1.39	−2.17	0.03	−5.75	−0.30
	Const	2.10	0.93	2.26	0.02	0.28	3.92
−7*σ	*X*	0.00	0.00	−1.04	0.30	−0.01	0.00
	Const	0.79	0.61	1.31	0.19	−0.40	1.98
−8*σ	*X*	0.00	0.00	−0.60	0.55	0.00	0.00
	Const	0.51	0.50	1.00	0.32	−0.48	1.49
−9*σ	*X*	0.00	0.00	−1.02	0.31	0.00	0.00
	Const	0.63	0.50	1.26	0.21	−0.35	1.61
−10*σ	*X*	0.00	0.00	−0.63	0.53	0.00	0.00
	Const	0.46	0.46	1.00	0.32	−0.44	1.37

As a result, the analysis revealed a negative correlation between WFE of market and thickness of the left tail of the distribution of logarithmic daily returns at −5 and −6 sigmas. In other cases, the model was not significant.

Thickness of the tails was divided by 1 million, for more interpretable model results (Table 5).

Therefore, it illustrates that the greater the risk of large losses in the stock market, deviating from the corresponding value for the normal distribution, the higher the probability that the market itself would not satisfy the hypothesis of a weak effectiveness.

4. Conclusion

A negative correlation between the distribution of returns of the stock indexes and WFE of the stock market was observed. Using the rate of convergence to the normal distribution, instead of the normal distribution, the investors will be able to significantly improve their risk assessment and bring it to market.

The first hypothesis was proved for all investigated indices of 24 countries.

The second hypothesis was partially proved. Two groups of countries were identified: the former satisfies the conditions of weak effectiveness and the latter is not effective. While testing the hypothesis of weak effectiveness, a sub-hypothesis of cointegration of time series, consisting of log returns of surveyed countries' stock indexes, was highlighted. This result suggests that, despite the random distribution of the yield indexes of these countries, there is a long-term dependence between them, which leads to their change. The studies of cointegration of time series on the stock markets are very important. The movement of the prices for derivative financial instruments is rather difficult to predict, but if the investor had a strategy based on a linear combination of several financial instruments, creating a stationary time series, it would greatly facilitate the formation of financial strategy.

Based on the third hypothesis, a negative relationship between the weak effectiveness of the stock market and the thickness of the left tail of the profitability distribution was explored. Thus, the thicker the left tail of the distribution of returns stock index, the higher the risk of large losses in the market, so the greater is the likelihood that the market is not effective. Hence, the closer the risk of losses on the stock market to the corresponding risk of loss for a normal distribution, the higher the probability that the market is weak effective. For non-effective markets, the probability of large losses is much higher than for a weak effective.

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