Are commodity price shocks important? A Bayesian estimation of a DSGE model for Russia

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Abstract: This paper constructs a DSGE model for an economy with commodity exports. We estimate the model using Russian data, making a special focus on quantitative effects of commodity price dynamics. There is a widespread belief that economic activity in Russia crucially depends on oil prices, but quantitative estimates are scarce. We estimate an oil price effect on the Russian economy in a general equilibrium framework. Our setup is similar to those of Kollmann (2001) and Dam and Linaa (2005), but we extend their models by explicitly accounting for oil revenues. In addition to standard supply, demand, cost-push, and monetary policy shocks, we include the shock of commodity export revenues. The main objective of the paper is to identify the contribution of structural shocks to business cycle fluctuations in the Russian economy. We found that despite a strong impact on GDP from commodity export shocks, business cycles in Russia are mostly domestically based.

Keywords: DSGE; business cycles; Bayesian estimation; oil price effect; commodity prices.

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1 Introduction

In the economic literature, there is a widespread belief that economic activity in Russia crucially depends on oil price dynamics. This perception is based on the fact that Russia is one of the world's largest oil producers, with oil and gas exports amounting to $342 bln in 2011, accounting for 18.5% of Russian GDP and one-half of federal budget revenues. In this situation, it seems evident that oil price shocks could dominate Russian business cycles and long-run dynamics of macroeconomic variables. However, quantitative estimates of oil price effects are scarce. For example, Rautava (2002) analyses the impact of oil prices on the Russian economy using the VAR methodology and cointegration techniques and discovers that, in the long run, a 10% increase in oil prices is associated with a 2.2% growth in Russian GDP. Their sample covered the period from Q1 1995 to Q3 2001. Jin (2008) uses a similar methodology and claims that in the 2000s, a permanent 10% increase in oil prices was associated with a 5.16% growth in Russian GDP. In both papers, the authors use quarterly data, so the time series seem to be too short for cointegration analysis to have good estimation properties. Moreover, neither of these papers raises questions about the short-run impact of oil prices on macroeconomic variables and the role of oil prices as a potential factor of the business cycle.

Since the 1990s, there has been a growing interest in dynamic stochastic general equilibrium (DSGE) models for macroeconomic analysis from both academia and central banks. Contrary to VAR, DSGE models provide a theoretical explanation of different interdependencies among variables in the economy. These models allow to determine the factors of business cycles, forecast macroeconomic variables, identify the impact of structural changes, etc., Sosunov and Zamulin (2007) analyse an optimal monetary policy in an economy sick with Dutch disease in a general equilibrium framework. They calibrate their model on Russian data, but they assume that the shock to the terms of trade is the only source of uncertainty in the economy, and they do not consider the relative importance of this kind of shock in real data. Semko (2013) estimates a modified version of the model by Dib (2008) using Russian data with a focus on optimal monetary policy. He mentions that his results indicate that the impact of oil price shock on GDP is small, as a rise in output in the oil production sector is associated with an output decline in manufacturing and non-tradable sectors, but quantitative estimates of the impact are not provided in the paper.

The purpose of our paper is twofold. The first goal is to elaborate a theoretical model with a special focus on commodity-exporting countries that is suitable as a basis for policy implications. The second goal is to determine the main sources of volatility of key macroeconomic variables in Russia and answer the question that we raised in the title of the paper: are commodity prices important as a factor of business cycles in an export-oriented economy?

Our paper has some policy implications. The belief that economic activity in Russia is mostly determined by oil price dynamics was an argument for the exchange rate management policy. Recently the Central Bank of Russia announced a new course of monetary policy based on an inflation targeting policy from 2015 onwards. It is crucial to understand what role commodity exports play in business cycles in order to assess the potential success of this policy switch. While the traditional Mundell-Fleming model states that flexible exchange rates dominate fixed exchange rates if foreign real shocks prevail, this prescription is called into doubt when an adjustment requires a substantial devaluation or revaluation of exchange rates (Cespedes et al., 2004). In this case, an exchange rate peg may be desirable.

In this paper, we modify the Kollmann's model (Kollmann, 2001) and assume external habit formation, a cashless economy, and CRRA preferences of households as in Smets and Wouters (2003) and Dam and Linaa (2005). The model contains a number of real and nominal frictions, like sticky prices and wages, local currency pricing, and capital adjustment costs. It is known from previous research that rigidities play a key role in the fitting and forecasting performance of DSGE models (Christiano et al., 2005; Smets and Wouters, 2007). Additionally, we assume that the nominal interest rate is an instrument of monetary policy and increase the number of structural shocks under consideration. We introduce 10 structural shocks. Nine of them are relatively standard, while the tenth is a commodity export shock. Next, we estimate the model on Russian data using Bayesian methods. Our results show that, while this shock contributes a lot to GDP variation, the most important factors of business cycles in Russia are domestically based.

We proceed as follows: Section 2 presents the model. For the sake of convenience, we present the full set of equations. In Section 3, we review our estimation techniques and discuss our results. Section 4 concludes.

2 Model

In this section, we present the model that we estimate in the next section. We assume two types of firms that produce intermediate and final goods. The final sector is competitive, and intermediate sector is monopolistic competitive. Households can own capital and rent it, as well as labour services to intermediate goods firms. They can optimise both intertemporally and intratemporally. Prices and wages are rigid due to a mechanism à la Calvo. A final good can be used for consumption and for investment. The final good is aggregated from domestic and imported intermediate ones. Export and import are possible only for intermediate products and are priced in local currency. Financial markets are incomplete and households can own domestic and foreign bonds (or issue debt). The core of our model is that by Kollmann (2001)[1] but we have made some important modifications. First of all, we assume external habit formation, a cashless economy and CRRA preferences. Secondly, we include revenues from oil exports which are assumed to increase households' wealth exogenously. Finally, we assume that monetary policy follows an interest rate rule.

2.1 Production sector

2.1.1 Final goods production

We assume that the only final good is produced by combining intermediate domestic and imported aggregates using Cobb–Douglas technology:

$$Q_t = \left(\frac{1}{\alpha_d} Q_t^d\right)^{\alpha_d} \left(\frac{1}{\alpha_{im}} Q_t^{im}\right)^{\alpha_{im}}, \quad 0 < \alpha_d < 1, \alpha_{im} = 1 - \alpha_d. \tag{1}$$

Q_t denotes the final output index. Q_t^d and Q_t^{im} are indices of aggregate domestic and foreign intermediate goods production, respectively, and they are defined as Dixit–Stiglitz aggregates:

$$Q_t^d = \left(\int_0^1 q_t^d(j)^{\frac{1}{1+\upsilon_t}} \mathrm{d}j\right)^{1+\upsilon_t} \qquad Q_t^{im} = \left(\int_0^1 q_t^{im}(j)^{\frac{1}{1+\upsilon_t}} \mathrm{d}j\right)^{1+\upsilon_t} \tag{2}$$

where $q_t^d(j)$ and $q_t^{im}(j)$ are quantities of type j intermediate goods produced domestically and abroad, respectively, and sold on domestic market, and υ_t is a random net mark-up rate. In other words, in the intermediate goods market, there is a continuum (of unit measure) of producers, and we use index j to indicate them. Each producer sells her own variety (also indicated by j) in the monopolistic competitive market. The final sector is perfectly competitive and does not incur any cost above the value of the intermediate bundles.

A cost-minimisation problem for the final producer can be written as:

$$\min TC_{\text{final}} = \int_0^1 p_t^d(j) q_t^d(j) \mathrm{d}j + \int_0^1 p_t^{im}(j) q_t^{im}(j) \mathrm{d}j \tag{3}$$

subject to constraints (1) and(2) where $p_t^d(j)$ and $p_t^d(j)$ represent prices of domestic and imported type j intermediate products respectively, both expressed in domestic currency. The demand functions for any variety (domestic or imported) of intermediate products as well as for intermediate aggregates are derived as a solution of the cost-minimisation problem. They are given by:

$$q_t^d = Q_t^d \left(\frac{p_t^d(j)}{P_t^d}\right)^{-\frac{1+\upsilon_t}{\upsilon_t}} \qquad q_t^{im} = Q_t^{im} \left(\frac{p_t^{im}(j)}{P_t^{im}}\right)^{-\frac{1+\upsilon_t}{\upsilon_t}} \tag{4}$$

and

$$Q_t^d = \alpha_d \frac{P_t}{P_t^d} Q_t \qquad Q_t^{im} = \alpha_{im} \frac{P_t}{P_t^{im}} Q_t \tag{5}$$

letting P_t^d and P_t^{im} be the price indices of intermediate domestic and foreign bundles sold in the domestic market, respectively, and P_t representing the final good price index. We postulate that intermediate goods are packed in a bundle at no cost, and the value of a bundle is equal to the value of its ingredients. The total revenue of the final producers is equal to their total costs as they are competitors and operate on a zero-profit bound. This means that:

$$P_t^d Q_t^d = \int_0^1 p_t^d(j) q_t^d(j) \mathrm{d}j \qquad P_t^{im} Q_t^{im} = \int_0^1 p_t^{im}(j) q_t^{im}(j) \mathrm{d}j. \tag{6}$$

So we get:

$$P_t^d = \left(\int_0^1 p_t^d(j)^{-\frac{1}{\upsilon}t} \mathrm{d}j \right)^{-\upsilon_t} \qquad P_t^{im} = \left(\int_0^1 p_t^{im}(j)^{-\frac{1}{\upsilon}t} \mathrm{d}j \right)^{-\upsilon_t}. \tag{7}$$

A zero-profit condition for the final good sector requires:

$$P_t^d Q_t^d + P_t^{im} Q_t^{im} = P_t Q_t. \tag{8}$$

Hence, the final good price index is determined by a weighted geometric mean of domestic and imported aggregates price indices:

$$P_t = \left(P_t^d\right)^{\alpha_d} \left(P_t^{im}\right)^{\alpha_{im}}. \tag{9}$$

2.1.2 Intermediate sector

An intermediate good j is produced from labour and capital with Cobb–Douglas technology:

$$y_t(j) = A_t K_t(j)^{\psi} L_t(j)^{1-\psi}, \qquad \text{where } 0 < \psi < 1 \tag{10}$$

where $y_t(j)$ is an output of an intermediate type j firm, A_t is a technology parameter, $K_t(j)$ is capital stock that firm j holds (capital utilisation is assumed to be equal to one), and $L_t(j)$ is the amount of labour services utilised by firm j and represents a Dixit–Stiglitz aggregate of different varieties of labour services provided by households:

$$L_t(j) = \left(\int_0^1 l_t(h,j)^{\frac{1}{1+\gamma}} \mathrm{d}h \right)^{1+\gamma} \tag{11}$$

where $l_t(h,j)$ is the amount of labour services of household h employed by firm j. Here we assume that there is a continuum (of unit mass) of households (indexed by h), their labour services are differentiated, and the labour market is monopolistic competitive. So each household is a monopolistic supplier of its labour and sets the wage on its own (we describe the mechanism of wage-setting below). On the contrary, capital is homogenous. The law of motion of the technology process is declared below. This, the total costs of firm j are the following:

$$TC_t(j) = R_t^K K_t(j) + \int_0^1 w_t(h) l_t(h,j) \mathrm{d}h, \tag{12}$$

where R_t^K is the rental rate of capital, and $w_t(h)$ is the wage of household h. The problem of an intermediate firm consists in minimising $TC_t(j)$ s.t. (10). The first-order conditions imply that demand functions for aggregate labour and capital can be written as:

$$L_t(j) = \frac{y_t(j)}{A_t} \left(\frac{\psi}{1-\psi} \cdot \frac{W_t}{R_t^K} \right)^{-\psi} \tag{13}$$

$$K_t(j) = \frac{y_t(j)}{A_t} \left(\frac{\psi}{1-\psi} \cdot \frac{W_t}{R_t^K} \right)^{1-\psi}. \tag{14}$$

Additionally,

$$l_t(h,j) = L_t(j) \left(\frac{w(h)}{W_t} \right)^{-\frac{1+\gamma}{\gamma}}. \tag{15}$$

As far as the total labour costs for intermediate firm j are concerned, they are equal to labour expenses for all varieties:

$$W_t L_t(j) = \int_0^1 w_t(h) l_t(h,j) \mathrm{d}h, \tag{16}$$

the aggregate wage index is

$$W_t = \left(\int_0^1 w_t(h)^{-\frac{1}{\gamma}} \mathrm{d}h \right)^{-\gamma}. \tag{17}$$

