Консорциум экономических исследований и образования
Economics Education and Research Consortium
Working Paper Series

ISSN 1561-2422

MONOPOLISTIC COMPETITION UNDER UNCERTAINTY

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Working paper No E 14/02

This project (No 12-5281) was supported
by the Economics Education and Research Consortium
and funded by GDN

Monopolistic competition under uncertainty

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Abstrct

Inspired by advances in general equilibrium modelling with monopolistic competition we re-consider the problem of the choice of firms under uncertainty, explore it in the framework of general equilibrium modelling, and develop a theory of monopolistic competition under demand uncertainty. We distinguish between two cases of uncertainty. In the first case the uncertainty disappears by the moment of trade and the output but not the prices are chosen under uncertainty. Then the uncertainty is established not to affect the equilibrium. The trade under uncertainty, considered in the second case, causes market imperfections. The supply is bigger (smaller) than the expected demand when the goods are good (bad) substitutes. In contrast to previous study, we show that uncertainty affects basically the prices and demand, but not the output.

Keywords: Monopolistic competition, Uncertain demand, Expected equilibrium, Good and bad substitutes, Market imperfection

JEL Codes: D81,D11,D41,L11

1 Introduction

It is now widely agreed upon that information imperfection significantly affects macroeconomic indicators [Friedman, 1968, Phelps, 1968]. Modern macroeconomics considers uncertainty a result of sequential shocks and treats it as an integral component of ongoing economic development. Modeling economy under uncertainty can be tackled, for example, using the theory of rational inattention [Sims, 2003] that links uncertainty to a limited rate of data processing or the theory of sticky information [Ball et al., 2005, Mankiw and Reis, 2007] that assumes delays in data availability.

Description of uncertainty with micro-level foundations starts with adjusting a firm's optimization problem. Assuming that risk-averse perfectly competitive firms maximize their expected profits given demand distribution, Sandmo [1971] establishes that production falls with increase of uncertainty measured by standard deviation of the demand distribution. In this model uncertainty leads to the emergence of risk that constitutes additional costs. Similar results (for a naturally defined, broad class of uncertainties) hold for monopolies [Leland, 1972]. However, a monopolist reduces its output less than a perfectly competitive firm [Appelbaum and Lim, 1982]. Dana [1999] assumes that firms are able to price the same product differently (say, at different time moments). Then under both perfect and imperfect competition the demand uncertainty leads to uncertainty in equilibrium prices. If the random variable underlying demand uncertainty has finite support, the fraction of goods sold at each price follows a particular optimal distribution law. Standard deviation of this distribution increases when the number of oligopolistic firms rises.

Ireland [1985] develops a theory of monopolistic competition under demand uncertainty. In his model the uncertainty is incorporated into prices and therefore appears in the demand implicitly. Ireland formulates partial equilibrium as a profit maximization problem. In this setting with costs of risks, uncertainty leads to additional costs and lowers potential profits which, at first sight, seems to be able to reduce not only the output but also the number of firms in the economy. According to [Ireland, 1985], market response to changes in uncertainty is ambiguous: it may well happen that the output and the number of firms both decline, but opposite movements in these two quantities may also occur. Moreover, [Ireland, 1985] shows that, under large elasticity of substitution between the goods, uncertainty growth can lead to social welfare increase.

Ireland's model produces a number of questions to be answered. For example, it is not clear

whether the predictions for the response of the output and the number of firms to changes of uncertainty are stable with respect to the model parameters. The unexpected result about possible positive influence of uncertainty on social welfare also needs further clarification. Ireland [1985] argues that less efficient firms exit the market under uncertainty growth and this positive effect dominates an increase in the cost of risk. This conclusion should be verified by using Melitz's approach [Melitz, 2003] to general equilibrium modeling with a heterogeneous production sector. Finally, the role of elasticity of substitution between different goods in the economy with demand uncertainty should be elaborated upon with greater scrutiny.

In this paper we aim at estimating the response of some important macroeconomic indicators to the emergence of (or changes in) uncertainty. Source of uncertainty is understood to originate in numerous sequential shocks. To attain our goal we combine Ireland's ideas with some new advances in the theory of monopolistic competition.

Since the publication of Ireland's model in 1985 the theory of monopolistic competition has expanded over different fields. P. Krugman won the Nobel prize for his international trade models explaining why export and import flows show unexpected similarities. Melitz [2003] introduced a model of trade with heterogeneous firms (recent advances are reported in [Melitz and Redding, 2012, Bernard et al., 2007, Bertoletti and Epifani, 2012, Mrazova and Neary, 2012]). Amiti and Pissarides [2005] show that more qualified workers find more effective firms with greater ease when economic space enlarges. The theory of monopolistic competition underlies decisions on taxes and subsidies [Andersson and Forslid, 2003, Borck and Pfluger, 2006], estimation of optimal city sizes [Glaeser and Kohlhase, 2003, Tabuchi et al., 2012] and possibilities of regional integration [Ludema and Wooton, 2000]. Inspired by the impressive advances outlined above we revisit a firm's optimization problem under uncertainty exploiting recently developed tools from the theory of monopolistic competition.

This paper assumes that the consumer preferences are described by a two-tier utility function. On the upper tier the consumers endowed by a power Cobb–Douglas utility choose between composite varieties of goods. The exponents of the Cobb-Douglas function are supposed to be random. Based on the lower-tier utility with constant elasticity substitution (CES) the consumers choose between concrete products of each variety and generate a demand with a multiplicative uncertainty. Firms are supposed to take their decision about output, prices, and wages as the best response to all possible values of uncertain demand.

We show that the response of economy to appearance of uncertainty is more ambiguous than previous studies reported. First, fundamental changes are exhibited by prices not by

output. Second, uncertainty generates a bias in demand. Third, the firms decide to produce a surplus of the goods with large elasticity of substitution and insufficient amount of the goods with small elasticity of substitution. This mismatch between the supply and demand is a direct consequence of trade under uncertainty.

