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On the A-equilibria Properties in Multicriteria Extensive Games

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Abstract

Using A-optimality concept for vector-valued maximization, we propose a refinement of Pareto equilibria in n-person multicriteria games. The theorems on existence of A-equilibria and subgame perfect A-equilibria are derived. Time consistency of A-equilibria in extensive multicriteria games with perfect and incomplete information is proved.

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1 Introduction

The concept of (weak) equilibria (or Pareto equilibria) in the games with vector payoffs was offered by Shapley in [11]. Some interesting properties of equilibria in different classes of multicriteria n-person games we established in [13, 6, 7, 2]. However, the set of all Pareto equilibriums may be "too large" in general case. Some reasonable refinements of Pareto equilibria were investigate in [1, 12, 8].

The main purpose of this paper is to present a new refinement of Pareto equilibria, which is based on so-called "A-optimality" concept for vector-valued maximization [14].

Section 2 contains main definitions and the general existence theorem. In section 3 we establish the existence of subgame perfect A- equilibrium (in pure strategies) in extensive multicriteria game with perfect information. Time consistency of A-equilibria is investigated in section 4.

2 A-equilibria in n-person multicriteria game

Let us consider n-person games where the player's payoff is given by a vector instead of a scalar (so-called multicriteria games). Pure strategy profile $\varphi = (\varphi_1, \ldots, \varphi_n) \in \prod_{j=1}^{n} \Phi_j$ provides to each player $i \in N = \{1, \ldots, n\}$ "payoffs" given by an $r(i)$-vector valued function $H_i : \prod_{j=1}^{n} \Phi_j \longrightarrow R^{r(i)}$, i.e. player i takes $r(i)$ criteria $H_i(\varphi) = H_{i|1}(\varphi), \ldots, H_{i|r(i)}(\varphi))$ into account. We denote by $MG(n, r(1), \ldots, r(n))$ the class of all n-person multicriteria games.

For all $x, y \in R^t$ we will use the notation $y > x$ if and only if $y_i > x_i$ for all $i \in \{1, \ldots, t\}$.

Definition 2.1. The strategy profile $\hat{\varphi} = (\hat{\varphi}_1, ..., \hat{\varphi}_n)$ is called (weak) equilibrium (multicriteria equilibrium or Pareto equilibrium) [11, 1, 13, 6] in multicriteria game $\Gamma \in MG(n, r(1), \ldots, r(n))$, iff

$$\forall i \in N \;\rceil \exists \; \varphi_i \in \Phi_i : H_i(\varphi_i, \widehat{\varphi}_{-i}) > H_i(\widehat{\varphi}_i, \widehat{\varphi}_{-i}), \tag{1}$$

where $\widehat{\varphi}_{-i} = (\widehat{\varphi}_j), j \in N \backslash \{i\}$.

Let $ME(\Gamma)$ denote the set of all equilibriums in multicriteria game Γ. Actually the set ME(Γ) may be "too large" in general case (see, for instance [1, 13, 8]).

One can use A-optimality concept [14] for reasonable refinement of ME set in n-person multicriteria game $\Gamma \in MG(n, r(1), \ldots, r(n))$.

Let $A^k = (a^k_{ij})$ be the $r(k) \times r(k)$ player k matrix with positive elements.

Definition 2.2. The strategy profile $\bar{\varphi} = (\bar{\varphi}_1, \ldots, \bar{\varphi}_n)$ is called A-equilibrium in multicreteria game $\Gamma \in MG(n, r(1), \ldots, r(n))$ iff

$$\forall \; k \in N \;\rceil \exists \; \varphi_i \in \Phi_i : \; A^k H_k(\varphi_k, \bar{\varphi}_{-k}) > A^k H_k(\bar{\varphi}_k, \bar{\varphi}_{-k}). \tag{2}$$

Using the notation

$$\widehat{H}_{k|j}(\varphi) = \sum_{j=1}^{r(k)} a_{ij}^k H_{k|j}(\varphi) \tag{3}$$

the vector inequality (2) could be written in the following form:

$$\begin{cases} \widehat{H}_{k|j}(\varphi_k, \bar{\varphi}_{-k}) > \widehat{H}_{k|j}(\bar{\varphi}_k, \bar{\varphi}_{-k}), \\ i = 1, \ldots, r(k). \end{cases} \tag{4}$$

Hence, the A-equilibrium could be thought as Pareto equilibrium (1) in the auxiliary multicriteria game with new vector-payoffs (3).