The marginal cost of firm j is equal to:

$$MC_t(j) = A_t^{-1} W_t^{1-\psi} {R_t^K}^{\psi} \psi^{-\psi} (1-\psi)^{\psi-1}. \tag{18}$$

Therefore, the marginal cost is the same for all firms in the market; it allows us to omit an index of a firm in what follows. Moreover, the total cost is a linear function of output, and the marginal cost is independent of output. This lets us consider problems of setting domestic and export prices separately. We assume that intermediate goods are sold on domestic and international markets:

$$y_t(j) = q_t^d(j) + q_t^{ex}(j), \tag{19}$$

where $q_t^d(j)$ and $q_t^{ex}(j)$ are quantities of intermediate good j sold on the domestic market and exported, respectively. All the intermediate goods sold in the domestic market are bought by the final producer. We postulate that intermediate firms can practice price discrimination between domestic and foreign markets. In general, this means that:

$$S_t p_t^{ex}(j) \neq p_t^d(j) \tag{20}$$

where $p_t^d(j)$ and $p_t^{ex}(j)$ are price indices of intermediate good j sold in the domestic market and exported, respectively, and S_t is a nominal exchange rate (expressed as a domestic currency price of foreign currency). The assumption about price discrimination and, consequently, the violation of the law of one price is motivated by a great number of theoretical and empirical papers (see, for example, Balassa (1964) and Taylor and Taylor (2004)) which show that the absolute PPP does not hold, at least, in the short-run. In the NOEM literature there are several microfounded approaches to model deviations from the PPP, and Ahmad et al. (2011) offer a very good review of them.[2] In this paper, we assume that intermediate firms – both domestic and foreign – and households carry out staggered price and wage setting, respectively, and the exporting and importing activity is characterised by price-to-market behaviour (Knetter, 1993). This means that the prices are set in the local (buyer's) currency. The staggered price and wage setting is implemented à la Calvo (Calvo, 1983). The probability of a price-changing signal is equal to $1 - \theta_d$. Because the number of firms is large, in accordance with the law of large numbers, we can define the share of firms reoptimising their prices each period as equal to $1 - \theta_d$, as well. All the firms are obliged to meet the demand for their products at the set price. Suppose a firm gets a signal and is allowed to adjust its price. In this case, the price chosen by the producer is one that maximises an expected discounted flow of her future profits:

$$\widetilde{p}_t^d(j) = \underset{p_t^d(j)}{\arg\max}\, E_t \left[\sum_{\tau=0}^{\infty} \theta_d^\tau \lambda_{t,t+\tau} \Pi_{t+\tau}^{d,j} \left(p_t^d(j) \right) \right] \tag{21}$$

where $\widetilde{p}_t^d$ is a reset price; $\Pi_{t+\tau}^{d,j}$ is the profit of intermediate firm j from selling its product in the domestic market (superscript d) at time $t + \tau$; $\lambda_{t,t+\tau}$ is a stochastic discount factor of nominal income (pricing kernel). It is assumed to be equal to the intertemporal marginal rate of substitution in consumption between periods t and $t + \tau$ and is given by:

$$\lambda_{t,t+\tau} \equiv \beta^\tau \frac{U'_{C,t+\tau}}{U'_{C,t}} \cdot \frac{P_t}{P_{t+\tau}}. \tag{22}$$

While solving its problem of profit maximisation, the firm takes into account all the expected profits until the next price-changing signal comes. As the number of periods to be taken into account is not known in advance, the producer maximises her discounted profit over an infinite horizon, and each profit is multiplied by the probability that the firm has not received a new price-changing signal before. The instantaneous profit of intermediate producer j from selling her variety in the domestic market is defined as:

$$\Pi_t^{d,j} = \left(p_t^d(j) - MC_t \right) q_t^d(j) = \left(p_t^d(j) - MC_t \right) \left(\frac{p_t^d(j)}{P_t^d} \right)^{-\frac{1+\upsilon_t}{\upsilon_t}} Q_t^d. \tag{23}$$

Therefore, the problem facing the producer is to maximise equation (21) subject to equation (23). The first order conditions result in the following equation for the optimal price:

$$E_t \sum_{\tau=0}^{\infty} \theta_d^{\tau} \lambda_{t,t+\tau} \frac{1}{\upsilon_{t+\tau}} (P_{t+\tau}^d)^{\frac{1+\upsilon_{t+\tau}}{\upsilon_{t+\tau}}} Q_{t+\tau}^d \widetilde{p}_t^d(j)^{-\frac{1+\upsilon_t}{\upsilon_t}-1}$$
$$\times \left(\widetilde{p}_t^d(j) - (1+\upsilon_{t+\tau}) MC_{t+\tau}\right) = 0. \tag{24}$$

2.2 Foreign sector

2.2.1 Export

We assume that the structure of a foreign economy is the same as the structure of a domestic one. Similar to the demand for domestic intermediate goods, the export demand is assumed to be defined as:

$$Q_t^{ex} = \alpha_{ex} \left(\frac{P_t^{ex}}{P_t^f} \right)^{-\eta} Y_t^f \tag{25}$$

where P_t^{ex} is the price index of the intermediate domestic bundle exported abroad, P_t^f is an aggregate price level in the foreign economy, and Y_t^f is a quantity of final goods produced in the foreign economy. Both prices are expressed in foreign currency. Similar to the demand for a particular type of intermediate goods in the domestic economy, export demand for a variety j $(q_t^{ex}(j))$ is given by:

$$q_t^{ex}(j) = Q_t^{ex} \left(\frac{p_t^{ex}(j)}{P_t^{ex}} \right)^{-\frac{1+\upsilon_t}{\upsilon_t}} \tag{26}$$

with the same elasticity of substitution that characterises the domestic demand:

$$Q_t^{ex} = \left(\int_0^1 (q_t^{ex}(j))^{\frac{1}{1+\upsilon_t}} \mathrm{d}j \right)^{1+\upsilon_t}. \tag{27}$$

The fact that the value of the exported bundle is equal to the value of its components

$$P_t^{ex} Q_t^{ex} = \int_0^1 p_t^{ex}(j) q_t^{ex}(j) \mathrm{d}j \tag{28}$$

gives the following equation for the price of the aggregate exported:

$$P_t^{ex} = \left(\int_0^1 (p_t^{ex}(j))^{-\frac{1}{\upsilon_t}} \right)^{-\upsilon_t} \mathrm{d}j. \tag{29}$$

As in the case of the domestic market, the intermediate producer must receive a price-changing signal to be able to reset her export price. The probability of this signal is equal to $1-\theta_{ex}$, and the signal is completely independent of the one allowing for the reoptimisation of the domestic price. The reset price is the price that maximises the expected discounted profit from export activity:

$$\widetilde{p}_t^{ex} = \underset{p_t^{ex}(j)}{\arg\max}\, E_t \left[\sum_{\tau=0}^{\infty} \theta_{ex}^{\tau} \lambda_{t,t+\tau} \Pi_{t+\tau}^{ex,j} \left(p_t^{ex}(j) \right) \right] \tag{30}$$

where the instantaneous profit from export activity is given by the following equation:

$$\Pi_t^{ex,j} = \left(S_t p_t^{ex}(j) - MC_t \right) q_t^{ex}(j) = \left(S_t p_t^{ex}(j) - MC_t \right) \left(\frac{p_t^{ex}(j)}{P_t^{ex}} \right)^{-\frac{1+\upsilon_t}{\upsilon_t}} Q_t^{ex}. \tag{31}$$

The first-order conditions for the optimal export reset price yield:

$$E_t \sum_{\tau=0}^{\infty} \theta_{ex}^{\tau} \lambda_{t,t+\tau} (P_{t+\tau}^{ex})^{\frac{1+\upsilon_{t+\tau}}{\upsilon_{t+\tau}}} Q_{t+\tau}^{ex} \frac{1}{\upsilon_{t+\tau}} (\widetilde{p}_t^{ex})^{-\frac{1+\upsilon_t}{\upsilon_t}-1} \times (S_{t+\tau} \widetilde{p}_t^{ex} - (1+\upsilon_{t+\tau}) MC_{t+\tau}) = 0. \tag{32}$$

2.2.2 Import

The importing of intermediate products is implemented by foreign companies.[3] Like domestically produced intermediate goods, all imported varieties are imperfect substitutes. The cost (in domestic currency) of importing firm j is $S_t P_t^f$, and its income is $p_t^{im}(j)$. P_t^f stands for the average cost (in foreign currency) of producing any variety abroad. Domestic prices of imported goods are also rigid due to the Calvo mechanism with price-changing probability equal to $1 - \theta_{im}$. If the foreign producer is allowed to reset her price in the domestic market, she chooses the optimal level so that to maximise her expected discounted future profits (in foreign currency):

$$\widetilde{p}_t^{im} = \underset{p_t^{im}(j)}{\arg\max}\, E_t \left[\sum_{\tau=0}^{\infty} \theta_{im}^{\tau} \lambda_{t,t+\tau}^{f} \frac{\Pi_{t+\tau}^{im,j} \left(p_t^{im}(j) \right)}{S_{t+\tau}} \right] \tag{33}$$

where the instantaneous profit of importing firm j is given by:

$$\Pi_t^{im,j} = \left(p_t^{im}(j) - S_t P_t^f \right) q_t^{im}(j) = \left(p_t^{im}(j) - S_t P_t^f \right) \left(\frac{p_t^{im}(j)}{P_t^{im}} \right)^{-\frac{1+\upsilon_t}{\upsilon_t}} Q_t^{im} \tag{34}$$

where foreign importers are assumed to be risk-neutral, so they discount their profits at the international risk-free rate:

$$\lambda_{t,t+\tau}^{f} = \prod_{j=t}^{t+\tau-1} \left(1 + i_j^f \right)^{-1} \tag{35}$$

where i_t^f is a foreign risk-free rate that is defined exogenously.

The first-order conditions for the problem facing the foreign importers result in the following equation for the optimal import price:

$$E_t \sum_{\tau=0}^{\infty} \theta_{im}^{\tau} \lambda_{t,t+\tau}^{f} \frac{1}{S_{t+\tau} \upsilon_{t+\tau}} (P_{t+\tau}^{im})^{\frac{1+\upsilon_{t+\tau}}{\upsilon_{t+\tau}}} Q_{t+\tau}^{im} \widetilde{p}_t^{im}(j)^{-\frac{1+\upsilon_{t+\tau}}{\upsilon_{t+\tau}}-1} \times \left(\widetilde{p}_t^{im}(j) - (1+\upsilon_{t+\tau}) S_{t+\tau} P_{t+\tau}^{f}\right) = 0. \quad (36)$$

As cost functions are identical for any firm in the intermediate goods and foreign sectors, all producers that have the opportunity to reoptimise their prices at time t, set them at the same level ($\widetilde{p}_t^d(j) = \widetilde{p}_t^d$, $\widetilde{p}_t^{ex}(j) = \widetilde{p}_t^{ex}$ and $\widetilde{p}_t^{im}(j) = \widetilde{p}_t^{im}$ for all j). Therefore, the price indices of domestic, export and import aggregates are given by the following equations:

$$\left(P_t^d\right)^{-\frac{1}{\upsilon}} = \theta_d \left(P_{t-1}^d\right)^{-\frac{1}{\upsilon}} + (1-\theta_d)\left(\widetilde{p}_t^d\right)^{-\frac{1}{\upsilon}} \quad (37)$$

$$\left(P_t^{ex}\right)^{-\frac{1}{\upsilon}} = \theta_{ex} \left(P_{t-1}^{ex}\right)^{-\frac{1}{\upsilon}} + (1-\theta_{ex})\left(\widetilde{p}_t^{ex}\right)^{-\frac{1}{\upsilon}} \quad (38)$$

$$\left(P_t^{im}\right)^{-\frac{1}{\upsilon}} = \theta_{im} \left(P_{t-1}^{im}\right)^{-\frac{1}{\upsilon}} + (1-\theta_{im})\left(\widetilde{p}_t^{im}\right)^{-\frac{1}{\upsilon}}. \quad (39)$$

2.3 Households

The population is assumed to consist of a continuum of households of unity measure. Any representative household maximises its expected discounted utility over an infinite horizon subject to its budget constraints. The utility function is increasing in consumption and decreasing in labour efforts. Only final good can be consumed.

