# PROTOTYPE-BASED CATEGORIZATION OF STRUCTURAL COMPLEXITY ESTIMATES OF SIMPLICIAL COMPLEXES

Konstantin Y. Degtiarev

Faculty of Business Informatics, School of Software Engineering The National Research University – Higher School of Economics, Moscow / Russia
e-mail: kdegtiarev@hse.ru

**Keywords:** simplex, simplicial complex, structural complexity (connectivity), Q-analysis, proximity, concepts (categories), prototype of category, extended point (F-point), extended line, categorization

## 1. Introduction

Growing interest in problems related to the field of man-made and natural complex systems led, in particular, to the need for application of formal approaches to study structural features of systems and obtain estimates of their *complexity*. It should be stressed that the terms «complexity» and «structure» are widely used and interpreted differently in various fields of scientific and practical activities. The material of the present paper is grounded on the holist algebraic method (*Q-analysis*) proposed by English mathematician and physicist R.H.Atkin [3,4,5]. At its core, the approach is aimed at both analysis of systems structures and calculation of numeric estimates of structural complexity of systems based on the results of such analysis. The essence of the approach is as follows: the initial matrix-based representation of system's structure is used with a purpose of obtaining its geometric and algebraic equivalent models in the form of *simplicial complex* K, which is formed by a set of properly adjoined objects called *simplexes*. Being a complex entity to visualize, each simplex is formed by *vertices* (points), the number of which determines dimensionality of a particular simplex. The existence of simplexes of different dimensions in K opens up a possibility of analyzing system's model (K) at each dimensional level through studying so-called *chains of connectivity*. They arise as a consequence of the presence of vertices shared by contiguous K's simplexes, which enables to group simplexes at each dimensional level into *connectivity components* following straightforward systematic procedure. Finally, numeric estimate of structural complexity (viz. *connectivity*) of simplicial complex K is obtained on the basis of aforesaid results by using simple analytic expression.

A concept of complex system (or, complexity in general) and its interpretations are really multifaceted (rf. comprehensive source «Principia Cybernetica Web» at http://pespmc1.vub.ac.be); we distinguish only *structural features*, which could bring a valuable information at initial stages of systems studying. Classification of systems as simple or complex ones usually takes into account several factors – the number of elements and relationships between them are among the most important. In other words, what is meant in the paper are those aspects of *hypothetical complexity*, which are manifested in system's structure and «arise through *connectivity* and the inter-relationships of a system's constituent elements» [9].

Section II of the paper summarizes some basic constituents of term system used in topological studies [2,10,18] and straightly in Q-analysis procedure; discussion of complex K's structural complexity estimate (SCE) $\Psi(K)$ as proposed by J.Casti is also covered by this section. Modified SCE that is based on notions of distance and similarity within psychological space (P-space) forms the main topic of Section III. Important practical issues of categorization of calculated numeric values of modified SCE (all dimensional levels of complex K's analysis), representation forms of prototypes and categories in a whole using extended points and lines are amply presented in Sections IV-V. Concluding remarks are drawn in Section VI.

## 2. Simplex. Simplicial Complex. Structural Complexity Estimate

*Simplicial complex* K can be represented as a pair (V,S), where $V=\{\upsilon_1,...,\upsilon_m\}$ is a finite set of vertices, and S are simplexes of complex K, satisfying the following properties:

1) $\forall\upsilon_i \in V$, $\{\upsilon_i\}\in S$ (any i-th vertex is a simplex of complex K),
2) $\forall\sigma\in S$, $\forall\underline{\sigma}\subset\sigma \mid \underline{\sigma}\neq\varnothing$, $\underline{\sigma}\in S$ (non-empty simplex $\underline{\sigma}$ being a *face* of simplex $\sigma$ is also a simplex of K) – see also item 5),
3) complex K is formed by regularly adjoining simplexes – the intersection of two simplexes is «either empty or a common face of each»; the union of simplexes of K (result of «gluing them together» along their faces) is called a *polyhedron*,
4) q-dimensional simplex $(\sigma_q \mid \dim(\sigma_q)=q,\ q\geq 0)$, or just *q-simplex* is a convex hull (inner area included) of its (q+1) vertices. This means that a point is 0-dimensional simplex, line segment is 1-simplex, a two-dimensional simplex is a triangle considered together with the interior region it bounds, $\sigma_3$ is a tetrahedron, etc.,
5) simplex $\sigma_q$ (q-simplex) is a q-dimensional face (or, in short, *q-face*) of p-simplex $\sigma_p$ ($q\leq p$), if each vertex of $\sigma_q$ is also a vertex of $\sigma_p$,
6) two simplexes $\sigma^{(\alpha)}$ and $\sigma^{(\beta)}$ are *q-connected* in complex K, if there is a sequence of intermediate simplexes $\sigma^{(\alpha)},\sigma^{(\tau_1)},...,\sigma^{(\tau_n)},\sigma^{(\beta)}$ such that any pair of simplexes in the sequence share q-face (thus, $\sigma^{(\alpha)}$ and $\sigma^{(\beta)}$ are connected by means of q-chain «links» that correspond to intermediate simplexes mentioned); if two simplexes $\sigma^{(\alpha)}$ and $\sigma^{(\beta)}$ are q-connected, they are also (q-1)-, (q-2)-connected, etc. Besides, any q-simplex $\sigma_q$ is q-connected to itself,
7) complex K is viewed as a formal representation (model) of the system under study; it is an aggregate S of simplexes $\sigma_q^{(i)}$ of different dimensions $q=\overline{0,N}$, $i=1,...,n_q$; *dimension of* K ($N=\dim(K)$) is the maximal dimensionality of its simplexes.

Consequently, Q-analysis procedure pursues the object to analyze consecutively simplicial complex K at each dimensional level q, $q=\dim(K),...,1,0$, and reveal the number of groups (clusters) of q-connected simplexes. Thereby, structural vector $Q=(Q_N,...,Q_1,Q_0)$, where $Q_q$ is the number of connectivity components at the dimensional level q, obtained as an effect of procedure's realization can be considered in the capacity of a global characteristic of K's structural organization. In addition, we may implicitly assume that at the level $q=0$ complex as system's model is completely connected, i.e. $Q_0=1$.

The value of each individual component of vector Q, as well as the number of such components, provide meaningful information about structural features (connectivity) of complex

K. It should be recognized that the presence of a large number of dimensional levels creates overt difficulties in obtaining reliable opinions concerning the level of complexity of K's connected structure in tote. To overcome such inconvenience the expression for calculation of structural complexity $\Psi(K) = \frac{2}{(N+1)\cdot(N+2)} \cdot \sum_{q=0}^{N} (q+1)\cdot Q_q$, $N = \dim(K)$, of the system (simplicial complex as its model) has been proposed by J.L.Casti [7,8]. The form of the expression is determined by underlying axioms, which can be summarized as follows:

1) $\Psi(K)$ is equal to 1 for the complex K consisting of a single simplex,
2) $\Psi(\overline{K}) \le \Psi(K)$ for subcomplex $\overline{K} \subset K$; combination of two complexes $K_1$ and $K_2$ leads to obtaining complex K, for which $\Psi(K) \le \Psi(K_1) + \Psi(K_2)$.

Despite the simplicity, convenience and attractiveness of the expression to calculate the estimate $\Psi(K)$, purely «mechanistic» approach to the use of components $Q_q$ does not allow to reveal relevant information masked in vector Q [11]. Turning complexity estimate of system's structure into a real number creates additional difficulties in the comparison of two different complexes because there is no real verbal scale, which would have been accustomed to human beings and would allow a group of experts to express opinions and draw easily conclusions about degree of complexity of K at each particular dimensional level $q = \overline{0, \dim(K)}$ of its analysis. Therefore, subsequent part of the paper deals with consideration of the approach that is more focused on human perception of characteristics obtained, mental comprehension and formation/comparison of personal constructs in psychological space (or, *P-space*).