2 Demand

An economy is assumed to consist of n manufacture sectors and one agricultural sector. Each manufacture sector produces a variety of N_i goods, $i = 1, \ldots, n$ (or, more accurately, a continuous set x_i of goods that has the mass N_i). A representative consumer forms a demand A for the agricultural good and a demand $Q_i(x_i)$, $i = 1, \ldots, n$, for the manufacture goods $x_i \in [0, N_i]$ maximizing her utility function

$$U = M_1^{\hat{\alpha}_1} M_2^{\hat{\alpha}_2} \ldots M_n^{\hat{\alpha}_n} A^{\hat{\alpha}_0} \longrightarrow \max \tag{1}$$

where

$$M_i = \left(\int_{N_i} (Q_i(x_i))^{\gamma_i} dx_i \right)^{1/\gamma_i} \tag{2}$$

$\hat{\alpha}_i \in (0,1)$, $\hat{\alpha}_0 = 1 - \sum_{i=1}^{n} \hat{\alpha}_i$, $\gamma_i \in (0,1)$. Under preferences (1) the manufacture goods in each sector have a constant elasticity substitution (CES) equalled to $1/(1-\gamma_i)$.

We assume that the exponents $\hat{\alpha}_i$, $i = 1, \ldots, n$, are affected by a random factor. More precisely, Let $\alpha_i \in (0,1)$, $i = 0, \ldots, n$, be some numbers such that their sum is equal to 1. Put, $\hat{\alpha}_i = \alpha_i \zeta$, $i = 1, \ldots, n$, where the random variable

$$\zeta \in \big(0, 1/(1-\alpha_0)\big) \tag{3}$$

has a given cumulative distribution function $\mathcal{F}(z)$ and mean 1. A boundedness of the random variable ζ assures inequalities $\hat{\alpha}_i \in (0,1)$ for all $i = 1, \ldots, n$. Thus, a general random variable describes the exponents $\hat{\alpha}_i$.

The consumer problem is formulated as it is faced by firms. The firms have incomplete information about exponents $\hat{\alpha}_i$, $i = 0, \ldots, n$. They know the cumulative distribution function $\mathcal{F}(z)$ but don't observe the concrete value of the random variable ζ. Therefore they solve the consumer problem with an uncertain demand. Technically, the exponents $\hat{\alpha}_i$ are treated by the firms as parameters. In contrast to the firms, the consumers know the value of ζ and solve a standard optimization problem with a fixed $\hat{\alpha}_i$.

Given prices p_A and $p_i(x_i)$ for the agricultural good A and for the manufacture goods x_i, $i = 1, \ldots, n$, $x_i \in [0, N_i]$, the budget constrain of the representative consumer with income Y is

$$\sum_{i=1}^{n} \int_0^{N_i} p_i(x_i) Q_i(x_i) dx_i + p_A A \leq Y. \tag{4}$$

For any concrete value of the random variable ζ the optimization problem (1), (4) is standard. Its solution is

$$\hat{Q}_i(x_i) = (p_i(x_i))^{\frac{1}{\gamma_i - 1}} \hat{\alpha}_i Y P_i^{\frac{\gamma_i}{1-\gamma_i}}, \tag{5}$$

where

$$P_i = \left(\int_0^{N_i} \left(p_i(x_i)\right)^{\frac{\gamma_i}{\gamma_i - 1}} dx_i \right)^{\frac{\gamma_i - 1}{\gamma_i}}. \tag{6}$$

is interpreted as the price index of the i-th variety.

3 Supply

3.1 Agricultural sector

In agricultural sector technology is characterized by a constant return to scale and perfect competition. Therefore the firms price their goods at the marginal cost: $p_A = m_A w_A$, where w_A and m_A are the wages and inverse productivity in the agricultural sector. The agricultural good is chosen as a numeraire so that $p_A = 1$. Without loss of generality the productivity in the agricultural sector is assign to one too. Therefore

$$w_A = p_A = 1 \tag{7}$$

in the agricultural sector.

3.2 Firm optimization problem in the manufacture sector

We assume that each firm produces exactly one good. Based on demand (5) the firms choose the amount $s_i(x_i)$ of the goods to produce and set their price $p_i(x_i)$. The firms are assumed to produce their goods and trade them under uncertain demand. It means that the firms, in contrast to the consumers, know only the cumulative distribution function of ζ but not its value z. In such a case the firms are constrained to choose their $s_i(x_i)$ and $p_i(x_i)$, which are independent on z, as a response to all possible values of ζ. The entry and exit decisions are supposed to be based on the expected profit. These choices lead to an expected equilibrium in the manufacture sectors.

A real demand, observed finally, in general differs from the supply. Potential gains and loses of the firms are implemented into balances. The description of this idea is postponed to section 5.

In this section we introduce the firm optimization problem. Given x_i, the firm observes the aggregate demand

$$q_i(x_i,\zeta) = (p_i(x_i))^{\frac{1}{\gamma_i-1}}\alpha_i\zeta y P_i^{\frac{\gamma_i}{1-\gamma_i}} \tag{8}$$

obtained from formula (5) by substitution of the aggregate income y for the individual income Y. By $\bar{q}_i(x_i)$ denote the mean value of $q_i(x_i,\zeta)$: Then $q_i(x_i,\zeta) = \bar{q}_i(x_i)\zeta$ and, by (8),

$$\bar{q}_i(x_i) = (p_i(x_i))^{\frac{1}{\gamma_i-1}}\alpha_i y P_i^{\frac{\gamma_i}{1-\gamma_i}}. \tag{9}$$

Let $s_i(x_i)$ be (yet unknown) quantity of the good x_i produced by the firm corresponding to this good. Then with the probability $\mathcal{P}\{s_i(x_i) < \zeta q_i(x_i,\zeta)\} = \big(1 - \mathcal{F}(s_i(x_i)/\bar{q}_i(x_i))\big)$ the average demand exceeds the supply. This supply is sold for a price $p_i(x_i)$ with a variable cost $m_i(x_i)w_i(x_i)$ for each item, where $m_i(x_i) = m_i$ is the i-th sector inverse productivity, which is independent on a concrete good of the i-th variety, and $w_i(x_i)$ is interpreted as a wage. A fixed cost $\varphi w_i(x_i)$ faced by the firm is measured in the wages.

If $\zeta = z < s_i(x_i)/\bar{q}_i(x_i)$, then the output exceeds the demand. We assume that each firm considers a storage of the unsold goods being impossible. Then the firm sells only $\bar{q}z$ goods, gets the revenue $p\bar{q}z$, but bears the variable cost mws over whole production. This event has the probability $dF(z)$.