We follow many other papers (Erceg et al., 2000; Gali, 2008) in assuming that labour services of different households are imperfect substitutes, as indicated above. Every household holds monopoly power in the market over its variety of labour and acts as a wage-setter. A wage-setting process is also rigid à Calvo with the probability of a wage-changing signal equal to $1-\theta_w$.

Each period, a representative household makes its consumption and portfolio choices. A household can own domestic and foreign bonds[4] as well as capital. If a household receives a wage-changing signal, it also makes a decision about a new reset price. A household faces only one kind of uncertainty – when it will be allowed to change its wage for the next time – and this shock is idiosyncratic. Therefore, different households can work different amounts of time and have different incomes (Christiano et al., 2005). But, as was shown in Woodford (1996) and Erceg et al. (2000), we can consider households to be homogenous with respect to the amount of consumption and wealth allocation among different types of bonds and capital owing to state-contingent assets. It allows us to drop a household index h for consumption in the utility function.

A household h maximises its expected discounted utility (subject to the budget constraint to be specified below):

$$V_0(h) = \max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, l_t(h)\right) \quad (40)$$

where C_t represents consumption, $l_t(h)$ is the labour services supplied by household h, and β is a subjective discount factor. As indicated above, the household manages three kinds of assets: domestic bonds, foreign bonds and capital stock. In addition to interest on bonds and capital, a household receives labour income, dividends from non-competitive intermediate firms, and revenues from commodity exports.

The capital accumulation equation can be written as:

$$K_{t+1} = (1-\delta)K_t + I_t - \chi\left(K_{t+1} - K_t\right) \tag{41}$$

where I_t is investment, and δ is the depreciation ratio. The last term in equation (41) stands for the capital adjustment cost, and the function χ is defined as follows:

$$\chi\left(K_{t+1}, K_t\right) = \frac{\Phi}{2}\frac{\left(K_{t+1} - K_t\right)^2}{K_t}. \tag{42}$$

We follow Smets and Wouters (2003) in defining the preferences, which are assumed to be described by an additively separable instantaneous utility function with CRRA form:

$$U\left(C_t, l_t(h)\right) = \epsilon_b \left(\frac{\left(C_t - \nu \widetilde{C}_{t-1}\right)^{1-\sigma_1}}{1-\sigma_1} - \epsilon_l \frac{l(h)^{1+\sigma_2}}{1+\sigma_2} \right) \tag{43}$$

letting $\widetilde{C}_{t-1}$ be external habits in consumption (Abel, 1990) and letting ν be a positive parameter of force of habits. The budget constraint of household h in period t is represented by the following equation:

$$\begin{aligned} P_t(C_t + I_t(h)) + D_t(h) + S_t D_t^*(h) = & \int_0^1 w_t(h) l_t(h,j)\mathrm{d}j + D_{t-1}(h)\left(1 + i_{t-1}\right) \\ & + S_t D_{t-1}^*(h)\left(1 + i_{t-1}^*\right) + R_t^K K_t(h) \\ & + \Pi_t^d(h) + \Pi_t^{ex}(h) + S_t O_t(h). \end{aligned} \tag{44}$$

The commodity production is assumed to be constant and normalised to unity, so all the fluctuations of commodity export revenues are due to changes of the commodity price (denoted by O_t in this paper). D_t^* denotes foreign bonds (credit from the foreign sector if D_t^* is negative), i_t is the nominal domestic interest rate, and i_t^* is the nominal foreign interest rate (including the risk premium). The financial markets are assumed to be imperfect, and the imperfections create a deviation of nominal interest rate on foreign bonds from the international risk-free rate i_t^f. This deviation can be interpreted as a risk premium:

$$1 + i_t^* = \rho\left(1 + i_t^f\right). \tag{45}$$

Like Lindé et al. (2009) and Curdia and Finocchiaro (2005), we assume that this risk premium can be specified by a decreasing function of net foreign assets of the economy. However, unlike the cited papers, we modify the function of risk premium and normalise net foreign assets to the total export (including commodity export income) in steady state:

$$\rho_t = \exp\left(-\omega\left(\frac{\bar{P}^f D_t^*}{\bar{P}^{ex}\bar{Q}^{ex} + \bar{O}}\right) + \epsilon_t^{\rho}\right) \tag{46}$$

where ϵ_t^ρ is a stochastic shock of the risk premium, ω is a normalising constant, and barred variables denote steady-state values of the corresponding variables without bars. Therefore, if the amount of debt of domestic households increases, the interest rate (with premium) increases as well. The technical reason for including the endogenous risk premium is that it guarantees the existence of stationary equilibrium (Schmitt-Grohe and Uribe, 2003).

During each period, a representative household maximises its expected discounted utility (equation (40)) subject to the sequence of dynamic constraints: (44) and (41).

The first-order conditions for this problem yield the following equations:

$$U_C' = P_t \mu_t \tag{47}$$

$$\beta E_t \mu_{t+1}(1+i_t) = \mu_t \tag{48}$$

$$\beta E_t \mu_{t+1} S_{t+1}(1+i_t^*) = \mu_t S_t \tag{49}$$

$$\beta E_t \mu_{t+1} R_{t+1}^K + \beta E_t P_{t+1}\mu_{t+1}\left(1-\delta-\chi_{2,t+1}'\right) = \mu_t(1+\chi_{1,t}')P_t \tag{50}$$

where μ_t is the Lagrange multiplier on the budget constraint. As indicated above, the household decides on consumption, investment, and portfolio distribution every period, but it chooses an optimal wage only on occasion when a wage-changing signal occurs. To derive the optimal reset wage for the firm reoptimising in period t, we reproduce the relevant parts of the maximisation problem written above. We take into account the probability that a new wage-changing signal does not come until $t+s$ is θ_w^s. In periods of wage resetting, the household maximises the expected discounted utility:

$$V_t^w(h) = \max E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau U\left(C_{t+\tau|t}, l_{t+\tau|t}(h)\right) \tag{51}$$

subject to the sequence of labour demand and budget constraints:

$$l_{t+\tau|t}(h,j) = L_{t+\tau|t}(j)\left(\frac{\widetilde{w}_t(h)}{W_{t+\tau|t}}\right)^{-\frac{1+\gamma}{\gamma}} \tag{52}$$

$$\begin{aligned} P_{t+\tau|t}&(C_{t+\tau|t} + I_{t+\tau|t}(h)) + D_{t+\tau|t}(h) + S_{t+\tau|t}D_{t+\tau|t}^*(h) \\ &= \int_0^1 w_{t+\tau|t}(h) l_{t+\tau|t}(h,j)\mathrm{d}j + D_{t+\tau-1|t}(h)\left(1+i_{t+\tau-1|t}\right) \\ &\quad + S_{t+\tau|t}D_{t+\tau-1|t}^*(h)\left(1+i_{t+\tau-1|t}^*\right) + R_{t+\tau|t}^K K_{t+\tau|t}(h) + \Pi_{t+\tau|t}^d(h) \\ &\quad + \Pi_{t+\tau|t}^{ex}(h) + S_{t+\tau|t}O_{t+\tau|t}(h). \end{aligned} \tag{53}$$

The first-order conditions for this problem result in the following equation for the reset wage:

$$\widetilde{w}_t(h)^{\frac{1+\gamma}{\gamma}\sigma_2+1} = (1+\gamma)\frac{E_t\sum_{\tau=0}^{\infty}\beta^\tau\theta_w^\tau\epsilon_{t+\tau}^b\epsilon_{t+\tau}^l L_{t+\tau}^{1+\sigma_2} W_{t+\tau}^{\frac{1+\gamma}{\gamma}(1+\sigma_2)}}{E_t\sum_{\tau=0}^{\infty}\beta^\tau\theta_w^\tau\mu_{t+\tau}L_{t+\tau}W_{t+\tau}^{\frac{1+\gamma}{\gamma}}} \tag{54}$$

where μ_t is the Lagrange multiplier associated with the budget constraint as given above. As the function of optimal wage does not depend on h, all households that have the opportunity to reoptimise their wages at time t, set them at the same level ($\widetilde{w}_t(h) = \widetilde{w}_t$). With the premise that, in each period, the ratio of households adjusting their wage is equal to $1 - \theta_w$, the law of motion for aggregate wages can be derived as:

$$(W_t)^{-\frac{1}{\gamma}} = \theta_w (W_{t-1})^{-\frac{1}{\gamma}} + (1 - \theta_w) (\widetilde{w}_t)^{-\frac{1}{\gamma}}. \tag{55}$$

2.4 Central bank

Because the goal of this paper is to estimate a DSGE model for the Russian economy, it is very important to use a monetary rule that actually describes the strategy of the Bank of Russia. However, at the moment, there is no scientific consensus regarding the monetary policy rule of the Bank of Russia. In the economic literature, the absence of a common opinion is indicated by the existence of different points of view regarding the best way to model the central bank's activity. For example, Vdovichenko and Voronina (2006) show that from 1999 to 2003, the Bank of Russia regulated the money supply, while the use of monetary instruments was limited by interventions on exchange markets and the sterilisation of excess liquidity with deposit operations. The authors claim that, unlike most central banks in developed countries, the discount rate in Russia plays a minor role. Hence they opt for the money supply rule. On the contrary, Benedictow et al. (2013) estimate an econometric model of the Russian economy based on data from 1995 to 2008. They suppose that monetary policy follows a simple Taylor rule, and the interest rate is set in response to unemployment and inflation changes. The authors claim that this kind of rule fits the data well even though the assumptions in the basis of the rule are hardly relevant to the Russian economy. In line with Benedictow et al. (2013) and Taro (2010) successfully estimate a non-linear interest rate rule on data from 1997 to 2007 under the assumption that the reaction of the central bank to an output gap and inflation is asymmetric. Finally, Yudaeva et al. (2010) aim to determine a monetary policy target for the Bank of Russia. They show that a forward-looking Taylor rule, as well as a money supply rule, can adequately describe Russian data from 2003 to 2010. Their results demonstrate that the central bank sets its instrument in response to the expected movements of inflation, output, and exchange rate, and uses interest rate smoothing. The authors do not opt for either of these rules. However, the fact that there are econometric papers that show that an interest rate rule can describe monetary policy in Russia allows us to use this kind of rule in our structural model. We therefore assume that monetary policy follows a modified Taylor rule with interest rate smoothing:

$$1 + i_t^d = (1 + i_{t-1}^d)^{z_1} (1 + \bar{i}^d)^{1-z_1} \left(\frac{\pi_t}{\bar{\pi}}\right)^{z_2(1-z_1)} \left(\frac{Y_t}{\bar{Y}}\right)^{z_3(1-z_1)} \epsilon_t^m. \tag{56}$$

2.5 Market clearing conditions and exogenous processes

During each period, an equilibrium in goods and financial markets must be maintained and a balance-of-payment identity must hold. Domestically produced intermediate goods are consumed within the economy or exported:

$$Y_t = Q_t^d + Q_t^{ex}. \tag{57}$$

The final good is divided among consumption and investment:

$$Q_t = C_t + I_t. \tag{58}$$

The balance-of-payment identity is derived from the household's budget constraint (44) and the equation of final good allocation (equation (57)). The balance-of-payment identity takes the form of:

$$P_t^{ex} Q_t^{ex} + O_t - \frac{1}{S_t} P_t^{im} Q_t^{im} - D_t^* + \left(1 + i_{t-1}^f\right) D_{t-1}^* = 0. \tag{59}$$