## 3. Vectors in Psychological Space. Modified Complexity Estimate

Without doubt, the amount of information at the expert's disposal within the scope of Q-analysis is rather scanty, but even so, in parallel with the calculation of Q-vector's values, significant information granules are obtained and … doomed to oblivion. Among these are the number $s_q$ of simplexes having dimension q or greater (all of them are considered when calculating $Q_q$ numeric values) and the total number $s(K)$ of non-empty simplexes in complex K. Current dimensional level q, $q = \overline{0, N}$, and $Q_q$ should be also included in this list, but at the moment they are both left aside, because of active involvement in the calculation of $\Psi(K)$. As mentioned in [11,12], the description of connectivity (structural complexity) modified estimate $\Psi_{MOD}(K)$ can be put into effect on the strength of simple term dictionary composed of q-level feature vectors $\widehat{A}_q = \left(a_q^{(1)}, a_q^{(2)}\right)$.

What is the reason for the transition to consider vectors instead of individual calculated values? Small number of information granules obtained at each q-level of complex's K analysis should not be considered separately, because direct interpretation of such emerging values seems problematic – the actual «weight» of each separate granule may become apparent only in close liaison with other supplementing granules. Elements of feature vectors $a_q^{(1)} = \frac{s_q}{s(K)}$

and $a_q^{(2)} = \dfrac{Q_q}{s_q}$ can be interpreted in quite natural way, and the shift towards the idea to represent objects as points in *psychological space* (P-space that can be also endowed with metrics) provides the opportunity to move away from a purely «narrow» perception of Q-values calculated in the course of performed analysis. P-space with metrics allows us to calculate *semantic distances* (in the form of Euclidean distance) between points in this space.

Thus, vectors $\widehat{A}_q$ constructed for each value of q lay a ground for alternative description of K's connectivity by means of novel geometrical units. But do they give us any advantages? Well, when dealing with the problem of categorization of complex K (viz. separate q-levels of analysis) according to specified verbal drawn up categories of structural complexity, vectors enable to carry out such categorization on the ground of important *similarity* concept. The latter has a huge significance in mathematics, psychology, technical sciences, etc. – in our case, such approach can be regarded as a certain level of cognition's expression that reflects the natural form of perception of characteristics revealed at the stage of K's analysis.

Vectors $\widehat{A}_q$, $q = \overline{0, N}$, can be considered as a peculiar abstractions that are directly related to the modeling approach in use. Virtually, domain engineers (group of experts) bring into play the perceptual mechanism aimed at highlighting certain numeric characteristics with clear subject semantics that become available as a result of Q-analysis realization [1]. Such vector approach allows to determine so-called Idealized Cases (IC) at each particular q-level of simplicial complex K's analysis – IC are directly linked to limiting values of vector's $A_q$ components that are used in determining $\Psi_q$-*squiggle rules* [11,12]:

$$\widetilde{A}_q = \left(\tilde{a}_q^{(1)}, \tilde{a}_q^{(2)}\right) = \left(1, \frac{1}{s_q}\right)\Bigg|_{Q_q=1,\ s_q=s(K)} \rightarrow \widetilde{\Psi}_q = 1 \qquad (1)$$

For each $0 \le q \le N$, $N = \dim(K)$, these rules can be verbalized as follows: if $s_q$ is equal to the total number of non-empty simplexes in K, and the number of connectivity components ($Q_q$) is equal to one, then we imply (i.e. associate in our mind) structural complexity of q-level to be equal to 1. Besides, the *actual* (calculated) *values* of vector $\widehat{A}_q$ as shown earlier may be associated with some yet unknown value *x* of $\widehat{\Psi}_q$, $0 < x \le 1$. Minute description of further computational steps are omitted in the paper deliberately (they are explained in sufficient detail in [11,12]), giving only general idea of vectors ($\widetilde{A}_q$ and $\widehat{A}_q$) and associated estimates ($\widetilde{\Psi}_q$ and $\widehat{\Psi}_q$) of K's connectivity at q-level processing. This condensed description based on rather influential geometrical model of perceived similarity can be set out in writing as follows:

1) *distance* between two vectors can be calculated using r-Minkowski metric:

$$D_q^{[r]}\left(\widetilde{A}_q, \widehat{A}_q\right) = d(q) = \left[\sum_{k=1}^{2}\left|\tilde{a}_q^{(k)} - \hat{a}_q^{(k)}\right|^r\right]^{1/r}$$

2) *similarity* (proximity) between two vectors can be determined [13] inversely by way of the parameterized distance obtained at the step 1),

i.e. $P_q\left(\widetilde{A}_q, \widehat{A}_q\right) = \exp\left[-a \cdot \left(D_q^{[r]}(\widetilde{A}_q, \widehat{A}_q)\right)^n\right]$, where $a > 0$ is a sensitivity parameter specified by expert group, and n value is often assumed to be equal to two [19],

3) refer to (1) – the actual value of structural complexity (connectivity) is obtained under reasonable assumption of existence of latent dependency $\widehat{\Psi}_q = P_q\left(\widetilde{A}_q, \widehat{A}_q\right)$, i.e. $\widehat{\Psi}_q$ is decreasing gradually towards zero with the growth of the distance d(q).

## 4. Concepts and Categories. Representation of Categories

Thus, turning acceptable numeric values of $\widehat{\Psi}_q$ into unit interval – essentially it's a formation of a usable scale – leads to an important issue, which is the main subject of the paper. We are interested to provide a verbal assessment of structural complexity (connectivity) of each q-level considered in the analysis of K. From the standpoint of expert such information makes more sense as compared to manipulations with ordinary "taciturn" numbers. «Upper» limit (not greater than 1) imposed on possible values of $\widehat{\Psi}_q$ can be interpreted as the structuring of our phenomenal space. After calculation of each $\widehat{\Psi}_q$ value in accord with the scheme outlined above, we are trying to put it into one of categories that come out though discussion of the following concepts $ct_i$, $i = \overline{1, M}$ (shown as an example) within the expert group:

- $[ct_1]$ «very weak connectivity (or, very low structural complexity)»,
- $[ct_2]$ «weak connectivity (or, low structural complexity)»,
- $[ct_3]$ «strictly moderate connectivity (or, average structural complexity)»,
- $[ct_4]$ «moderately strong connectivity (or, moderately high structural complexity)»,
- $[ct_5]$ «strong connectivity (or, high structural complexity)»

It can be reasonably assumed that for all existing differences of opinion, members of the expert group share (nearly) the same core mental space; the group comes to definite conclusions regarding the number of concepts in use (we may cautiously imply 4 to 5 concepts put forward) and their appropriate naming (linguistic labels listed above). During the discussion each expert uses his individual mental space utilizing the elements of visual perception (in particular, with regard to arisen scale of values), experience, knowledge, etc. Stability of concepts across individuals (domain experts) is a good question to draw attention to – it's easy to imagine that perception of particular q dimensional level ($q = \overline{0, N}$), at which, for instance, simplexes geometrically represented as *n*-polytopes ($n \geq 5$) differs from the case of $q = 1$ (less intricate objects are accounted). This topic is beyond the scope of the paper.

Categorization process that is initiated thereafter stipulates that each calculated $\widehat{\Psi}_q$ «falls» into corresponding category (concept) – concepts per se work as «pattern-recognition devices to classify novel entities» [20]. $\widehat{\Psi}_q$ values already incorporate through the instrumentality of geometric model; calculations attributed to similarity/proximity in vector psychological space (*vP-space*) – all those properties that are needed to categorize them. And this is exactly the point, to which intent attention should be paid.

The principal question can be formulated as follows: *what is the workable and intuitively apprehensible way to represent concept (category) within the scope of formal approach under* *study?* Unfortunately, quite an easy question is not associated with a simple answer – despite the availability of empirical information, visual and formal representations cannot be yet called rather straightforward. Perception and experts understanding of such forms of concept's representation that does not require exact measuring and calculations come to the foreground. Such situation can be considered as one of those practical cases when potential costs of exact (or, almost exact) form of information objects elicitation are really not needed [24].