Let $ME^A(\Gamma)$ denote the set of all A-equilibriums in multicreteria game Γ.

Note that $ME^A(\Gamma) \subset ME(\Gamma)$. Let us use the following notation:

$$\lambda = (\lambda_1, \ldots, \lambda_{r(k)}) \in \Lambda^{r(k)} = \{\lambda \in R^{r(k)} \mid \lambda_j \geq 0,\ \sum_{j=1}^{r(k)} \lambda_j = 1\},$$

$$\mu_j(\lambda) = \lambda_1 a_{1j} + \lambda_2 a_{2j} + \ldots + \lambda_{r(k)} a_{r(k)j},\ j = 1, \ldots, r(k). \tag{5}$$

Lemma 2.3. Given $\varphi_{-k} \in \prod_{j \neq k} \Phi_j$ suppose there exists such $\lambda \in \Lambda^{r(k)}$ and $\bar{\varphi}_k \in \Phi_k$ that

$$\sum_{j=1}^{r(k)} \mu_j(\lambda) H_{k|j}(\bar{\varphi}_k, \varphi_{-k}) = \max_{\varphi_k \in \Phi_k} \sum_{j=1}^{r(k)} \mu_j(\lambda) H_{k|j}(\varphi_k, \varphi_{-k}). \tag{6}$$

Then

$$\rceil \exists\ \varphi_k \in \Phi_k :\ A^k H_k(\varphi_k, \varphi_{-k}) > A^k H_k(\bar{\varphi}_k, \varphi_{-k}). \tag{7}$$

Proof. Using notations (3) and (5) one can write the condition (6) in the following form:

$$\lambda_1 \widehat{H}_{k|1}(\bar{\varphi}_k, \varphi_{-k}) + \ldots + \lambda_{r(k)} \widehat{H}_{k|r(k)}(\bar{\varphi}_k, \varphi_{-k}) =$$

$$= \max_{\varphi_k \in \Phi_k} \sum_{j=1}^{r(k)} \lambda_j \cdot \widehat{H}_{k|j}(\varphi_k, \varphi_{-k}) \tag{8}$$

Suppose that condition (7) is violated, i.e. there exists such strategy φ_k that

$$A^k H_k(\varphi_k, \varphi_{-k}) > A^k H_k(\bar{\varphi}_k, \varphi_{-k})$$

or

$$\begin{cases} \widehat{H}_{k|j}(\varphi_k, \varphi_{-k}) > \widehat{H}_{k|j}(\bar{\varphi}_k, \varphi_{-k}), \\ i = 1 \ldots, r(k). \end{cases} \tag{9}$$

If we multiply the inequality i from system (9) by λ_i and then add all inequalities, we get

$$\sum_{i=1}^{r(k)} \lambda_i \cdot \widehat{H}_{k|j}(\varphi_k, \varphi_{-k}) > \sum_{i=1}^{r(k)} \lambda_i \cdot \widehat{H}_{k|j}(\bar{\varphi}_k, \varphi_{-k})$$

The latter inequality contradicts (8).

Theorem 2.4. If the players' strategy sets $\Phi_k \in \mathrm{Comp}\, R^{m_k}$, and payoffs functions $H_k(\varphi)$, $\varphi \in \prod_{k=1}^{n} \Phi_k$, are continuous, $k = 1, \ldots, n$, then the multicriteria n-person game $\Gamma \in MG(n, r(1), \ldots, r(n))$ possesses A-equilibrium (for every given $r(k) \times r(k)$ positive matrixes A^k, $k = 1, \ldots, n$).

Corollary 2.5.. Let $\Gamma \in MG(n, r(1), \ldots, r(n))$ be a finite n-person multicriteria game (every player k has a finite number m_k of pure strategies). Then the set of A-equilibriums in mixed extension of multicriteria game Γ is non-empty.