This equation implies that the exchange rate is floating. We are aware of the fact that this is not the case in Russia, but we think that complicating of the model may not make the estimation more accurate. We assume that all exogenous processes, except mark-up and monetary policy shocks, are given by AR(1) and the mark-up shock and monetary policy shocks are i.i.d processes:

$$\log A_t = \rho_a \log A_{t-1} + (1 - \rho_a) \log \bar{A} + \varepsilon_t^a \tag{60}$$
$$\log O_t = \rho_o \log O_{t-1} + (1 - \rho_o) \log \bar{O} + \varepsilon_t^o \tag{61}$$
$$\log Y_t^f = \rho_{y^f} \log Y_{t-1}^f + (1 - \rho_{y^f}) \log \bar{Y}^f + \varepsilon_t^y \tag{62}$$
$$\log \pi_t^f = \rho_{\pi^f} \log \pi_{t-1}^f + (1 - \rho_{\pi^f}) \log \bar{\pi}^f + \varepsilon_t^\pi \tag{63}$$
$$\log (i_t^f + 1) = \rho_{i^f} \log (1 + i_{t-1}^f) + (1 - \rho_{i^f}) \log (1 + \bar{i}^f) + \varepsilon_t^i \tag{64}$$
$$\log \epsilon_t^b = \rho_b \log \epsilon_{t-1}^b + (1 - \rho_b) \log \bar{\epsilon}^b + \varepsilon_t^b \tag{65}$$
$$\log \epsilon_t^l = \rho_l \log \epsilon_{t-1}^l + (1 - \rho_l) \log \bar{\epsilon}^l + \varepsilon_t^l \tag{66}$$
$$\log \epsilon_t^\rho = \rho_z \log \epsilon_{t-1}^\rho + (1 - \rho_\rho) \log \bar{\epsilon}^\rho + \varepsilon_t^\rho \tag{67}$$
$$\log \upsilon_t = \log \bar{\upsilon} + \varepsilon_t^\upsilon \tag{68}$$
$$\log \epsilon_t^m = \varepsilon_t^z. \tag{69}$$

Finally, our measure of real GDP in the model is:

$$GDP_t = Q_t + \frac{S_t P_t^{ex} Q_t^{ex} + S_t O_t - P_t^{im} Q_t^{im}}{P_t}. \tag{70}$$

3 Estimation

To find a solution for the model, we normalise all the nominal variables to national or foreign price levels (see Appendix A) and log-linearise the non-linear system around a non-stochastic steady state. We assume that in a steady state the current account is equal to zero; we also assume that $\eta = 1$. These assumptions are sufficient to derive an analytical solution for all the variables in a steady-state. We present the steady-state derivation in Appendix B and the final log-linearised model in Appendix C. We solve the model in Dynare and estimate it using Bayesian techniques. We think that calibration is unsuitable in our case because of a lack of microeconomic and macroeconomic papers that could have served as references for assigning values to hyperparameters.

3.1 Solution and data

As in most recent studies involving estimation of DSGE models, such as Smets and Wouters (2003), we carry out a so-called strong econometric interpretation of the DSGE model (Geweke, 1999). Following the terminology of Geweke, we distinguish between strong and weak interpretations of the DSGE model. The weak econometric interpretation implies that the DSGE model parameters are calibrated to make some theoretical moments as close as possible to the sample moments of the time series. The advantage of this approach is that the estimates are (often) more robust. It also allows focusing on those elements of the model that are of the most interest to the researcher.

In contrast, a strong econometric interpretation means that the whole probability space for the model is chosen. This allows for production of a likelihood function and the use of the method of maximum likelihood or Bayesian techniques for estimating parameters. This in turn makes it possible to obtain a full description of the data-generating process to test the specification of the model and to produce forecasts.

Bayesian estimation is a combination of maximum likelihood estimates (determined by the structure of the model and the data) with some prior knowledge described with prior distributions in order to construct a posterior distribution for the parameters of interest. Certainly, the use of prior information may raise questions about the origin of the prior knowledge and its credibility. However, from a practical point of view, using prior distributions improves parameters estimates. Pre-sample information is particularly necessary when one deals with emerging economies . When the sample size is limited, the maximum likelihood function is often almost flat, and its combination with some reasonable non-flat prior can help achieve identification (Fernandez-Villaverde, 2010). Nevertheless, we try to avoid using too tight priors in order to reduce the distortion effect on the final results and to let the likelihood dominate the posterior whenever it is possible. After choosing the prior, we combine it with the sample information described by the likelihood function and we receive the posterior distribution, by employing Bayes theorem. For the sake of simplicity, we characterise the posterior distribution by its mode, median, and variance. The posterior distribution is estimated in two steps. First, the posterior mode and approximate covariance matrix are calculated. The covariance matrix is computed numerically as the inverted (negative) Hessian at the posterior mode. Thereafter, the posterior distribution of model parameters is generated with a random-walk Metropolis–Hastings algorithm.

There are 10 shocks in the model: technology shock, commodity export revenues shock, monetary policy shock, mark-up shock, preference shock, labour supply shock, foreign interest rate shock, foreign prices shock, foreign output shock, and risk premium shock. For our estimation, we use nine time series. This guarantees the absence of stochastic singularity without resorting to measurement errors. Thus, we implicitly assume that all the observed volatility is caused by structural shocks. The variables that we consider to be observed for estimation are consumption, domestic inflation, domestic interest rate, real wages, the real exchange rate, oil revenues, foreign inflation, foreign interest rate, and foreign output.

The source for most of the data is the International Financial Statistics database. Other sources will be indicated below. All the series are quarterly, starting in the third quarter of 1999 and ending in the third quarter of 2011. We take into account the fact that the series are quite short, but we intentionally avoided using the earlier data on account of the severe financial crisis of 1998. By the third quarter of 1999, the impact of the financial crisis of

1998 on the Russian economy was reduced substantially. This allows us to consider our sample period (at least before 2008) as relatively homogenous both in terms of policy and hitting shocks. We are aware of the fact that the Bank of Russia changed its monetary policy after the financial crisis of 2008, but we do not restrict the sample intentionally to the end of 2008 to avoid making our time series even shorter. To convert nominal variables (consumption, output) to real terms, we use the GDP deflator. The series were seasonally adjusted with Census X12.

As an observable series for consumption, we use nominal private final consumption expenditures per capita. After seasonal adjustment, we take the logarithm of the series and detrend it linearly. The series of linearly detrended producer price inflation stands for an observable series of domestic inflation (π_t^d). The interest rate is assumed to correspond to the money market rate. The quarterly values were calculated by dividing the annual (detrended) interest rate (in percentage points) by 400. The series for wages was taken from the Rosstat database. The series is already seasonally adjusted, so we make no additional adjustment; we just take the logarithm and detrend the series linearly. For the real exchange rate series $(\mathcal{E}_t)$[5] we take the weighted average of EUR/RUR and USD/RUR exchange rate series. The weights are 0.45 and 0.55, respectively. These are the same weights that the Bank of Russia has used for calculating the currency basket (the operational benchmark for the exchange rate policy) since February 2007.[6] The series of real dollar and euro exchange rates were calculated on the basis of consumer price indices. Finally, we take the logarithm and detrend linearly the series of the exchange rate.

Next, we take per capita revenues from the export of crude oil, oil products, and natural gas to stand for the observable variable of commodity export. The data source on commodity exports is Balance of Payments statistics provided by the Bank of Russia. The series is expressed in terms of the bi-currency basket; we take the logarithm of the series and detrend it linearly.

All foreign variables are also expressed in terms of the bi-currency basket. The CPI inflation series for both the US and the euro area are combined to stand for the foreign inflation variable in the model. We use money market rates for the euro area and the US (federal funds rate) to calculate the series for the foreign interest rate. The annual series (in percentage points) are divided by 400. To calculate the series of observable output for the foreign economy, we use weighted per-capita GDP values. We seasonally adjust the series, take logarithms, and detrend it linearly. In our calculations, we consider 2005 as a base year, which does not affect the calculations, as for all series (except interest rates and inflation), we take the logarithm of the series and detrend them.

3.2 Priors

When choosing the prior distributions, we follow the common practice in the literature. We fix the same subset of parameters that is usually fixed in similar studies. In a Bayesian sense, this means that we assign zero variance of prior distribution, so we set the discount factor β at 0.99 and the depreciation rate at 0.025. We fix ψ (the capital elasticity of the production function) at 0.33, which reflects the scientific consensus that the ratio of labour to overall income is about two-thirds. As P_{ex} and P_{im} are determined by the same shock as P_d, we assume that θ_{ex} and θ_{im} are equal to θ_d, which is estimated. We tried to estimate the ratio of domestic goods in consumption (α_d), but the estimated value was too low and the whole convergence deteriorated, so we fix α_d at 0.74 according to calculations made in our previous studies (Malakhovskaya, 2013). Following Dam and Linaa (2005), we also fix the net wage mark-up and steady-state value of the net price mark-up process at 0.2. Our system includes the value of the steady-state of oil revenues, which is not known. To overcome the problem, we rewrite the system in terms of $\tilde{o} = \frac{\bar{O}}{P^{ex}Q^{ex}}$ and calibrate $\tilde{o}$ as the mean value of the ratio of commodity exports (crude oil, oil products, and natural gas) to non-commodity exports (all exports besides revenues from crude oil, oil products, and natural gas) over the sample period.[7]

We estimate 28 parameters in total, which are the parameters of preference, production function, and capital adjustment cost, as well as autocorrelation coefficients and standard errors that determine structural shocks. While choosing the prior distributions, we follow common rules: we assume beta distribution for all the parameters that can take only values between 0 and 1, we assume gamma distribution for all preference parameters, normal distribution for parameters of the monetary policy rule, and inverted gamma distribution for standard errors of structural shocks. When choosing moments of prior distributions, we follow Smets and Wouters (2003), Smets and Wouters (2007) and Dam and Linaa (2005) whenever it is possible. The fact that the moments of posterior distribution are different from those in cited papers means that the estimates are primarily determined by data and not by prior distributions. For other variables, the mean values of prior distributions were chosen to be consistent with our econometric estimates.

We follow existing studies in assuming that θ_d and θ_w have a beta distribution with a mean set at 0.75 and a standard deviation at 0.1 (for example, Smets and Wouters (2003)). This implies that prices and wages are reset about once a year.

We assume that the mean values of prior distributions for utility parameters (σ_1 and σ_2) are equal to 1 and 2, respectively, following Smets and Wouters (2003). We also assume that the habit persistence parameter fluctuates around 0.6, with a standard deviation equal to 0.1. We intentionally choose a smaller mean value than in Smets and Wouters (2003) and Smets and Wouters (2007) to make the prior distribution more symmetric because of a lack of any previous estimates of this parameter in the Russian data. Our prior distribution for the capital adjustment cost parameter corresponds to that in Dam and Linaa (2005), but contrary to Dam and Linaa (2005), we estimate the capital mobility parameter, and we set the mean value of its prior distribution at 0.002 following Lane and Milesi-Ferretti (2001). The parameters of the monetary policy rule are assumed to be normally distributed. The mean values of prior distributions generally correspond to a simple Taylor rule and are the same as in Smets and Wouters (2007), for instance, but we assume greater standard deviations than in existing papers because it allows us to admit a wider range of possible parameters for the rule. To determine the mean values of prior distributions for autocorrelation parameters for the commodity export revenues process (ρ_o) and all exogenous processes describing the foreign economy ($\rho_{y^f}, \rho_{i^f}, \rho_{y^f}$), we use regressions on our data. For these four parameters, we choose small standard deviations to make their distributions tight. As for all remaining autocorrelation coefficients, we assume that they have a beta distribution with a mean value set at 0.5 and a standard deviation set at 0.2, in accordance with Smets and Wouters (2007). All standard errors of exogenous process are assumed to follow an inverted gamma distribution. We choose the same mean value for all distributions except one. For σ_{i^f}, we take a smaller value because of the convergence problem.

3.3 Estimation results

We summarise our assumptions about the prior distributions and present our estimation results in Table 1.