### *Representation of categories – Cases A and B: usual intervals and membership functions*

On one hand, it looks attractable to describe (to the extent possible) categories using notions with sharp boundaries – usual interval (line segment) as convex subsets of $\mathbb{R}$ are the simplest and most expressive candidates. Does it really look irreproachably and persuasive?

It should be observed that on the basis of conducted analysis, synthesis and generalization, experts verbalize and form the concerted structure of perceptual images $\rho_i = \rho(ct_i)$, $i = \overline{1, M}$, of concepts under consideration. For all the similarities of expert opinions, differences are unavoidable. Making a start from the tempting interval model, we may assume that in each of M images of concepts we may identify central categorical region $Cr_i$ together with accompanying parts that are located on each side of it. The lateral parts $Lp_{left}^{(i)}$ and $Lp_{right}^{(i)}$ lie partially within the area of intersection of adjacent categories (their representations). It can be also suggested that central regions of the above-mentioned representations do not overlap; if the number of potentially pithy concepts is small, such supposition does not seem excessively restrictive. In effect, representation of concepts (categories) can be reduced to trapezoidal membership functions ($MF \equiv \mu(x)$), which express the meaning of the linguistic variable $\chi$ «degree of structural complexity (connectivity) of a dimension level q». Linguistic values (labels) of $\chi$ are defined by experts in accordance with the number M of input categories. Perceptual images $\rho_i$ can be associated in their substantial manifestation with constructed MFs. This fact may be expressed in slightly coarsened form as the following transition:

$$\rho_i = \rho(ct_i) \to \mu^{(i)}(x) = f_i\left(G(\Omega), M(\Omega)\right) \,\Big|\, \Sigma \to \left(\chi, T(\chi)\right),\ i = \overline{1, M} \qquad (2)$$

where $G(\Omega)$ and $M(\Omega)$ in (2) are syntactic and semantic rules that are directly related to knowledge, skills and intuition (collectively designated by symbol $\Omega$) inherent to expert group members in the presence of the specifics $\Sigma$ of the given problem affecting the choice of $\chi$ and set $T(\chi)$ of its linguistic values. Functions $f_i$ in (2) express peculiarities of the process of MFs constructing according to rules – the presence of the subscript $'i'$ emphasizes the potential emergence of various kinds of nuances related to different images $\rho_i$ caused by processed values, information, perception of scale's parts, etc. With respect to a given problem set T can be represented as $T(\chi) = \{'\textit{very low}', '\textit{low}', '\textit{average}', '\textit{moderatly high}', '\textit{high}'\}$. Expanding the number of linguistic labels is not advisable because of the need for precise verbal expression of additional terms in T that may become semantically similar.

With regard to the situation concerned, the support of normal MF is defined as a region of the universe of discourse (UoD – unit interval, in which calculated value $\widehat{\Psi}_q$ falls to) having a length of $\left(\left|Lp_{left}^{(i)}\right| + \left|Cr_i\right| + \left|Lp_{right}^{(i)}\right|\right)\Big|_{i=\overline{1,M}}$, where M stands for the number of specified categories, the central area $Cr_i$ of the i-th interval, $i = \overline{1, M}$, stands for the core of corresponding MF. The lateral parts $Lp_{left}^{(i)}$ and $Lp_{right}^{(i)}$ cover those elements of UoD, which are characterized by some degree of fuzziness, i.e. the matter are those $x \in Lp_{left}^{(i)}$ and $x \in Lp_{right}^{(i)}$ that form MF boundaries.

Even if we accept such approach, «mental construction» of membership functions can be a challenge to determine their parameters and rather rigid stance of experts. It is clear that several groups of methods (reasoning, NN- and GA-approaches, etc.) to develop MFs proved their practical usefulness up to now. Representation of categories and their internal structure are based on both objective and subjective rules/properties that are rather intricate to express in clear and complete form. There are also additional issues to be taken into account – not fully intelligible specific character of intersection of intervals that represent categories, jagged boundaries of these intervals, no real need to spend extra efforts on sophisticated calculations and «fight» for unnecessary accuracy (in everyday situations people don't focus on explicit analytical relations and expressions), aim to attain relative simplicity of representation, to name a few. In the last analysis, perceptions and understanding of possible expressions of categories are asseverated by experts without accompanying procedures associated with measurements and calculations. In view of the aforesaid, does it make sense not to give immediate preference to the use of MFs, but to think about possible visual alternative?