Averaging over all possible values z of the random variable ζ gives the expression for the average profit:

$$\langle\pi(x_i)\rangle = (p_i(x_i) - m_i w_i(x_i))s_i(x_i)\left(1 - \mathcal{F}\left(\frac{s_i(x_i)}{\bar{q}_i(x_i)}\right)\right) + \\ + \int_0^{s_i(x_i)/\bar{q}_i(x_i)} (p_i(x_i)\bar{q}_i(x_i)z - m_i w_i(x_i)s_i(x_i))f(z)dz - \varphi w(x_i), \tag{10}$$

The change of the variables $t = s/\bar{q}$ simplifies the expression for the profit:

$$\langle\pi(x_i)\rangle = (p_i(x_i) - m_i w_i(x_i))t_i(x_i)\bar{q}_i(x_i) - p_i(x_i)\bar{q}_i(x_i)\int_0^{t_i(x_i)} \mathcal{F}(z)dz - \varphi w_i(x_i). \tag{11}$$

Thus, the firm maximizes the average profit

$$\langle\pi(x_i)\rangle \longrightarrow \max, \tag{12}$$

given by formula (11) choosing the optimal t and p.

Ignoring a possibility of storage the firms underestimate the profit. This underestimation could be interpreted as a risk cost that agrees with the sunk entry cost proposed by Melitz [2003] for his model with heterogeneous firms.

The free entry and exit condition closes the firm optimization problem. We assume that firms are ready to operate on markets until their *expected* profit disappears:

$$(p_i - m_i w_i)s_i - p_i s_i I_{\mathcal{F}(t_i)}/t_i - \varphi w_i = 0. \tag{13}$$

3.3 Expected equilibrium in the manufacture sectors

The first order conditions for optimization problem (12) are

$$\langle \pi(x_i) \rangle'_p = 0, \quad \langle \pi(x_i) \rangle'_t = 0. \tag{14}$$

The arguments x_i and the indices i are dropped in this section. Computing the partial derivative of the profit in p we use the standard assumption that each firm is so small with respect to the market that it fails to affect the price index. Technically, it means that computing the elasticity $E_{\bar{q},p}$ of the demand the firm sets $P'_p = 0$ and gets

$$\mathcal{E}_{\bar{q}}(p) = -1/(1-\gamma) \quad \forall p \tag{15}$$

Let $I_{\mathcal{F}}(t) = \int_0^t \mathcal{F}(z)dz$. Then the first condition in (14) is written as

$$\frac{\partial \langle \pi \rangle}{\partial p} = t - I_{\mathcal{F}}(t) + \left(\frac{(p - mw)t}{p} - I_{\mathcal{F}}(t) \right) \mathcal{E}_{\bar{q}} = 0, \tag{16}$$

By assumption (15), formula (16) is simplified to

$$t - I_{\mathcal{F}}(t) - \left(\frac{(p - mw)t}{p} - I_{\mathcal{F}}(t) \right) \frac{1}{1-\gamma} = 0. \tag{17}$$

The second condition in (14) links the markup to cumulative distribution of uncertainty:

$$\mathcal{F}(t) = \frac{p - mw}{p}. \tag{18}$$

After substitution of (18) into (17) we get a key equation for t:

$$\frac{\mathcal{F}_c(t)}{\frac{1}{t}\int_0^t \mathcal{F}_c(z)dz} = \gamma, \tag{19}$$

where $\mathcal{F}_c(t)$ is the complement cumulative distribution function of the random variable ζ. With the elasticity $\mathcal{E}_{\tilde{\mathcal{F}}}(t)$ of the function

$$\tilde{\mathcal{F}}(t) = \int_0^t \mathcal{F}_c(z)dz$$

equation (19) is written in a more compact way:

$$\mathcal{E}_{\tilde{\mathcal{F}}}(t) = \gamma, \tag{20}$$

By first order conditions (14), key equation (19), and zero-profit condition (13), a straightforward algebra leads to the following expression for the output:

$$s_i = \frac{\varphi\gamma_i}{m_i(1-\gamma_i)}. \tag{21}$$

The solution of the firm optimization problem defines an expected partial equilibrium in the economy because this solution reflects the expectation of the firms that maximize their expected profit. The existence and uniqueness of this expected partial equilibrium are determined by properties of equation (20). Indeed, let equation (20) have a unique solution t. Then the optimal price is derived from equation (18). Given this price, formula (21) defines the average the optimal output s.

Assume that the firms face identical costs inside each sector. Then all the terms of equation (20) depend on i but not on x_i. Whence, the optimal output, prices, and demand are symmetrical in an arbitrary sector i ($s_i(x_i) = s_i$, $p_i(x_i) = p_i$, $\bar{q}_i(x_i) = \bar{q}_i$).

Theorem 1 (Sufficient condition of the equilibrium)**.** *Let the elasticity of the function $\tilde{\mathcal{F}}$ monotonically decreases from* 1 *to* 0*. Then the expected equilibrium in each manufacture sector exists and it is unique.*

The proof of theorem 1 is evident.

Different distributions satisfy the conditions of theorem 1, in particular,

R1: uniform distributions;

R2: (truncated) exponential distributions;

R3: (truncated) log-normal distributions.

Items **R1** and **R2** are checked straightforward, item **R3** is exhibited by numeric computation (see appendix A), whereas the proof of existence and uniqueness of the equilibrium for the log-normal distributions is given in appendix B in a way that does not involve theorem 1.

The cutting of the exponential and log-normal random variables is introduced to see the random variable ζ lying on $(0, 1/(1-\alpha_0))$ and satisfying formula (3). A decrease of the elasticity of the function $\tilde{\mathcal{F}}$ does require this cutting. Multi-mode random variables contradict the assumptions of theorem 1.

The output s_i found in (21) coincides with the output under complete certainty. Whence, even under demand uncertainty the firms estimate output correctly. This observation is, probably, still valid under linear costs for a broad class of general utility functions described in [Zhelobodko et al., 2012, Bertoletti and Entro, 2013].

Appearance of uncertainty distorts the average demand $\bar{q}_i$ anticipated by the firms.

Theorem 2. *Let the elasticity $\mathcal{E}_{\tilde{\mathcal{F}}}(t)$ monotonically decrease in t. Let $\gamma^* = \mathcal{E}_{\tilde{\mathcal{F}}}(1)$. Then if $\gamma_i > \gamma^*$, the expected average aggregate demand $\bar{q}_i$ in the i-th sector exceeds the supply. On the contrary, if $\gamma_i < \gamma^*$, the supply exceeds the expected average aggregate demand. Finally, if $\gamma_i = \gamma^*$, then the expected average aggregate demand and the supply coincide.*

For log-normal uncertainty $\gamma^* = 1/2$ (see appendix B). So, producing the goods with a low elasticity of substitution (which is equal to $1/(1-\gamma)$) the firms rationally underestimate the demand. On the contrary, the firm optimally overestimate the demand of the goods with a high elasticity of substitution.