Table 1 Prior and posterior distribution of parameters

	Prior distribution			*Posterior estimate*		*Posterior distribution*		
Parameter	*Type*	*Mean*	*Std. dev.*	*Mode*	*Std. error*	*10%*	*Median*	*90%*
θ_d	Beta	0.75	0.1	0.507	0.084	0.424	0.546	0.687
θ_w	Beta	0.75	0.1	0.398	0.065	0.359	0.442	0.523
σ_1	Gamma	1	0.3	1.015	0.25	0.819	1.126	1.521
σ_2	Gamma	2.0	0.6	1.74	0.513	1.275	1.884	2.68
ϕ	Gamma	15	4	10.57	4.44	6.87	12.05	18.23
ν	Beta	0.6	0.1	0.661	0.086	0.549	0.662	0.757
ω	Normal	0.002	0.001	0.0033	8.6×10^{-4}	0.0023	0.0034	0.0045
z_1	Beta	0.8	0.1	0.861	0.029	0.823	0.862	0.894
z_2	Normal	1.5	0.3	1.597	0.246	1.295	1.607	1.929
z_3	Normal	0.12	0.075	0.103	0.051	0.047	0.11	0.187
ρ_{yf}	Beta	0.94	0.01	0.943	0.01	0.929	0.942	0.954
$\rho_{\pi f}$	Beta	0.28	0.01	0.28	0.01	0.268	0.28	0.293
ρ_b	Beta	0.5	0.2	0.351	0.141	0.202	0.367	0.537
ρ_l	Beta	0.5	0.2	0.888	0.078	0.68	0.853	0.933
ρ_a	Beta	0.5	0.2	0.862	0.069	0.694	0.829	0.918
ρ_o	Beta	0.75	0.05	0.787	0.043	0.728	0.785	0.837
ρ_{if}	Beta	0.98	0.01	0.972	0.013	0.95	0.969	0.983
ρ_{rp}	Beta	0.5	0.2	0.741	0.065	0.623	0.719	0.799
σ_{ea}	Inv. gam.	0.05	Inf	0.032	0.011	0.025	0.04	0.085
σ_{if}	Inv. gam.	0.02	Inf	0.002	3.8×10^{-4}	0.0024	0.0026	0.0029
$\sigma_{\pi f}$	Inv. gam.	0.05	Inf	0.007	6.8×10^{-4}	0.006	0.007	0.008
σ_{yf}	Inv. gam.	0.05	Inf	0.009	8.4×10^{-4}	0.008	0.009	0.01
σ_{eb}	Inv. gam.	0.05	Inf	0.081	0.02	0.067	0.091	0.126
σ_{el}	Inv. gam.	0.05	Inf	0.257	0.078	0.236	0.358	0.539
σ_{ez}	Inv. gam.	0.05	Inf	0.012	0.002	0.01	0.012	0.014
σ_{eo}	Inv. gam.	0.05	Inf	0.13	0.013	0.117	0.132	0.15
σ_{er}	Inv. gam.	0.05	Inf	0.017	0.004	0.015	0.019	0.025
σ_{ev}	Inv. gam.	0.05	Inf	0.016	0.04	0.013	0.01	0.03

First of all, we present means and standard deviations of prior distributions. Then we show the mode and standard error of posterior distributions, which are estimated by a numerical minimisation method. The standard error is calculated on the basis of Hessian estimated at the mode of distribution. Finally, we present the median and 80% interval for each parameter. These values were estimated with the MCMC algorithm. The Metropolis-Hastings algorithm was implemented with 400,000 iterations with two chains. But the convergence was achieved earlier, which can be confirmed by Brooks and Gelman's procedure.[8]

All the estimates are significantly different from zero. For all prior and posterior distributions, see Appendix C. They confirm that the convergence is good. For all autocorrelation coefficients for structural shocks except two (ρ_b and ρ_{π_f}), the mode values are higher than 0.7. This validates the hypothesis about the high persistence of structural shocks.

In addition, our estimations of nominal rigidity parameters (θ_d) and (θ_w) do not contradict economic logic (about 0.5 and 0.4, respectively). This means that prices and wages are not very rigid, with contracts lasting about 5 months for wages and 8 months for prices. It is noteworthy that our estimates differ from the estimates of nominal rigidity parameters in other papers, where the level of nominal rigidity turned out to be unreasonably high. For example, in the paper by Dam and Linaa (2005), which is very close to ours with regard to the theoretical model, the estimate of the nominal rigidity parameter is 0.94, which means that contracts are not reset for about four years. The fact that our estimates of the nominal rigidity parameter are not too big allows us not to resort to inflation indexation in the Calvo mechanism, as in Christiano et al. (2005). Besides, in the paper by Dam and Linaa (2005), the authors received a very high value of mark-up volatility. Our estimate of this parameter is completely reasonable.

All the remaining parameters also take reasonable values. For example, the habit formation parameter is estimated to be 0.66. This value is higher than estimates in Smets and Wouters (2003) for the euro area (0.541) and in (Dam and Linaa, 2005) for Denmark (0.433). This fact can be interpreted as a higher inertia of consumption in Russia. The parameters of preferences (σ_1 and σ_2) also took the plausible values of 1.01 and 1.73, respectively. This means that the labour supply elasticity is about 0.6 and the intertemporal elasticity of substitution is unity. It is worth noting that our estimate of labour supply elasticity is less than values usually used to calibrate macroeconomic models, but it corresponds well to microeconometric estimates of this parameter (Peterman, 2012). All parameter estimates of the monetary policy rule are also in line with economic logic.

3.4 Impulse response analysis and historical decomposition

3.4.1 Impulse response analysis

After estimating the model, we analysed its properties with impulse response functions.

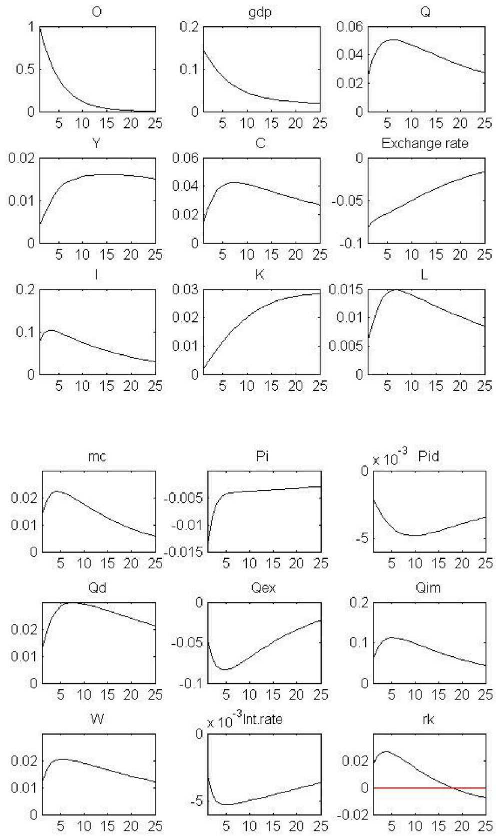
In Figure 1, we present the effects of a positive shock of commodity exports on the dynamics of the main macroeconomic variables.

The increase in households' income implies an increase in households' consumption and their demand in goods market. This is followed by an increase in labour demand, investments, capital, wages, the rate of return on capital, GDP, and domestic output (without oil). The rise in commodity export revenues results in a real appreciation of the exchange rate, encouraging non-commodity imports and discouraging non-commodity exports. In quantitative terms (in percentage points), a positive shock of commodity export revenues has the strongest influence on GDP, real exchange rate, investments, exports and imports. Domestic production changes positively, though only to a small extent, but the effect persists.

3.4.2 Historical decomposition

In this section, we investigate what the driving forces of the main macroeconomic variables in Russia are. The model can describe which shocks dominate the dynamics of all observed variables. Figures 2–4 show the historical contribution of all shocks to some variables of interest with columns of different patterns. In Figure 2, the historical decomposition of the detrended logarithm of consumption over the sample period is presented.

Figure 1 Oil price shock effect (see online version for colours)



It is noteworthy that, despite the fact that the level of openness of the Russian economy is rather high (primarily due to oil exports), the dynamics of consumption is explained, first of all, by domestic shocks. The shock of preferences and technology shocks are the most influential for consumption dynamics. As expected, the commodity export revenues shock is relatively important. This shock contributed to a great extent to consumption growth during the four years before the financial crisis of 2009.[9]

In Figure 3, we present the historical decomposition of the logarithm of the detrended real exchange rate. The figure shows that the commodity shock contributes more strongly to the RER error variance than to the consumption error variance. We pay attention to the fact that during the four years before the crisis, the commodity export shock contributed to real appreciation of the exchange rate. The figure also shows that the abrupt depreciation of the rouble in the first quarter of 2009 was caused by a sharp increase in risk premium, followed, consequently, by a small negative effect of commodity export shock. The monetary policy of the central bank probably helped to avoid even greater depreciation than could have taken place.

In Figure 4, the historical decomposition of a simulated series of GDP can be found. We simulate the series because we do not have it among our observable variables. Figure 4 shows that the commodity shock contributes much to GDP dynamics over the sample period. It is noteworthy that the commodity export shock explains the output growth before the financial crisis. The figure also shows that the output decrease in 2009 was caused by the joint pressure of negative commodity export shock and restrictive monetary policy (interest rate increase).

Figure 2 Historical decomposition of consumption

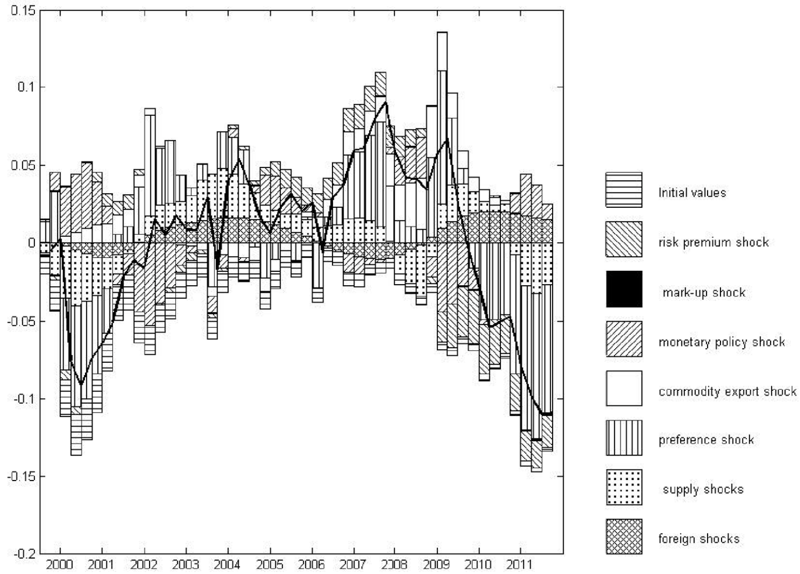


Figure 3 Historical decomposition of real exchange rate

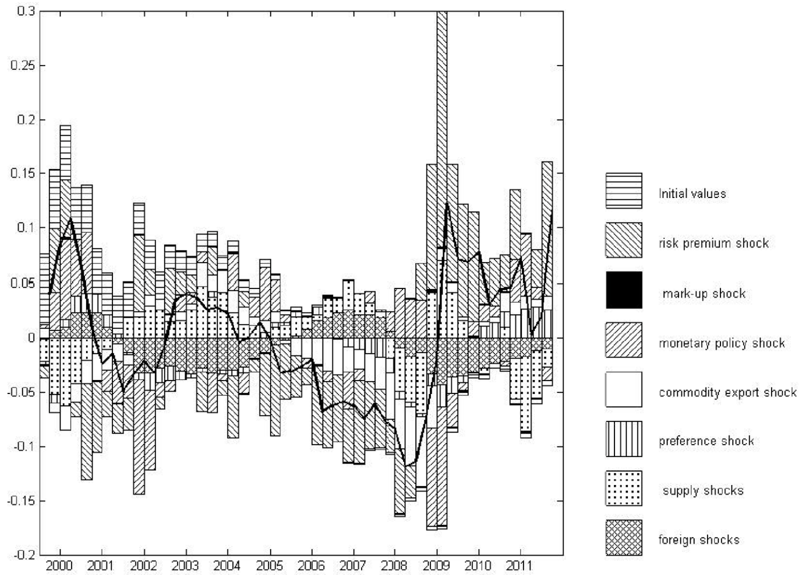
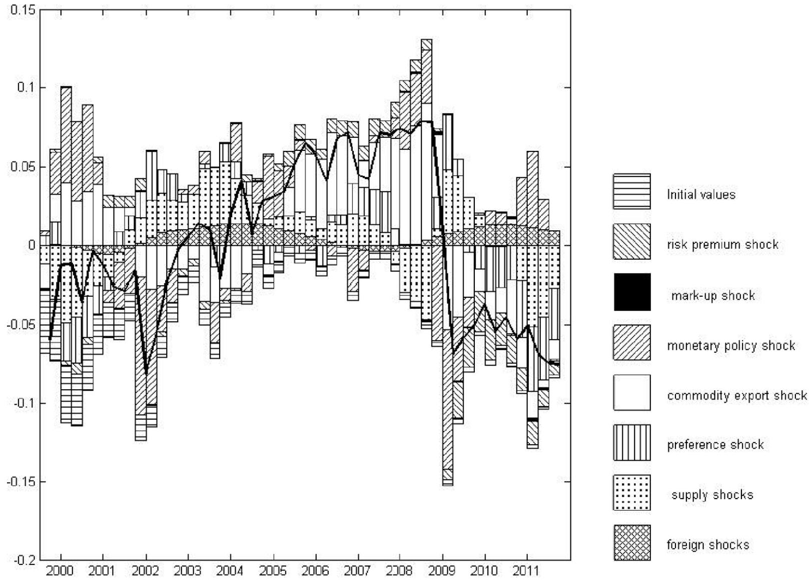


Figure 4 Historical decomposition of GDP (simulated series)



3.4.3 Forecast error variance decomposition

Table 2 shows a variance decomposition of forecast error at various horizons: in the short run (1 year), medium run (3 years), and long run (20 years). This allows us to come to several conclusions. First of all, technological and labour supply innovations explain a large part of all measures of output in the model, including GDP, output of final goods (without oil), and output of intermediate goods and at all horizons. Supply shocks account for 68% of the error variance of intermediate goods output in the short run and for more than 80% in the medium and long run. The part of the error variance of the final goods output explained by supply shocks is 43% in the short run and more than 60% in the medium and long run. The part of GDP error variance determined by supply shocks varies from 50% to 70% at various horizons. This result confirms the conclusions of a baseline RBC model in which a business cycle is driven primarily by a technological shock. The result is also in line with structural VAR models with long-run restrictions in which the output is determined by a supply shock in the long run (Blanchard and Quah, 1989).