3 Subgame-perfect A-equilibria in multicriteria extensive game with perfect information

Let $\Gamma = \{N, K, P, A, h\}$ be a finite multicriteria n-person game in extensive form [3, 9] with perfect information, where:

- K is the game tree (with initial node x_0) that consists of set Z of all terminal nodes (endpoints) and set $X = K \setminus Z$ of all intermediate nodes;
- $S(x)$ is the set of all node x immediate "successors"; $S(x) = \emptyset \quad \forall\, x \in Z$;
- $S^{-1}(x)$ is the unique immediate "precessor" of the node x: $x \in S\left(S^{-1}(x)\right)$, $S^{-1}(x_0) = \emptyset$;
- $Z(x)$ is the set of all terminal nodes, which can be reached from x;
- $\omega = \{x_0, x_1, x_2, \ldots, x_l\}$ — the play (or trajectory) of length l:

$$x_l \in Z, x_{j-1} = S^{-1}(x_j), \quad j = 1, \ldots, l.$$

- P_i is the set of all nodes where player i moves;
- A is the "choice partition", i.e. $A_j = \{x \in K \setminus Z \mid |S(x)| = j\}$;
- $h_i(z) = (h_{i/1}(z), \ldots, h_{i/r(i)}(z))$ is the player's i payoffs vector at the terminal node $z \in Z$.

The player's i pure strategy is a function (with domain P_i) that determines for every node $x \in P_i$ some choice or alternative $y \in S(x)$.

Let $\Phi_i, i \in N$ denote the set of all player's i pure strategies in Γ. The strategy profile $\varphi = (\varphi_1, \ldots, \varphi_n)$ determines a unique play $\omega = \{x_0, x_1, \ldots, x_l\}$ in Γ, where $\varphi_i(x_k) = x_{k+1}$, if $x_k \in P_i, x_l \in Z$, and, respectively, a collection of all players vector payoffs $\{h_i(x_l)\}_{i \in N}$.

Hence, we get the player's i vector payoff function:

$$H_i(\varphi) = h_i(\omega) = h_i(x_l). \tag{10}$$

Let us use $MG^P(n, K, r(1), \ldots, r(n))$ to denote the class of all finite n-person multicriteria extensive games with perfect information and vector payoffs (10).

In game Γ with perfect information every intermediate node $x \in K \backslash Z$ generates subgame $\Gamma_x = \{N^x, K^x, P^x, A^x, h^x\}$, which components are just the restrictions of corresponding components of original game Γ onto subtree K_x (the subgame Γ_x tree).

In particular,

$$h_i^x(y) = h_i(y) \quad \forall y \in Z(x) \quad \forall i \in N \tag{11}$$

Let Φ_i^x denote the set of all player's i pure strategies in subgame Γ_x. Strategy profile $\varphi^x \in \prod_{j=1}^n \Phi_i^x$ generates the unique play $\omega^x = \{x, \ldots, x_m\}$ in the subgame and, hence, the collection of players' vector payoffs:

$$H_i^x : \prod_{j=1}^n \Phi_j^x \longrightarrow R^{r(i)}, i \in N. \tag{12}$$

Suppose $x \in K \backslash Z, x \neq x_0$. For every strategy profile φ^x in subgame Γ_x let $\Gamma_D = \Gamma_D(\varphi^x)$ denote the so-called factor-game on the tree $K^D = \{x\} \cup K \backslash K^x$.

Note that $\{x\} \cup Z \backslash Z(x)$ is the set of terminal nodes in factor-game, and

$$h_i^D(x) = H_i^x(\varphi^x), i \in N. \tag{13}$$

Let Φ_i^D denote the set of all player's i pure strategies in factor-game Γ_D. Strategy profile $\varphi^D \in \prod_{j=1}^n \Phi_i^D$ generates unique play $\omega^D = \{x_0, \ldots, x_k\}$ in factor-game Γ_D and the collection of players' vector payoffs H_i^D.

The decomposition of original extensive game Γ at node x onto subgame Γ_x and factor-game Γ_D generates the corresponding decomposition of pure (and mixed) strategies [3, 9]. The pure strategy $\varphi_i \in \Phi_i$ decomposition at intermediate node x onto pure strategy $\varphi_i^x \in \Phi_i^x$ in subgame Γ_x and pure strategy $\varphi_i^D \in \Phi_i^D$ in factor-game Γ_D means that φ_i^x is the restriction of φ_i onto set P_i^x, and φ_i^D is the restriction of φ_i onto set P_i^D.

Note that $P_i = P_i^x \bigcup P_i^D$, and, hence, one can compose the player's pure strategy $\varphi_i = (\varphi_i^D, \varphi_i^x) \in \Phi_i$ in original game Γ from his strategies $\varphi_i^x \in \Phi_i^x$ and $\varphi_i^D \in \Phi_i^D$ in subgame Γ_x and factor-game Γ_D respectively.