4 Expected general equilibrium

4.1 Labour market and balances

We assume that the agricultural sector requires unskilled labour where as the manufacture sectors do skilled labour. The skilled workers are able to move from one firm to another inside a sector seeking for higher salaries. This opportunity equalizes the wages in each sector: $w(x_i) = w_i$. Then the costs faced by the firms are identical inside each sector too. The expected equilibrium in the manufacture sectors were considered in sector 3.2 under this merely assumption. The numbers L_A and L_i, $i = 1, \ldots, n$, of the workers in the agricultural and manufacture sectors are supposed to be given.

In firm's opinion, the aggregate income y of the consumers consists of the wages of the skilled and unskilled workers:

$$y = \sum_{i=1}^{n} w_i L_i + L_A. \tag{22}$$

Each firm of the i-th sector requires $m_i s(x_i) + \varphi$ labour units. Therefore the labour balance in the i-th sector is

$$N_i(m_i s_i + \varphi) = L_i, \quad i = 1, \ldots, n. \tag{23}$$

The characteristics of the general equilibrium expected by the firms, by definition, solve optimization problems of the consumers (1), (4) and firms (12) and satisfy balances (7), (13), (22), (23).

4.2 Characteristics of the expected equilibrium

Combining labour balance (23) and expression (21) for the supply we find the number N_i of the firms in the i-th sector:

$$N_i = \frac{L_i(1-\gamma_i)}{\varphi}. \tag{24}$$

Substituting into solution (9) of the consumer optimization problem expressions (22) for the aggregate income y, (21) for supply, (24) for the number of the firms, and (18) for the prices we get n equations

$$w_i L_i = \frac{\alpha_i t_i (1-\mathcal{F}(t_i))}{\gamma_i} \left(\sum_{j=1}^{n} w_j L_j + L_A \right), \quad i = 1, \ldots, n \tag{25}$$

with respect to the wages w_i. Denoting

$$G(t_i) = \frac{t_i(1-\mathcal{F}(t_i))}{\gamma_i}, \tag{26}$$

where t_i is the solution of key equation (19), and summing up equations (25) for all $i = 1, \ldots, n$ we find first the sum $\sum w_j L_j$ and then the wages:

$$w_i = \frac{L_A}{L_i} \frac{\alpha_i G(t_i)}{1 - \sum_{j=1}^{n} \alpha_j G(t_j)}. \tag{27}$$

Then the prices are derived from equation (18):

$$p_i = \frac{m_i L_A}{L_i} \frac{\alpha_i t_i}{\gamma_i (1 - \sum_{j=1}^{n} \alpha_j G(t_j))}. \tag{28}$$

Formulae (21), (24), (27), and (28), following from the first order condition of the corresponding maximization problems, determine equilibrium variables. Since the goal functions are concave, these formulae indeed lead to the optimal solution.

4.3 Comparative statics

The dependence of the equilibrium on the variance σ of the random variable ζ is linked to the solution t of equation (19), which is not treated analytically. We consider a log-normal uncertainty and reveal that small and large elasticities of substitution γ in the utility function

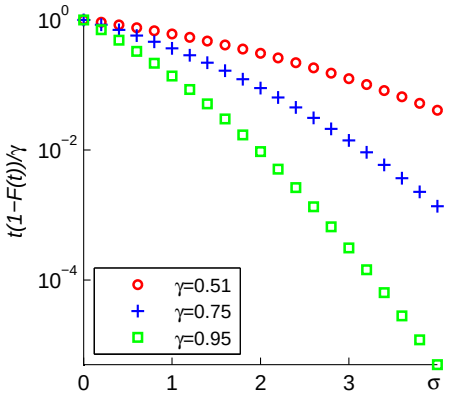

Figure 1: Function $t(1-\mathcal{F}(t))/\gamma$, where t is the solution of equation (19), as a function of the standard deviation σ for different γ_i.

generate different behaviour of t as a function of σ. Log-normal random variables give a natural example of a multiplicative noise. Let the density of ζ be

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma z} \exp\left(-\frac{\left(\log(z) + \sigma^2/2\right)^2}{2\sigma^2}\right). \tag{29}$$

Under this choice of the density the mean of the random variable ζ is 1 and the standard deviation, which parametrizes the uncertainty, equals to σ.

For the sake of simplicity merely non-truncated log-normal distribution is considered. The results of the analysis are still valid for the log-normal distribution narrowed onto an interval (z_1, z_2) because density (29) on this interval is normed by the integral $\int_{z_1}^{z_2} f(z)dz$ so that the solution of equation (19) is not changed.

A numerical analysis of equation (19) shows that the function $G(t) = t(1-\mathcal{F}(t))/\gamma$ decreases in σ, Figure 1. Therefore the nominal wages decrease with a growth of the uncertainty whereas the changes of the prices are ambiguous. The behaviour of t as a function of σ underlies the demand-uncertainty dependence. According to our numerical analysis, t decreases in σ if $\gamma < 0.5$ and increases if $\gamma > 0.5$. Index i, which indicates the sector number, is dropped here. Theoretical conclusions concerning the influence of the log-normal uncertainty on the demand, supply, prices, and wages are summarized in table 1.

5 Realization of uncertainty

Balance of money. Despite firms take their decision under uncertainty, the random variable ζ attains some value z. As a result, the firm's profit can be positive as well as negative. Skilled

Table 1: Sign of the derivative of the equilibrium variables with respect to σ for log-normal uncertainty.

	t'_σ	$\bar{q}'_\sigma$	s'_σ	N'_σ	p'_σ	w'_σ
$\gamma < 0.5$	< 0	> 0				
$\gamma = 0.5$	$= 0$	$= 0$	$= 0$	$= 0$	?	< 0
$\gamma > 0.5$	> 0	< 0				

workers are assumed to be the owners of the firms. We assume that non-zero profit changes the consumer aggregate income to

$$y = \sum_{j=1}^{n} w_j L_j + L_A + \sum_{j=1}^{n} N_j \pi_j. \tag{30}$$

The profit coincidence inside sectors is taken into account in (30).

Pay attention, that the firms are supposed to be myopic and solve their optimization problem with the aggregate income given by (22) instead of (30). This simplification allows one to solve the corresponding mathematical problem analytically.