However, contrary to the identified VAR literature, the monetary policy shock is a source of macroeconomic volatility at all horizons. In the short run, the monetary policy shock accounts for 18.5% of the error variance of consumption, 38% of the error variance of GDP, 18.7% of the real exchange rate error variance, and 38.5% of the error variance of CPI inflation. In the long run, the importance of the monetary policy shock as a driving factor of an economy decreases, yet still remains significant. For example, monetary policy accounts for 17% of the GDP error variance at the 20-year horizon. The result is not a surprise: monetary policy explains even a larger part of long-term output error variance in the euro area (Smets and Wouters, 2003).

The preference shock is a primary driving force of consumption volatility in the short run, as it explains 45% of the error variance at the one-year horizon. In the medium and long run, consumption is driven mostly by supply shocks, like all measures of output.

Table 2 Forward error variance decomposition

Shock	*C*	*Y*	*GDP*	*Q*	*Qd*	*Qex*	*Qim*	ε	*Pi*	*W*
				1 year						
Preference	45.3	3.9	7.3	10.2	5.8	2.2	17.3	1.1	2.1	0.7
Labour supply	11	26.6	19.3	16.7	23.8	24.1	0.4	11.2	19.8	71.5
Commodity export	1.9	0.1	23.8	2.2	0.5	4.9	10.5	4.2	0.4	1.0
Technology	17.7	42.5	31.1	27.4	38.3	36.5	0.4	16.4	32.3	20.8
Monetary policy	18.5	26.5	38.0	34.0	29.8	2.1	18.6	18.7	38.5	0.1
Price mark-up	0.2	0.3	0.4	0.3	0.2	0.2	0.4	0.2	1.4	0.8
Risk premium	2.7	0.1	1.1	5.0	0.7	17.7	31.5	37.0	3.9	3.3
Foreign output	0	0	0.1	0	0	2.1	0.2	0.2	0	0
Foreign interest rate	2.7	0.1	1.6	4.1	0.8	10.1	20.8	11.0	1.6	1.9
Foreign inflation	0	0	0	0	0	0.1	0	0.1	0	0
				3 years						
Preference	22.0	2.0	4.0	5.5	3.0	1.2	11.3	0.8	2.0	0.5
Labour supply	28.6	42.3	35.0	31.7	39.7	34.2	0.6	20.9	22.3	71.9
Commodity export	4.8	0.3	19.0	4.4	1.1	7.5	22.7	6.3	0.6	1.7
Technology	27.3	41.8	34.4	31.2	39.2	34.0	0.5	20.5	31.9	21.0
Monetary policy	10.5	13.0	20.0	17.9	14.9	1.4	11.7	13.4	36.5	0.1
Price mark-up	0.1	0.1	0.2	0.2	0.2	0.1	0.3	0.1	1.3	0.4
Risk premium	1.9	0.1	1.1	3.1	0.5	9.9	22.7	26.8	3.8	1.9
Foreign output	0	0	0.1	0	0	1.7	0.5	0.3	0	0
Foreign interest rate	5.0	0.4	2.9	5.9	1.5	10.0	29.8	10.9	1.7	2.3
Foreign inflation	0	0	0	0	0	0.1	0	0.1	0	0
				20 years						
Preference	17.1	2.0	3.8	5.1	2.9	0.9	8.8	0.7	2.1	0.9
Labour supply	31.9	45.4	37.3	33.2	42.5	35.1	0.5	23.4	23.3	68.2
Commodity export	8.5	1.2	18.5	7.3	2.5	7.7	25.7	6.6	0.9	3.4
Technology	25.7	39.4	32.3	28.7	36.9	30.6	0.5	20.3	31.6	21.6
Monetary policy	7.9	10.9	16.6	14.7	12.4	1.4	8.4	11.5	35.3	0.2
Price mark-up	0.1	0.1	0.2	0.1	0.1	0.1	0.2	0.1	1.2	0.4
Risk premium	2.5	0.2	1.5	3.7	0.7	9.9	22.0	24.2	3.7	2.1
Foreign output	0.1	0.1	0.3	0.1	0	1.6	0.8	0.3	0	0.1
Foreign interest rate	6.3	0.8	3.8	7.2	2.0	12.7	33.2	12.7	1.8	3.1
Foreign inflation	0	0	0	0	0	0.1	0	0	0	0

The commodity export shock contributes much to GDP and import volatility at all horizons. It accounts for 23.8% of the error variance of GDP in the short run and about 19% in the long run. The portion of import volatility explained by the commodity export shock varies from 10.5% to 25.7%. It is noteworthy that the commodity export shock is not an important source of volatility of non-commodity output. The real appreciation induced by a positive commodity export shock increases imports and decreases exports. It is the reason why the consumption growth following a positive commodity export shock does not affect the intermediate goods output (the portion of error variance explained by the commodity export shock is close to zero at all horizons). Thus, the model shows symptoms of the Dutch disease in Russia at least before 2012.

The risk premium shock is the most important source of volatility of the real exchange rate in the short-run, accounting for 31.5% of the error variance, and, along with supply shocks, contributes significantly to the RER variance in the long run.

Contrary to existing literature, we do find that the mark-up shock explains a large part of the error variance of inflation. Vice versa, the impact of the price mark-up shock on all variables in the model, including prices, is not significant. It would be interesting to verify if this result is robust in the the case of another monetary policy rule or model setup. We leave this question for our future research.

Therefore, although Russia is an open economy, our results show that the fluctuations of macroeconomic variables are determined primarily by domestic shocks. Domestically based shocks account for 88% and 81% of the error variance of final goods output in the short run and long run, respectively. The only measure of economic activity that shows a considerable dependence on commodity dynamics is GDP because it explicitly accounts for export revenues. This result has some implications for macroeconomic policy. In the paper, we do not discuss an optimal monetary policy issue, but it does seem reasonable for policy makers to switch to inflation targeting in the near future, as the Central Bank of Russia promised to do by 2015.

4 Conclusion

In this paper, we constructed a DSGE model for an economy with commodity exports. The parameters of the model were estimated using Bayesian techniques on Russian data. Our principal goal was to identify the contribution of structural shocks to the business cycle fluctuations in an economy with commodity exports. Our main interest was the quantitative estimate of the impact of the commodity export shock on macroeconomic volatility in Russia. However, the model is general and may be estimated or calibrated for any export-oriented economy.

The paper is also an important step toward a general equilibrium model suitable for policy analysis and for forecasting similar models that are currently in use by central banks in many countries.

Our model yields plausible estimates, and the impulse response functions are in line with empirical evidence. We made a historical decomposition of two observed time series (consumption and real exchange rate) and one simulated time series to identify which shocks were the most influential in any particular quarter. It is interesting to note that the financial crisis of 2009 in Russia is captured by the model as a joint influence of risk premium shock and commodity export shock, which seems reasonable.

Finally, we determine the contribution of structural shocks to forecast error variance of endogenous variables in the short, medium, and long run.

Our results show that non-commodity output both for final and intermediate goods is determined by domestic demand (monetary policy) and supply shocks (shock of technology and labour supply shock) at all horizons. The commodity export shock does not contribute much to non-commodity output volatility, accounting for only 7.3% of the error variance of final goods output at the 20-year horizon. The likely reason is that the positive commodity shock results in real exchange rate appreciation, thereby decreasing exports and increasing imports. The commodity revenues shock accounts for up to 7.73% of the error variance of non-commodity exports and up to 25.71% of the error variance of imports in the long run. So the model shows the symptoms of the Dutch disease in Russia at least before 2012. However, commodity export revenues shock does contribute much to GDP, since GDP explicitly accounts for all export revenues. The shock accounts for 24% of the error variance of GDP in the short run and about 19% in the medium and long run. Consumption is driven primarily by preference shock in short run and by supply shocks in medium and long run. The most influential shocks for the real exchange rate are risk premium shock (at all horizons), monetary policy shock (in the short run) and supply shocks (in the medium and long run).

Our main conclusion is the following: in spite of a strong impact by commodity export shock on GDP, the business cycle in Russia is mostly domestically based. Although we do not explicitly consider an optimal monetary policy issue in the paper, the conclusion implies that it is reasonable for policy makers to switch to inflation targeting as the Central Bank of Russia is supposed to do by 2015.

We admit that our model may underestimate the impact of commodity exports on a domestic economy for two reasons. First, we do not split public and private consumption, so we do not account for an increase in government spending when the situation in the oil market is favourable. This could be crucial in the case of a higher propensity to spend in the public sector than in the private one. Second, the model is stationary and cannot account for permanent shocks. In this paper, we leave aside these possible extensions for computational reasons. Elaborating these issues is left for future research.

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Appendix A: Normalisation

$$p_t^d = \frac{P_t^d}{P_t} \qquad p_t^{ex} = \frac{P_t^{ex}}{P_t^f} \qquad p_t^{im} = \frac{P_t^{im}}{P_t}$$

$$r_t^K = \frac{R_t^K}{P_t} \qquad w_t = \frac{W_t}{P_t^f} \qquad mc_t = \frac{MC_t}{P_t}$$

$$\pi_t = \frac{P_t}{P_{t-1}} \qquad \pi_t^d = \frac{P_t^d}{P_{t-1}^d} \qquad \pi_t^f = \frac{P_t^f}{P_{t-1}^f}$$

$$\widetilde{p}_t^d = \frac{\widetilde{P}_t^d}{P_t} \qquad \widetilde{p}_t^{ex} = \frac{\widetilde{P}_t^{ex}}{P_t^f} \qquad \widetilde{p}_t^{im} = \frac{\widetilde{P}_t^{im}}{P_t}$$

$$\widetilde{w}_t = \frac{\widetilde{W}_t}{P_t} \qquad \mathcal{E}_t = \frac{S_t P_t^f}{P_t} \qquad d_t^f = \frac{D_t^f}{P_t^f}$$

$$o_t = \frac{O_t}{P_t^f}$$

Appendix B: Steady-state derivation

$$p^d = (1+\upsilon)mc$$
$$p^{ex} = (1+\upsilon)\frac{mc}{\mathcal{E}}$$
$$p^{im} = (1+\upsilon)\mathcal{E}$$
$$p^d = (1+\upsilon)mc$$
$$p^{ex} = (1+\upsilon)\frac{mc}{\mathcal{E}}$$
$$1+i = \frac{1}{\beta}$$
$$1+i^* = \frac{1}{\beta}$$
$$\big(C(1-\nu)\big)^{-\sigma_1} = \mu$$
$$r^K = \frac{1}{\beta} - (1-\delta)$$
$$w = (1+\gamma)\big(C^{\sigma_1}(1-\nu)^{\sigma_1}\big)L^{\sigma_2}$$
$$A = 1$$
$$\begin{aligned} mc &= w^{1-\psi}\left(r^K\right)^{\psi}\psi^{-\psi}(1-\psi)^{\psi-1} \\ &= \left((1+\gamma)\left(C^{\sigma_1}(1-\nu)^{\sigma_1}\right)L^{\sigma_2}\right)^{1-\psi}\left(r^K\right)^{\psi}\psi^{-\psi}(1-\psi)^{\psi-1} \\ &= (1+\gamma)^{1-\psi}\left(\big(C^{\sigma_1}(1-\nu)^{\sigma_1}\big)L^{\sigma_2}\right)^{1-\psi}\left(r^K\right)^{\psi}\psi^{-\psi}(1-\psi)^{\psi-1} \end{aligned}$$
$$1 = \left(p^d\right)\left(p^{im}\right)^{\alpha_{im}} = ((1+\upsilon)mc)^{\alpha_d}((1+\upsilon)\mathcal{E})^{\alpha_{im}}$$

$$\mathcal{E} = (1+\upsilon)^{-\frac{1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}$$
$$p^{ex} = (1+\upsilon)\frac{mc}{\mathcal{E}} = \frac{(1+\upsilon)mc}{(1+\upsilon)^{-\frac{1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}} = (1+\upsilon)^{\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{\frac{1}{\alpha_{im}}}$$
$$p^{im} = (1+\upsilon)\mathcal{E} = (1+\upsilon)^{1-\frac{1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}} = (1+\upsilon)^{-\frac{\alpha_d}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}$$