### ***Representation of categories – Case C: prototypes of categories, perceived ranges***

Each member of the expert group is definitely able to specify central regions $Cr_i$, which can be rather narrow or wide enough. To certain degree of supposition, such central part bears resemblance with *prototype* known from the theory (horizontal organization of categories) of E.Rosch [22]. For instance, she underlines the fact that many «natural categories are internally structured into a prototype (clearest cases, best examples) of the category with non-prototype members tending towards an order from better to poorer examples». In other words, prototypes are members that «fit the closest to our bodily experience of the category» [16], i.e. central regions $Cr_i$ can be treated as representation of prototypes. The prototype reflects the fact that membership degrees of certain entities within the category are definitely not uniform. The use of textual bunch «membership degrees» here does not necessarily imply any tangible calculations or obtaining rough estimates. In this regards, many publications in the fields of linguistics, psychology, social sciences prefer to talk about *psychological saliency*, which can be associated generally with terms of primacy, uniqueness, priority, patency, etc.

Representation of concept's prototype (most typical instance of concept) as a singleton does not seem natural, especially when it comes to categorization tasks; in many situations expert groups may face with disagreements concerning particular value(s) that represent a category. This fact does not contradict with what is said earlier – domain experts really share nearly the same mental space, but their personal constructs that come into light under the same objectives may vary. Prototypes are viewed as a basis for categories discrete structuring and their encoding in mind – practical studies show that the understanding (perception) of prototype's boundaries in expertise is much more stable as compared to both boundaries of terms that stand for categories and the way these boundaries overlap [6,21]. As also stated in [16], «reasoning with prototypes is indeed so common that we could not function without it».

Thereby, the facts show that

1) concepts (categories) are fuzzy (vague),

2) prototype is a basis of category's representation, and
3) prototype's boundaries are considered as relatively stable – nevertheless, refinement, «fine-tuning», of prototypes is a persistent process associated with the activity of expert (in all fairness, concepts (categories) exhibit resembling «flexibility»).

In everyday life, in an attempt to give a clear and understandable explanation and presentation of ideas, solutions or specific plans, we often resort to various types of images (charts, simple drawings, etc.). Returning to the main subject, we can assume that the presentation of expert opinions regarding the formal expression of the obtained information on categories (concepts) may use graphic primitives (e.g. points and lines) and the incidence between them. Using very familiar elements, we are still talking about *perceived approximate ranges* (PAR) that are almost for sure at domain experts disposal. In effect, PAR can be regarded as a result of expert group's activity, natural graphic expression of confidence in the expert view of the categories under consideration (in light of convenience and clarity of pictorial forms).

Thus, in each of M intervals (conceptual areas identified by experts in the unit interval of $\widehat{\Psi}_q$ potential values) we can define central region, which is not reducible to a singleton. By analogy with Dirichlet tessellation, these central parts can be named as *generating areas*, and for the case under consideration we directly link them to prototypes of categories at hand. It can be thought that central areas of those intervals do not intersect; such assumption is not tight, since «new items are classified according to their relative similarity to learned prototypes» [15]. In the context of the problem the word «learned» can be replaced with «elicited (from the group of experts)». If $\widehat{\Psi}_q$ value falls into i-th prototypical area ($i = \overline{1,M}$), then the estimate of complex K's structural complexity (connectivity) at dimensional level q is automatically verbalized as $t_i$, where $T(\chi) = \{t_i\}$ – set of linguistic labels (values) of the variable $\chi$ (2). In this case we're talking about almost complete confidence in categorization of $\widehat{\Psi}_q$ value, classifying it as a value with «high validity index».

## 5. F-points and Extended Lines in Category's Representation

Presentation of a prototype, which expresses virtually full agreement of experts regarding the association of numeric interval considered with the label of a given category, may be based on the notion of *extended line*. Since the matter is i-th interval ($i = \overline{1,M}$), its boundaries can be formed by F-points $\widetilde{P}_i$ and $\widetilde{Q}_i$. Terms mentioned are divulged by the following definitions.