Given z, *the observed consumer demand and the firm's profit are remained to find.* These quantities solve the system of n equations

$$q_i = (p_i)^{\frac{1}{\gamma_i - 1}} \alpha_i z y P_i^{\frac{\gamma_i}{1-\gamma_i}}, \tag{31}$$

obtained from (8) by substitution of a concrete value z for the random variable ζ. The aggregate income y is given by (30). Pay attention that the right hand side of formula (31), which determines q_i, contains q_i too. This q_i, along with the other q_j, $j \neq i$, is "hidden" in the income (30). Therefore first we have to show that money balance (30) does not contradict the other model equations.

Exclude the profit of the i-th manufacture sector in the aggregate profit $\sum N_j \pi_j$ and denote the other part of the aggregate profit by Π_{-i}: $\Pi_{-i} = \sum_{j \neq i} N_j \pi_j$.

Lemma 1. *The system of equations* (30), (31) *is equivalent to*

$$q_i(z) = \frac{\alpha_i z}{N_i p_i} \left(\sum_{j=1}^{n} w_j L_j + L_A - \frac{\varphi w_i N_i}{1 - \gamma_i} + \Pi_{-i} \right) + \alpha_i z s_i, \quad q_i > s_i, \tag{32}$$

$$q_i(z) = \frac{\alpha_i z}{N_i p_i} \left(\sum_{j=1}^{n} w_j L_j + L_A - \frac{\varphi w_i N_i}{1 - \gamma_i} + \Pi_{-i} \right) \frac{1}{1 - \alpha_i z}, \quad q_i < s_i, \tag{33}$$

where Π_{-i} *depends on* $q_j(z)$, $j \neq i$.

The proof of the lemma is given in appendix C. The inequality $q_i(z) > s_i$ with $q_i(z)$ given by (32) and the opposite inequality $q_i(z) < s_i$ with $q_i(z)$ given by (33) are equivalent to two opposite inequalities with respect to z:

$$\frac{\alpha_i z}{N_i p_i (1-\alpha_i z)} \left(\sum_{j=1}^{n} w_j L_j + L_A + p_i q_i N_i - \frac{\varphi}{1-\gamma_i} w_i N_i + \Pi_{-i} \right) \gtrless s_i. \tag{34}$$

It means that the balance of money (30) leads to a mathematically correct problem.

Formal expressions for demand. Different expressions for the profits $\pi_i = p_i s_i - m_i w_i s_i - \varphi w_i$ as $q_i > s_i$ and $\pi_i = p_i q_i - m_i w_i s_i - \varphi w_i$ as $q_i < s_i$ create technical inconvenience to write out the solution of equations (31), (30). Let $z_{(1)}$, ..., $z_{(n)}$ be the solutions of the system of equations $s_i = q_i(z)$ that transform inequalities (34) into equalities. The indices of z are not to be confused with the sector numbers. Then one has to split z-axis into sub-intervals by points $z_{(1)}$, ..., $z_{(n)}$ to resolve for each i the uncertainty $s_i/q_i(z) \gtrless 1$. Then the choice between equations (32) and (33) is well defined on each z sub-interval. We describe in detail two the most important cases, which both correspond to a big deviation of the random variable ζ from its mean: $z > \max\{z_{(j)}\}$ implying that $s_i < q_i$ in each sector i and $z < \min\{z_{(j)}\}$ leading to $s_i > q_i$ for all i.

Lemma 2. *Let* $z > \max\{z_{(j)}\}$. *Then*

$$q_i(z) = \frac{\varphi \gamma_i z}{m_i(1-\gamma_i)t_i} \left(1 - \sum_{j=1}^{n} \alpha_j \big(G(t_j) - t_j \big) \right), \tag{35}$$

where the function $G(t)$ *is defined in* (26). *Let* $z < \min\{z_{(j)}\}$. *Then*

$$q_i(z) = \frac{\varphi \gamma_i z}{m_i(1-\gamma_i)t_i} \frac{1 - \sum_{j=1}^{n} \alpha_j G(t_j)}{1 - z \sum_{j=1}^{n} \alpha_j}. \tag{36}$$

The proof of the lemma is given in appendix C.

Imperfection caused by uncertainty. The excess of the demand over the supply, $q_i(z) > s_i$, is a source of unused money in the economy. Indeed, by (31) with $P_i^{-\gamma_i/(1-\gamma_i)} = N_i p_i^{-\gamma_i/(1-\gamma_i)}$, the inequality $q_i(z) > s_i$ is equivalent to $\alpha_i z Y/(N_i p_i) > s_i$ or to $\alpha_i z Y > N_i p_i s_i$. The left hand side of the last inequality is the budget for the goods of the i-th manufacture sector, whereas the right hand side represents actual spending for these goods. Therefore when the demand is larger than it has been expected by firms, the consumers reserve money for their needs but fail to buy the desired quantity of the goods. If $s_i > q_i(z)$, the budget $\alpha_i z Y$ is spent for $N_i q_i$ goods priced at p_i completely. In this case imperfection is reflected by the excess of the supply.

We do not expand our simple model to discuss paths to market clearance. Our model reveals the features of the firm decision making under uncertainty. Nevertheless further steps of modelling are clear enough. First, the money reserved for deficit goods could be spent for the goods that are unsold. This step shrinks the imperfections but does not eliminate it completely. Second, multi-period models can be introduced. Then either unused money is saved to be spent later or remained goods will be sold in the next periods.

6 Policy

Our theoretical model predicts that the gap between the demand and supply increases as a response to uncertainty reinforcement. The supply, as a rule, exceeds the expected demand in the manufacture sectors, in which the goods are characterized by a high elasticity of substitution. On the contrary, producing goods with a low elasticity of substitution the firms optimally underestimate the demand. The closer the elasticity of substitution to the boundary values 0 and 1 is the more the gap between the demand and the supply becomes.

Realizing these properties and aiming at market clearance regulators shouldn't either change the tax policy or encourage research and development (R&D). Indeed, in the framework of the expected equilibrium, changes of the tax policy affect the variable cost ws, which are absent in equation (19) determining the ratio t of the supply to the demand. Equation (19) does not also contain the fixed cost φw, which are naturally interpreted as investment into R&D [Judd, 1985]. Whence, changes of investment into R&D does not affect the relationship between the demand and the supply. Thus, the imbalance between the demand and the supply caused by market de-stabilization and following quasi-rational behaviour of the consumers are improved by measures intended for an integral normalization of the market. In other words, market errors caused by its diversity and variability cannot be corrected by interference into firm activity.