The current account equilibrium imply that:

$$p^{ex}Q^{ex} + o = \frac{p^{im}}{\mathcal{E}}Q^{im}$$
$$p^{ex}Q^{ex} = \alpha_{ex}Y^f$$
$$\alpha_{ex}Y_f + o = (1+\upsilon)Q^{im}$$
$$Q^{im} = \frac{\alpha_{ex}Y^f + o}{1+\upsilon}$$
$$Q^{im} = \alpha_{im}\frac{Q}{p^{im}}$$
$$Q = \frac{Q^{im}p^{im}}{\alpha_{im}} = \frac{\alpha_{ex}Y^f + o}{1+\upsilon}\cdot\frac{(1+\upsilon)\mathcal{E}}{\alpha_{im}}$$
$$= \frac{1}{\alpha_{im}}(\alpha_{ex}Y^f + o)(1+\upsilon)^{-\frac{1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}$$
$$\frac{Q^d}{Q^{im}} = \frac{\alpha_d p_{im}}{\alpha_{im} p_d} = \frac{\frac{\alpha_d}{\alpha_{im}}(1+\upsilon)^{-\frac{\alpha_d}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}}{(1+\upsilon)mc} = \frac{\alpha_d}{\alpha_{im}}(1+\upsilon)^{-\frac{1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}$$
$$Q^d = \frac{\alpha_{ex}Y^f + o}{1+\upsilon}\cdot\frac{\alpha_d}{\alpha_{im}}(1+\upsilon)^{-\frac{1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}$$
$$= (\alpha_{ex}Y^f + o)\frac{\alpha_d}{\alpha_{im}}(1+\upsilon)^{-1-\frac{1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}$$
$$Q^{ex} = \alpha_{ex}Y^f(p^{ex})^{-1} = \alpha_{ex}Y^f\left((1+\upsilon)^{\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{\frac{1}{\alpha_{im}}}\right)^{-1}$$
$$= \alpha_{ex}Y^f(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}$$
$$Y = Q^d + Q^{ex} = (\alpha_{ex}Y^f + o)\frac{\alpha_d}{\alpha_{im}}(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}$$
$$+\alpha_{ex}Y^f(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}$$
$$= \left(\alpha_{ex}Y^f + \frac{\alpha_d}{\alpha_{im}}\left(\alpha_{ex}Y^f + o\right)\right)(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}$$
$$= \alpha(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}$$

where $\alpha = \alpha_{ex}Y^f + \frac{\alpha_d}{\alpha_{im}}(\alpha_{ex}Y^f + o)$

Let us turn to labour and capital

$$K = Y\left(\frac{\psi}{1-\psi}\cdot\frac{w}{r^K}\right)^{1-\psi}$$

$$= \alpha(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}} \left(\frac{\psi}{1-\psi} \cdot \frac{(1+\gamma)\left(C^{\sigma_1}(1-\nu)^{\sigma_1}\right)L^{\sigma_2}}{r^K}\right)^{1-\psi}$$

We know that

$$mc = (1+\gamma)^{1-\psi}\left(C^{\sigma_1}(1-\nu)^{\sigma_1}L^{\sigma_2}\right)^{1-\psi}(r^K)^{\psi}\psi^{-\psi}(1-\psi)^{\psi-1}$$

so

$$(1+\gamma)^{1-\psi}\left(\left(C^{\sigma_1}(1-\nu)^{\sigma_1}\right)L^{\sigma_2}\right)^{1-\psi} = mc\big(r^K\big)^{-\psi}\psi^{\psi}(1-\psi)^{1-\psi} \Rightarrow$$

$$\begin{aligned}
K &= \alpha(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}\psi^{1-\psi}(1-\psi)^{\psi-1}\big(r^K\big)^{\psi-1} \\
&\quad \times(1+\gamma)^{1-\psi}\big(C^{\sigma_1}(1-\nu)^{\sigma_1}L^{\sigma_2}\big)^{1-\psi} \\
&= \alpha(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha}{}_{im}} mc^{-\frac{1}{\alpha_{im}}}\psi^{1-\psi}(1-\psi)^{\psi-1}\big(r^K\big)^{\psi-1}mc\big(r^K\big)^{-\psi} \\
&\quad \times\psi^{\psi}(1-\psi)^{1-\psi} \\
&= \alpha\psi(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}(r^K)^{-1} \\
L &= K\frac{1-\psi}{\psi}\cdot\frac{r^K}{w} = (1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}\frac{\alpha(1-\psi)}{(1+\gamma)(C^{\sigma_1}(1-\nu)^{\sigma_1}L^{\sigma_2}}.
\end{aligned}$$

We know that:

$$\begin{aligned}
&(1+\gamma)C^{\sigma_1}(1-\nu)^{\sigma_1}L^{\sigma_2} = mc^{\frac{1}{1-\psi}}r^{K^{-\frac{\psi}{1-\psi}}}\psi^{\frac{\psi}{1-\psi}}(1-\psi) \Rightarrow \\
&L = \alpha(1-\psi)(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}\frac{1}{mc^{\frac{1}{1-\psi}}r^{K^{-\frac{\psi}{1-\psi}}}\psi^{\frac{\psi}{1-\psi}}(1-\psi)} = \\
&\quad = \alpha(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}-\frac{1}{1-\psi}}\left(r^K\right)^{\frac{\psi}{1-\psi}}\psi^{-\frac{\psi}{1-\psi}} \\
&Y = K^{\psi}L^{1-\psi} \\
&\quad = \alpha\left(\psi(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}}(r^K)^{-1}\right)^{\psi} \\
&\quad\quad \times\left((1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}-\frac{1}{1-\psi}}(r^K)^{\frac{\psi}{1-\psi}}\psi^{-\frac{\psi}{1-\psi}}\right)^{1-\psi} \\
&\quad = \alpha(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{1}{\alpha_{im}}}.
\end{aligned}$$

Now we can determine steady state of consumption:

$$\begin{aligned}
C &= (1-\nu)^{-1}(1+\gamma)^{-\frac{1}{\sigma_1}} mc^{\frac{1}{(1-\psi)\sigma_1}}\left(r^K\right)^{-\frac{\psi}{1-\psi}\sigma_1}\psi^{\frac{\psi}{(1-\psi)\sigma_1}}(1-\psi)^{\frac{1}{\sigma_1}}L^{-\frac{\sigma_2}{\sigma_1}} \\
&= (1-\nu)^{-1}(1+\gamma)^{-\frac{1}{\sigma_1}} mc^{\frac{1}{(1-\psi)\sigma_1}}\left(r^K\right)^{-\frac{\psi}{(1-\psi)\sigma_1}}\psi^{\frac{\psi}{(1-\psi)\sigma_1}}(1-\psi)^{\frac{1}{\sigma_1}} \\
&\quad \times\left(\alpha(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}-\frac{1}{1-\psi}} r^{K^{\frac{\psi}{1-\psi}}}\psi^{-\frac{\psi}{1-\psi}}\right)^{\frac{-\sigma_2}{\sigma_1}} \\
&= (1-\nu)^{-1}(1+\gamma)^{-\frac{1}{\sigma_1}} mc^{\frac{1}{(1-\psi)\sigma_1}}\left(r^K\right)^{-\frac{\psi}{(1-\psi)\sigma_1}}\psi^{\frac{\psi}{(1-\psi)\sigma_1}}(1-\psi)^{\frac{1}{\sigma_1}} \\
&\quad \times\alpha^{-\frac{\sigma_2}{\sigma_1}}(1+\upsilon)^{(\frac{\alpha_{im}+1}{\alpha_{im}})\cdot\frac{\sigma_2}{\sigma_1}} mc^{\left(\frac{\alpha_d}{\alpha_{im}}+\frac{1}{1-\psi}\right)\cdot\frac{-\sigma_2}{\sigma_1}} r^{K^{(\frac{\psi}{1-\psi})\cdot\frac{(-\sigma_2)}{\sigma_1}}}\psi^{\frac{\psi}{1-\psi}\cdot\frac{\sigma_2}{\sigma_1}} = \\
&= (1-\nu)^{-1}(1+\gamma)^{-\frac{1}{\sigma_1}}(1-\psi)^{\frac{1}{\sigma_1}}\psi^{\frac{\psi(1+\sigma_2)}{(1-\psi)\sigma_1}}\left(r^K\right)^{-\frac{\psi(1+\sigma_2)}{(1-\psi)\sigma_1}} \\
&\quad \times\alpha^{-\frac{\sigma_2}{\sigma_1}}(1+\upsilon)^{\frac{\alpha_{im}+1}{\alpha_{im}}\cdot\frac{\sigma_2}{\sigma_1}} mc^{\frac{\alpha_d\sigma_2}{\alpha_{im}\sigma_1}+\frac{1+\sigma_2}{(1-\psi)\sigma_1}}.
\end{aligned}$$

Steady-state of investment:

$$I = \delta K = \alpha\delta\psi(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}} mc^{-\frac{\alpha_d}{\alpha_{im}}} (r^K)^{-1}.$$

Therefore, equation $Q = C + I$ takes the following form

$$\Upsilon_Q mc^{-\frac{\alpha_d}{\alpha_{im}}} = \Upsilon_C mc^{\frac{\alpha_d \sigma_2}{\alpha_{im}\sigma_1} + \frac{1+\sigma_2}{(1-\psi)\sigma_1}} + \Upsilon_I mc^{-\frac{\alpha_d}{\alpha_{im}}}$$

where

$$\begin{aligned}
\Upsilon_Q &= \frac{1}{\alpha_{im}}(\alpha_{ex}Y^f + o)(1+\upsilon)^{-\frac{1}{\alpha_{im}}} \\
\Upsilon_C &= (1-\nu)^{-1}(1+\gamma)^{-\frac{1}{\sigma_1}}(1-\psi)^{\frac{1}{\sigma_1}}\psi^{\frac{\psi(1+\sigma_2)}{(1-\psi)\sigma_1}} \\
&\quad \times (r^K)^{-\frac{\psi(1+\sigma_2)}{(1-\psi)\sigma_1}}\alpha^{-\frac{\sigma_2}{\sigma_1}}(1+\upsilon)^{\frac{\alpha_{im}+1}{\alpha_{im}}\cdot\frac{\sigma_2}{\sigma_1}} \\
\Upsilon_I &= \alpha\delta\psi(1+\upsilon)^{-\frac{\alpha_{im}+1}{\alpha_{im}}}(r^K)^{-1}.
\end{aligned}$$