Definition 3.1 *The strategy profile $\widehat{\varphi} \in ME^A(\Gamma)$ is called subgame perfect [10] A-equilibrium in Γ iff:*

$$\widehat{\varphi}^x \in ME^A(\Gamma^x) \quad \forall x \in K \backslash Z. \tag{14}$$

Let $SPME^A(\Gamma)$ denote the set of all subgame perfect A-equilibriums in Γ.

It was shown in [7] how one can use the auxiliary unicriterium game ("trade-off unicriterium game" [11]) and backwards induction procedure to construct all subgame perfect weak equilibriums (in pure strategies) in multicriteria extensive game with perfect information.

Using the same approach for A-equilibria we may prove the following propositions.

Theorem 3.2 *Every finite n-person extensive multicriteria game $\Gamma \in MG^P(n, K, r(1), ..., r(n))$ with perfect information possesses subgame perfect A-equilibrium $\hat{\varphi} \in SPME^A(\Gamma)$ in pure strategies.*

Corollary 3.3 *The set $ME^A(\Gamma)$ of all A-equilibriums (in pure strategies) in finite n-person multicriteria extensive game $\Gamma \in MG^P(n, K, r(1), ..., r(n))$ with perfect information is non-empty.*

4 Time consistency of A-equilibria in multicriteria extensive game with incomplete information

Now let us turn to the class $MG(n, K, r(1), ..., r(n))$ of finite n-person extensive games $\Gamma = \{N, K, P, A, U, h\}$ with incomplete information [3, 9, 4, 5] and with vector payoffs. We let U denote the collection of all players informational sets. The mixed strategy profile μ in extensive game Γ with incomplete information generates a whole set $\Omega(\mu)$ of plays (trajectories) ω on game tree K, and let $p(\omega, \mu)$ denote the probability of the play ω realization in Γ if all players use mixed strategies $\mu_i, i \in N$.

Note, that intermediate node x generates subgame Γ_x of the game Γ with incomplete information if and only if every informational set in Γ is included in K_x or does not intersect with K_x.

The decomposition of extensive game Γ with incomplete information at node x onto factor-game Γ_D and subgame Γ_x generates corresponding decomposition of mixed strategies. In that case the following proposition holds [3, 9].

Lemma 4.1 *Every pair μ_i^x and μ_i^D of player's i mixed strategies in Γ_x and Γ_D can be obtained as the result of decomposition of some mixed strategy μ_i*

in original game Γ. *Moreover, for each play* $\omega \in \Gamma$ *which contains* x, *the following condition holds:*

$$p(\omega, \mu) = p(\bar{\omega}_x, \mu^D) \cdot p(\omega^x, \mu^x), \tag{15}$$

where $\mu^D = (\mu_1^D, ..., \mu_n^D)$ *— the strategy profile in* Γ_D, $\mu^x = (\mu_1^x, ..., \mu_n^x)$ *— the strategy profile in subgame* Γ_x, $\omega = \{x_0, ..., x, ..., x_l\}, x_l \in Z$ *— the play in* Γ, $\bar{\omega}_x = \{x_0, ..., x\}$ *— the play in* Γ_D, $\omega^x = \{x, ..., x_l\}$ *— the play in* Γ_x, $p(\bar{\omega}_x, \mu^D) = p(x, \mu^D)$ *— the probability of reaching node* x *if all players use mixed strategies* $\mu_i^D, i \in N$ *in factor-game* Γ^D.

Let $\hat{\mu} \in ME^A(\Gamma)$ and strategy profile $\hat{\mu}$ generates set $\Omega(\hat{\mu})$ of optimal plays ω on game tree K. Let $G(\hat{\mu})$ be the set of all possible subgames Γ_x along the "optimal game evolution", i.e. $x \in \omega, \omega \in \Omega(\hat{\mu})$.

Definition 4.2 *The set* $ME^A(\Gamma)$ *(the optimality principle* ME^A*) satisfies the time consistency property [9, 4, 5] if for every A-equilibrium* $\hat{\mu} \in ME^A(\Gamma)$ *and every subgame* $\Gamma_x \in G(\hat{\mu})$ *the following inclusion holds:* $\hat{\mu}^x \in ME^A(\Gamma_x)$.