Firms are able to operate in the model because their potential losses are secured by the consumers, formula (30). In the real world market regulators have to bring consumer saving into firm activity. They can strengthen financial system and stimulate small and medium-size business. Otherwise, when a real balance of money is closer to (22) than to (30), an unexpected fall of demand makes firms end with negative profits but unsold goods whereas the demand of the consumers is not satisfied fully. This conclusion can be easily checked by repeating the arguments of section 5 with balance of money (22). Thus, without intervention of regulators the firms are induced to return to natural economy proposing their goods as salary under a fall

of demand.

Finally, firms choose their strategy under uncertainty when they have a limited information (or, according to the theory of rational inattention [Sims, 2003] they cannot process the complete information). In other words, the firms face the demand superimposed on noise (that is a random variable). Then firms are able to decrease the uncertainty analysing the demand at a cost. Under a rational strategy they get more and more precise information about the demand until the marginal profit provided by additional information coincides with the marginal cost. As a result an output strategy is undertaken under incomplete information as before and therefore the imbalance between the demand and supply found in section 3.3 must be still existing.

7 Conclusion

The paper returns to the problem of influence of uncertain demand onto the output and pricing strategy of firms. In order to exhibit this influence we modify the Dixit–Stiglitz approach to modelling of general equilibrium with monopolistic competition. Following this approach we define a closed economy that consists of the agricultural sector with perfect competition and constant return to scales and n manufacture sectors with monopolistic competition and increasing return to scales. The upper-tier consumer preferences reflecting the choice of manufacture varieties and agricultural goods follow the Cobb–Douglas utility function. The lower-tier inter-sector preferences are given by a power function. Its exponent γ reflects the elasticity of substitution $1/(1-\gamma)$ between the representatives of each variety.

The firms are myopic in the model setting. They know the demand as a function of prices up to a random multiplier with a given distribution. The firms constrained by labour and money balances as well as by free entry and exit condition maximize their expected profit and fix the optimal output, prices, and wages. These variables together with the induced number of the firms and expected demand constitute the *expected* equilibrium, which exhibits a response of economy to uncertainty. Since the observed demand almost surely deviates from the supply under general conditions, the observed profit of the firms is different from zero. These potential gains and loses are implemented into the money balance.

We explore the simplest uncertainty in the firm optimization problem, that is a multiplicative noise with a finite support. This uncertainty is realized by consumer preferences given by the Cobb–Douglas utility at the upper tier, formula (1). The firm optimization problem with

other uncertainty could be "embedded" into general equilibrium modelling too, but with more complicated consumer preferences.

The demand uncertainty implemented in our model leads to uncertainty of the prices index. Thereby we specify and make more real a key assumption of the Dixit–Stiglitz theory. Firms not only fail to affect the price index (as in the Dixit–Stiglitz theory) but also have incomplete information about it.

Originality of our approach is based on merely embedding of the firm optimization problem into a general equilibrium model with monopolistic competition. As a result we are able to specify conclusions of papers [Sandmo, 1971, Appelbaum and Lim, 1982, Ireland, 1985] and introduce a theory of monopolistic competition under uncertain demand. We find a condition, which uncertainty satisfies to, leading to existence and uniqueness of the expected equilibrium. A typical random variables (uniform, exponential, log-normal) satisfy this condition. A rational strategy of the firms is qualitatively described in terms of the elasticity substitution between the goods. There is only one value γ^* of the exponent of the lower-tier utility function such that the supply is equal to the expected aggregate demand. If $\gamma > \gamma^*$, i.e. the goods in a sector have relatively small elasticity of substitution, then the firms rationally under-estimate the demand. In this case a deficit of these goods are expected. On the contrary, the output of the goods with a high elasticity of substitution ($\gamma < \gamma^*$) is larger than the expected aggregate demand. Yet Ireland [1985] indicated at a specific role of the elasticity of substitution between the goods under uncertainty. However only this paper gives a complete description of the inter-relation between the elasticity of substitution and the response of economy to uncertainty changes.

Certain mismatch between demand and supply sometimes surprises public opinion. A shortage of specific new iphones observed during the last years in Europe, lines for some cars in Russia during the last crisis in 2009, and numerous other "unexpected" examples are understandable due to our theory that predicts an underestimation of the demand for bad substitutes by rational firms.

We establish that under linear cost, appearance of uncertainty does not affect the output (and the number of the firms) in a sector but alters the demand (if $\gamma \neq \gamma^*$) and prices. So, a price change as a response to uncertainty is a general property. The details of this change depend on a concrete type of the uncertainty. One has to relax the assumption about linear costs to study a response of the output (and the number of the firms) to uncertainty changes.

Based on uncertain demand, the firms' decisions generate imperfection of the economy. If the observed demand is small then a part of the output remains unsold. On the contrary, if the

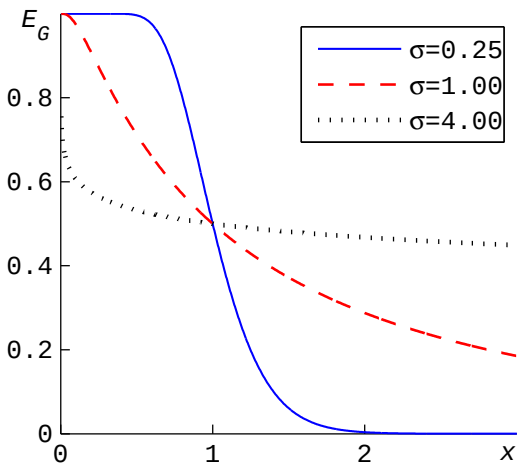

Figure 2: The right hand side of equation (19) for σ given in the legend.

observed demand is large then some money is not spent fully. The expected equilibrium does not clear the market and therefore does not meet standard equilibrium conditions. In order to tackle this problem one could introduce a multi-period model, in which surpluses of each time period go the following period. Then our expected equilibrium appears to be a point lying on an equilibrium trajectory. The equilibrium trajectories are defined and explored by Grandmont [1977].

The uncertainty studied in this paper reflects variability and diversity of markets. Firms are badly informative because markets change and old information is no more precise. Our approach reveals imperfections that can collapse the economy under absence of regulators.

A Integral of complement distribution function for log-normal random variable: decrease of elasticity

Figure (2) introduces the elasticity $\mathcal{E}_I(t)$ of the integral $I(t) = \int_0^t \mathcal{F}_c(z)dz$ of the complementary distribution function of the log-normal random variable. Straightforward computation shows that the limit values of $\mathcal{E}_I(t)$ as t goes to 0 and $+\infty$ are 1 and 0 respectively. Therefore a solution of equation (19) exists.