We solve the equation $Q = C + I$:

$$\begin{aligned}
\Upsilon_C mc^{\frac{\alpha_d\sigma_2}{\alpha_{im}\sigma_1}+\frac{1+\sigma_2}{(1-\psi)\sigma_1}+\frac{\alpha_d}{\alpha_{im}}} &= \Upsilon_Q - \Upsilon_I \\
mc^{\frac{\alpha_d\sigma_2}{\alpha_{im}\sigma_1}+\frac{1+\sigma_2}{(1-\psi)\sigma_1}+\frac{\alpha_d}{\alpha_{im}}} &= \frac{\Upsilon_Q - \Upsilon_I}{\Upsilon_C} \\
mc &= \left(\frac{\Upsilon_Q - \Upsilon_I}{\Upsilon_C}\right)^{\left(\frac{\alpha_d\sigma_2}{\alpha_{im}\sigma_1}+\frac{1+\sigma_2}{(1-\psi)\sigma_1}+\frac{\alpha_d}{\alpha_{im}}\right)^{-1}} \\
&= \left(\frac{\Upsilon_Q - \Upsilon_I}{\Upsilon_C}\right)^{\frac{(1-\psi)\sigma_1\alpha_{im}}{\alpha_{im}+\sigma_2+\alpha_d\sigma_1-\alpha_d\psi\sigma_1-\alpha_d\psi\sigma_2}}
\end{aligned}$$

Appendix C: Log-linearised model

$$\widehat{Q}_t^d = \widehat{Q}_t - \widehat{p}_d \tag{71}$$
$$\widehat{Q}_t^{im} = \widehat{Q}_t - \widehat{p}_{im} \tag{72}$$
$$\widehat{Q}_t^{ex} = \widehat{Y}_t^f - \widehat{p}_{ex} \tag{73}$$
$$\alpha_d\widehat{p}_t^d + \alpha_{im}\widehat{p}_t^{im} = 0 \tag{74}$$
$$\widehat{L}_t = \widehat{r}_t^K - \widehat{w}_t + \widehat{K}_t \tag{75}$$
$$\widehat{K}_t = -\widehat{A}_t + (1-\psi)\left(\widehat{w}_t - \widehat{r}_t^K\right) + \widehat{Y}_t \tag{76}$$
$$\widehat{m}c = -\widehat{A}_t + (1-\psi)\widehat{w}_t + \psi\widehat{r}_t^K \tag{77}$$
$$\widehat{p}_t^d - \theta_d\widehat{p}_{t-1}^d + \theta_d\widehat{\pi}_t = (1-\theta_d)(1-\beta\theta_d)\widehat{m}c_t + \beta\theta_d E_t(\widehat{p}_{t+1}^d - \theta_d\widehat{p}_t^d + \widehat{\pi}_{t+1}) \tag{78}$$
$$\begin{aligned}
\widehat{p}_t^{ex} - \theta_{ex}\widehat{p}_{t-1}^{ex} + \theta_{ex}\widehat{\pi}_t^f &= (1-\theta_{ex})(1-\beta\theta_{ex})(\widehat{m}c_t - \widehat{\mathcal{E}}_t) \\
&\quad + \beta\theta_{ex}E_t(\widehat{p}_{t+1}^{ex} - \theta_{ex}\widehat{p}_t^{ex} + \widehat{\pi}_{t+1}^f)
\end{aligned} \tag{79}$$
$$\widehat{p}_t^{im} - \theta_{im}\widehat{p}_{t-1}^{im} + \theta_{im}\widehat{\pi}_t = (1-\theta_{im})(1-\beta\theta_{im})\widehat{\mathcal{E}}_t$$

$$+\beta\theta_{ex}E_t(\widehat{p}^{im}_{t+1} - \theta_{im}\widehat{p}^{im}_t + \widehat{\pi}_{t+1}) \tag{80}$$

$$\widehat{w}_t - \theta_w(\widehat{w}_{t-1} - \widehat{\pi}_t) = \frac{(1-\theta_w)(1-\beta\theta_w)\gamma}{(1+\gamma)\sigma_2+\gamma}(\widehat{\epsilon}^l_t$$
$$+\frac{1+\gamma}{\gamma}\sigma_2\widehat{w}_t + \sigma_2\widehat{L}_t + \frac{\sigma_1}{1-\nu}(\widehat{C}_t - \nu\widehat{C}_{t-1}))$$
$$+\beta\theta_w(\widehat{w}_{t+1} - \theta_w(\widehat{w}_t - \widehat{\pi}_{t+1})) + \beta\theta_w E_t(1-\theta_w)\widehat{\pi}_{t+1} \tag{81}$$

$$U_{c,t} = \widehat{\epsilon}^b_t - \frac{\sigma_1}{1-\nu}(\widehat{C}_t - \nu\widehat{C}_{t-1}) \tag{82}$$

$$E_t(\widehat{U}_{c,t+1} - \widehat{\pi}_{t+1}) + \widehat{\imath} = \widehat{U}_{c,t} \tag{83}$$

$$E_t(\widehat{U}_{c,t+1} + \widehat{\mathcal{E}}_{t+1} - \pi^f_{t+1}) + \widehat{\imath}^* = \widehat{U}_{c,t} + \widehat{\mathcal{E}}_t \tag{84}$$

$$\widehat{U}_{c,t} + \phi(\widehat{K}_{t+1} - \widehat{K}_t) = E_t(\widehat{U}_{c,t+1} + \beta r^K \widehat{r}^K_{t+1} + \beta\phi\widehat{K}_{t+2} - \beta\phi\widehat{K}_{t+1}) \tag{85}$$

$$\widehat{K}_{t+1} = (1-\delta)\widehat{K}_t + \delta \tag{86}$$

$$\widehat{I}_t\widehat{\rho}_t = -\frac{\omega P^f}{P^{ex}Qex}\widehat{d}^f_t + \widehat{\epsilon}_\rho \tag{87}$$

$$\widehat{\imath} = z_1\widehat{\imath}_{t-1} + (1-z_1)z_2\widehat{\pi}_t + (1-z_1)z_3\widehat{Y}_t + \epsilon_z; \tag{88}$$

$$\widehat{Y}_t = \frac{Q^d}{Y}\widehat{Q}^d_t + \frac{Q^{ex}}{Y}\widehat{Q}^{ex}_t \tag{89}$$

$$\widehat{Q}_t = \frac{C}{Q}\widehat{C}_t + \frac{I}{Q}\widehat{I}_t \tag{90}$$

$$p^{ex}Q^{ex}(\widehat{p}^{ex}_t + \widehat{Q}^{ex}_t) + o\widehat{o}_t - \frac{p^{im}Q^{im}}{\mathcal{E}}(\widehat{p}^{im}_t + \widehat{Q}^{im}_t - \widehat{\mathcal{E}}_t) - \widehat{d}^f_t + \widehat{\imath}^*\widehat{d}^f_{t-1} = 0 \tag{91}$$

$$\widehat{gdp_t} = \widehat{Q}_t + \frac{\mathcal{E}p^{ex}Q^{ex}}{Q}(\widehat{\mathcal{E}}_t + \widehat{p}^{ex}_t + \widehat{Q}^{ex}_t) + \frac{\mathcal{E}o}{Q}(\widehat{\mathcal{E}}_t + \widehat{7}o_t)$$
$$-\frac{p^{im}Q^{im}}{Q}(\widehat{p}^{im}_t + \widehat{Q}^{im}_t) \tag{92}$$

$$\widehat{\imath}^* = \widehat{\imath}^f_t + \widehat{\rho}_t \tag{93}$$

$$\widehat{\pi}_d = \widehat{p}^d_t - \widehat{p}^d_{t-1} + \widehat{\pi} \tag{94}$$

$$\widehat{\epsilon}^b_t = \rho_b\widehat{\epsilon}^b_{t-1} + \varepsilon^b_t \tag{95}$$

$$\widehat{\epsilon}^l_t = \rho_l\widehat{\epsilon}^l_{t-1} + \varepsilon^l_t \tag{96}$$

$$\widehat{A}_t = \rho_a\widehat{A}_{t-1} + \varepsilon^A_t \tag{97}$$

$$\widehat{\imath}^f_t = \rho_{if}\widehat{\imath}^f_{t-1} + \varepsilon^{if}_t \tag{98}$$

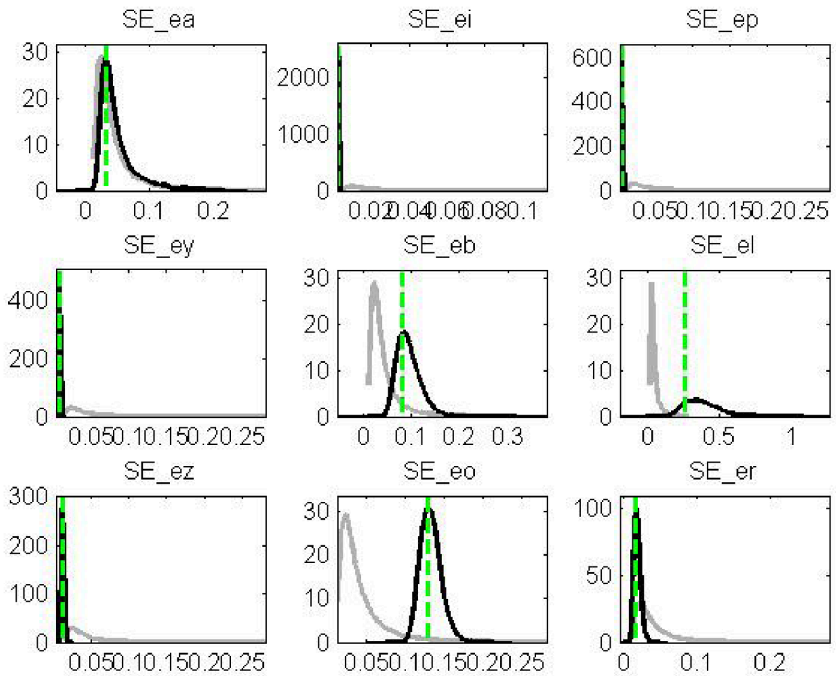
$$\widehat{\pi}^f_t = \rho_{\pi_f}\widehat{\pi}^f_{t-1} + \varepsilon^{\pi_f}_t \tag{99}$$

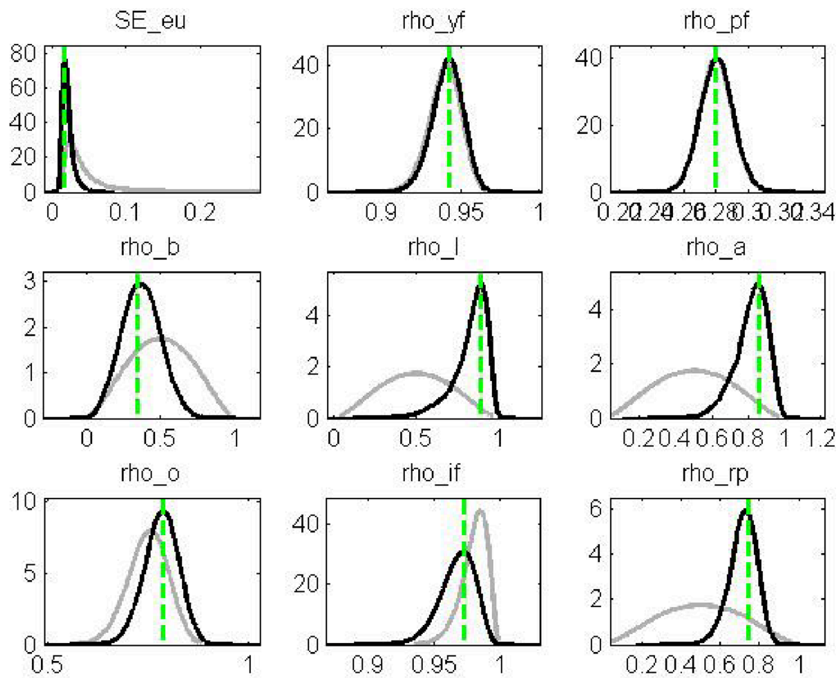
$$\widehat{Y}^f_t = \rho_{Yf}\widehat{Y}^f_{t-1} + \varepsilon^{Y_f}_t \tag{100}$$

$$\widehat{o}_t = \rho_o\widehat{o}_{t-1} + \varepsilon^o_t \tag{101}$$

$$\widehat{\epsilon}_{\rho,t} = \rho_\rho\widehat{\epsilon}_{\rho,t-1} + \varepsilon^\rho_t \tag{102}$$

Appendix D: Priors and posteriors (see online version for colours)





Appendix D: Priors and posteriors (see online version for colours) (continued)