Theorem 4.3 *The set* $ME^A(\Gamma)$ *of all A-equilibriums (in mixed strategies) in game* $\Gamma \in MG(n, K, r(1), ..., r(n))$ *with incomplete information satisfies the time consistency property.*

Proof. $\hat{\mu} \in ME^A(\Gamma)$ iff no player i has such mixed strategy μ_i that

$$A^i H_i(\mu_i, \hat{\mu}_{-i}) > A^i H_i(\hat{\mu}_i, \hat{\mu}_{-i}). \tag{16}$$

Suppose $\Gamma_x \in G(\hat{\mu})$, i.e. $x \in \omega_n, \omega_n \in \Omega(\hat{\mu}), x \neq x_0$. The set of all optimal trajectories $\{\omega_n\}$, generated by $\hat{\mu}$ can be divided onto two subsets: $\{\eta_m\} = \{\omega | x \in \omega\}$ and $\{\chi_k\} = \{\omega | \omega \text{ does not contain } x\}$, and $\{\eta_m\} \bigcap \{\chi_k\} = \emptyset$.

The player's i expected vector payoff could be written in the following form:

$$H_i(\hat{\mu}) = \sum_m p(\eta_m, \hat{\mu}) \cdot h_i(\eta_m) + \sum_k p(\chi_k, \hat{\mu}) \cdot h_i(\chi_k) \tag{17}$$

Let $\hat{\mu}^D = (\hat{\mu}_1^D, ..., \hat{\mu}_n^D)$ be the result of strategy profile μ decomposition, corresponding to factor-game $\Gamma_D = \Gamma_D(\hat{\mu}^x)$, and

$$p(\bar{\eta}_x, \hat{\mu}^D) = p(x, \hat{\mu}^D) = p(x, \hat{\mu})$$

be the probability of reaching node x (or the probability of play $\bar{\eta}_x = \{x_0, ..., x\}$) in factor-game Γ_D, when all players use strategies $\hat{\mu}_i^D, i \in N$.

Suppose that the time consistency condition is violated in subgame Γ_x, i.e. $\hat{\mu}^x \notin ME^A(\Gamma_x)$. Then for some player $i \in N$, there exists such strategy μ_i^x in Γ_x that

$$A^i H_i^x(\mu_i^x, \hat{\mu}_{-i}^x) > A^i H_i^x(\hat{\mu}_i^x, \hat{\mu}_{-i}^x). \tag{18}$$

Suppose strategy profile $(\mu_i^x, \hat{\mu}_{-i}^x)$ generates the set of plays $\{\xi_\alpha^x\}$ in the subgame, which are realized with positive probabilities $p(\xi_\alpha^x, (\mu_i^x, \hat{\mu}_{-i}^x))$. Then we can rewrite the vector inequality (18) in the following form:

$$\sum_\alpha p(\xi_\alpha^x, (\mu_i^x, \hat{\mu}_{-i}^x)) \cdot A^i \cdot h_i^x(\xi_\alpha^x) > \sum_m p(\eta_m^x, \hat{\mu}^x) \cdot A^i \cdot h_i^x(\eta_m^x). \tag{19}$$

Taking lemma 4.1 into account, pair μ_i^x and $\hat{\mu}_i^D$ of player's i mixed strategies in Γ_x and Γ_D can be obtained as the result of decomposition of some strategy $\beta_i = (\hat{\mu}_i^D, \mu_i^x)$ in Γ. Moreover:

$$A^i H_i(\beta_i, \hat{\mu}_{-i}) =$$

$$= \sum_\alpha p(\bar{\eta}_x, \hat{\mu}^D) \cdot p(\xi_\alpha^x, (\mu_i^x, \hat{\mu}_{-i}^x)) \cdot A^i \cdot h_i^x(\xi_\alpha^x) + \sum_k p(\chi_k, \hat{\mu}) \cdot A^i \cdot h_i(\chi_k). \tag{20}$$

Now let us multiply both parts of inequality (19) by positive value $p(\bar{\eta}_x, \hat{\mu}^D)$ and then add $\sum_k p(\chi_k, \hat{\mu}) \cdot A^i \cdot h_i(\chi_k)$ to both parts of obtained vector inequality.

Taking (19) and (17) into account, we will finally have the following vector inequality:

$$A^i H_i(\beta_i, \hat{\mu}_{-i}) > A^i H_i(\hat{\mu}_i, \hat{\mu}_{-i}).$$

Obviously, this vector inequality contradicts (16).

Hence, the set $ME^A(\Gamma)$ of all A-equilibriums in mixed strategies satisfies the time consistency property in n-person multicriteria extensive games with incomplete information.

Corollary 4.4 *The set $ME^A(\Gamma)$ of all A-equilibriums in pure strategies in n-person multicriteria extensive game $\Gamma \in MG^P(n, K, r(1), ..., r(n))$ with perfect information satisfies time consistency.*

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