The elasticity decreases from 1 to 0 such that the more the standard deviation σ is the slower (in x) the curve goes its way from one to zero. Thus, Figure (2) gives evidence that equation (19) has a unique solution.

B Equation (19): uniqueness of solution for log-normal distribution

Lemma 3. *The equation* (19) *is equivalent to the following equation*

$$\ln\left(K\frac{\Phi(u-\sigma_1)}{\Phi(-u-\sigma_1)}\right)-2u\sigma_1=0, \tag{37}$$

where $\Phi(\cdot)$ *is a normal cumulative distribution function and*

$$K=\frac{\gamma}{1-\gamma},\quad \sigma_1=\sigma/2,\quad u=\frac{\log t}{\sigma}. \tag{38}$$

Proof. With the definition of $f(z)$ the left-hand side of (19) is reduced to

$$\int_0^t zf(z)dz=\frac{1}{\sqrt{2\pi\sigma}}\int_0^t\exp\left(-\frac{\left(\log z+\sigma^2/2\right)^2}{2\sigma^2}\right)dz$$

After the change of the variables

$$v=\frac{\log z+\sigma^2/2}{\sigma},\quad z=e^{v\sigma-\sigma^2/2},\, dz=\sigma e^{v\sigma-\sigma^2/2}\,dv \tag{39}$$

we get

$$\int\limits_0^t zf(z)dz=\frac{1}{\sqrt{2\pi\sigma}}\int\limits_{-\infty}^{\frac{\log t+\sigma^2/2}{\sigma}}e^{-\frac{v^2}{2}}\sigma e^{v\sigma-\frac{\sigma^2}{2}}dv=$$

$$=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\frac{\log t+\sigma^2/2}{\sigma}}e^{-\frac{(v-\sigma)^2}{2}}dv.$$

Finally, if $w=v-\sigma$ then

$$dw=dv,\quad \frac{\log t+\sigma^2/2}{\sigma}-\sigma=\frac{\log t-\sigma^2/2}{\sigma}$$

and

$$\int\limits_0^t zf(z)dz=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\frac{\log t-\sigma^2/2}{\sigma}}e^{-\frac{w^2}{2}}dw=\Phi\left(\frac{\log t}{\sigma}-\frac{\sigma}{2}\right) \tag{40}$$

Analogously, the right-hand side of (19) can be rewritten in the following way

$$t(1-\mathcal{F}(t))=t\int\limits_t^{+\infty}f(z)dz=\frac{t}{\sqrt{2\pi}\sigma z}\int\limits_t^{+\infty}\frac{1}{2}e^{-\left(\frac{\log(z)+\sigma^2/2}{\sigma}\right)^2}dz.$$

After the change of variables (39) we have

$$t\int\limits_{t}^{+\infty} f(z)dz = \frac{t}{\sqrt{2\pi}\sigma}\int\limits_{\frac{\log t+\sigma^2/2}{\sigma}}^{+\infty} e^{-v\sigma+\sigma^2/2}e^{-v^2/2}\sigma e^{v\sigma-\sigma^2/2}\,dv =$$

$$\frac{t}{\sqrt{2\pi}}\int\limits_{\frac{\log t+\sigma^2/2}{\sigma}}^{+\infty} e^{-v^2/2}\,du \quad (41)$$

Thus

$$t(1-\mathcal{F}(t)) = t\left(1-\Phi\left(\frac{\log t}{\sigma}+\frac{\sigma}{2}\right)\right) = t\Phi\left(-\frac{\log t}{\sigma}-\frac{\sigma}{2}\right) \quad (42)$$

With (42) and (40) equation (19) can be reduced to the form

$$K\Phi\left(\frac{\log t}{\sigma}-\frac{\sigma}{2}\right) = t\Phi\left(-\frac{\log t}{\sigma}-\frac{\sigma}{2}\right)$$

or, with the help of (38),

$$K\Phi(u-\sigma_1) = e^{2\sigma_1 t}\Phi(-u-\sigma_1).$$

Lemma is proved. □

Put,

$$g(u) = \ln\left(K\frac{\Phi(-\sigma_1+u)}{\Phi(-\sigma_1-u)}\right)$$

Lemma 4. *Inequality*

$$g'(u) > 2\sigma_1 \quad (43)$$

is valid uniformly in u.

Proof. As $g(u)$ can be written in the form

$$g(u) = \ln K + \ln\Phi(-\sigma+u) - \ln\Phi(-\sigma-u) - \sigma u$$

it is easy to see that

$$g'(u) = \frac{\phi(-\sigma_1+u)}{\Phi(-\sigma_1+u)} + \frac{\phi(-\sigma_1-u)}{\Phi(-\sigma_1-u)}, \quad (44)$$

where $\phi(\xi) = \Phi'(\xi)$.

Prove that

$$\frac{\phi(\xi)}{\Phi(\xi)} \geq -\xi, \quad \forall\xi. \quad (45)$$

If $\xi > 0$ we have nothing to show because the left hand side of inequality (45) is positive, where as the right hand side is negative. Assume that $\xi \leq 0$. Then

$$-\xi\Phi(\xi) = -\xi\int_{-\infty}^{\xi}\frac{1}{\sqrt{2\pi}}e^{-v^2/2}dv \leq -\int_{-\infty}^{\xi}\frac{v}{\sqrt{2\pi}}e^{-v^2/2}dv = \phi(v).$$

The last inequality proves (45). Substitution of (45) for (44) results in

$$g'(u) \geq -(-\sigma_1 + u) - (-\sigma_1 - u) = 2\sigma_1.$$

Formula (43) is proved.

□

Theorem 3. *For any $\gamma \in [0, 1]$, $\sigma > 0$ equation (19) has a unique solution.*

Proof. According to lemma 3, equation (19) is equivalent to (37). From (43) it follows that the left hand side of the equation (37) increases monotonically in u and therefore crosses zero no more than at one point. The existence of a solution is proved above. □

C Proofs of lemmata 1 and 2

Proof of lemma 1. Evidently,

$$\pi_i = p_i \min\{s_i, q_i\} - m_i w_i s_i - \varphi w_i. \tag{46}$$

Formula (32) follows directly from formulae (46) with $\min\{s_i, q_i\} = s_i$ and (21). Let $q_i < s_i$. By using $p_i^{1/(\gamma_i - 1)} P_i^{\gamma_i/(1-\gamma_i)} = 1/(N_i p_i)$, which comes from (21), and substituting (46) into formula (31), we get

$$q_i = \frac{\alpha_i z}{N_i p_i} \left(\sum_{j=1}^{n} w_j L_j + L_A + p_i q_i N_i - \frac{\varphi}{1 - \gamma_i} w_i N_i + \Pi_{-i} \right).$$

Merely its solution is given by expression (33). □

Proof of lemma 2. The proof of the first statement consists of a routine substitution of the model variables into formula (31). We stop at the main steps. Since $s_j = \min\{s_j, q_j(z)\}$ for all j, the right hand side of (31) does not depend on the demands:

$$q_i(z) = \frac{\alpha_i z}{N_i p_i} \left(\sum_{j=1}^{n} w_j L_j + L_A + \sum_{j=1}^{n} N_j \big(p_j s_j - (m_j s_j + \varphi) w_j \big) \right)$$

By using expressions (21) and (24) for the optimal supply and the number of the firms, we see that the sum with $w_j L_j$ goes off and

$$q_i(z) = \frac{\alpha_i z}{N_i p_i} \left(L_A + \sum_{j=1}^{n} N_j p_j s_j \right).$$

With (24) and (28) the last formula is transformed into (35).

Under the condition of the lemma the expression for the profit is

$$\pi_i = p_i q_i - m_i w_i s_i - \varphi w_i.$$

Substituting this profit into formula (31) we have

$$q_i = \frac{\alpha_i z}{N_i p_i}\left(\sum_{j=1}^{n} w_j L_j + L_A + \sum_{j=1}^{n} N_j (p_j q_j - w_j(m_j s_j + \varphi))\right).$$

As above, $\sum w_j L_j$ goes off because $N_j w_j (m_j s_j + \varphi) = w_j L_j$. Then

$$q_i = \frac{\alpha_i z}{N_i p_i}\left(L_A + \sum_{j=1}^{n} N_j p_j q_j\right) \tag{47}$$

The demands q_i solve the system of n equations (47). First, multiplying (47) by $N_i p_i$ and summing them up we find

$$\sum_{j=1}^{n} N_j p_j q_j = \frac{\sum_{j=1}^{n} \alpha_j z L_A}{1 - \sum_{j=1}^{n} \alpha_j z} \tag{48}$$

Substituting (48) into (47) and taking into account expressions (28) and (24) for the prices and the number of firms, we prove the lemma. □

Acknowledgements . We are thankful for Professors M. Alexeev (Indiana University) and S. Weber (SMU and NES) for useful discussion. This work is supported by EERC grant C 12-581.

References

M. Amiti and C. A. Pissarides. Trade and industrial location with heterogeneous labor. *Journal of International Economics*, 67:392–412, 2005.

F. Andersson and R. Forslid. Tax Competition and Economic Geography. *Journal of Public Economic Theory*, 5:279–303, 2003.

A. Appelbaum and C. Lim. Monopoly versus Competition under Uncertainty. *Canadian Journal of Economics*, 15:355–363, 1982.

L. Ball, G. Mankiw, and R. Reis. Monetary policy for inattentive economies. *Journal of Monetary Economics*, 52:703–725, 2005.

A. B. Bernard, S. J. Redding, and P.-K. Schott. Comparative Advantage and Heterogeneous Firms. *Review of Economic Studies*, 74:31–66, 2007.

P. Bertoletti and F. Entro. Monopolistic Competition: A Dual Approach with an Application to Trade. 2013. URL http://www.intertic.org/Theory\%20Papers/Bertoletti-Etro.pdf.

P. Bertoletti and G. Epifani. Consumer Behavior, Monopolistic Competition, and International Trade. 2012. URL http://www.fondazionemasi.it/UploadDocs/382_Epifani____Bertoletti.pdf.

R. Borck and M. Pfluger. Agglomeration and tax competition. *European Economic Review*, 50:647–668, 2006.

J. D. Dana. Equilibrium price dispersion under demand uncertainty: the roles of costly capacity and market structure. *RAND Journal of Economics*, 30:632–660, 1999.

M. Friedman. The role of monetary policy. *Am. Econ. Rev.*, 58:1–17, 1968.

E. Glaeser and J. Kohlhase. Cities, regions and the decline of transport costs. *Papers in Regional Science*, 83:197–228, 2003.

J.-M. Grandmont. Temporal general equilibrium theory. *Econometrica*, 45:535–572, 1977.

N. J. Ireland. Product Diversity and Monopolistic Competition Under Uncertainty. *The Journal of Industrial Economics*, 33:501–513, 1985.

K. L. Judd. On the performance of Patents. *Econometrica*, 53:567–586, 1985.

E. S. Leland. Theory of the firm facing uncertainty demand. *American Economic Review*, 62: 278–291, 1972.

R. D. Ludema and I. Wooton. Economic geography and the fiscal effects of regional integration. *Journal of International Economics*, 52:331–357, 2000.

G. Mankiw and R. Reis. Sticky information in general equilibrium. *Journal of the European Economic Association*, 2:603–613, 2007.

M. J. Melitz. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica*, 71:1695–1725, 2003.

M. J. Melitz and S. J. Redding. Heterogeneous Firms and Trade. *NBER Working Paper No. 18652*, 2012. URL http://www.nber.org/papers/w18652.

M. Mrazova and J. P. Neary. Selection effects with heterogeneous firms. 2012. URL `http://www.eea-esem.com/files/papers/eea-esem/2012/1883/MrazovaNeary2012.pdf`.

G. I. Ottaviano and D. Puga. Agglomeration in the Global Economy: A Survey of the "New Economic Geography". *The World Economy*, 21:707–731, 1998.

E. S. Phelps. Money-wage dynamics and labor market equilibrium. *J. Polit. Econ.*, 76:678–711, 1968.

A. Sandmo. On the Theory of the Competitive Firm under Price Uncertainty. *American Economic Review*, 61:65–73, 1971.

C. A. Sims. Implications of rational inattention. *Journal of Monetary Economics*, 50:665–690, 2003.

T. Tabuchi, J.-F. Thisse, and D.-Z. Zeng. On the number and size of cities. *Journal of Economic Geography*, 5:423–448, 2012.

E. Zhelobodko, S. Kokovin, M. Parenti, and J.-F. Thisse. Monopolistic Competition: Beyond the Constant Elasticity of Substitution. *Econometrica*, 80:2765–2784, 2012.