**Definition 1**. *F-point* can be viewed as a disk, i.e. convex subset $fp \subset \mathbb{R}^2$ such that $fp = \left(X \in \mathbb{R}^2 \middle| d\left(c_{fp}, X\right) \le \varphi\right)$, where $d(\cdot)$ is a distance between the center of F-point ($c_{fp}$) and any arbitrary $X \in \mathbb{R}^2$, $\varphi > 0$ is a diameter of the disk.

In fact, Def.1 expands the case of more habitual notion of the point, so the disk mentioned above can be associated with so-called *extended point* that serves as an obvious imprint of uncertainty aureole around ordinary point in the Cartesian space [14,17,23].

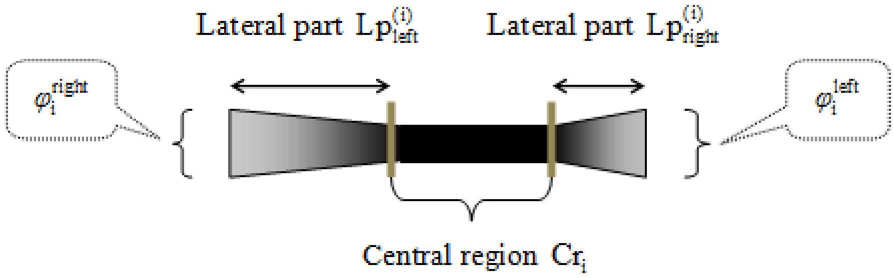


**Figure 1**. General graphical representation of i-th category based on a combination of extended lines

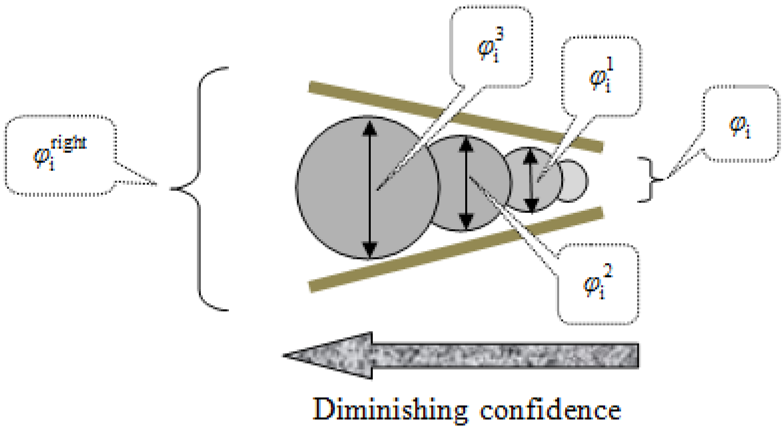


**Figure 2**. Imaginary set of disks within trapezoid-like area (lateral parts)

**Definition 2**. *Extended line* (EL) is a line that passes through two extended points $\tilde{P} = fp_1$ and $\tilde{Q} = fp_2$ (it can be regarded as a semblance of a tube connecting disks $\tilde{P}$ and $\tilde{Q}$). Definition does not impose extra constraints on diameters $\varphi_1$ and $\varphi_2$ of both points, therefore EL can be defined as a widening stripe bounded by two tangents to the boundaries of disks.

Consequently, if i-th prototype of a category – central area $Cr_i$ – is represented by the extended line (segment) $\left[\tilde{P}_i, \tilde{Q}_i\right]$, having a width of $\varphi_i$, $i = \overline{1, M}$, then we may assume that all these tubes (rf. Def.2) are rather narrow. Why do we make such assertion? Suppose particular parameter $\varphi_i$ is inversely proportional to the degree of expert's confidence in the resulting evaluation of category's prototype. Probably, such confidence (it is high for prototype parts) should not be considered in complete isolation from doubts and minor variances, and it leads to acceptance of $0 < \varphi_i < \varepsilon$. Extended line segments can be drawn using diminutive spray-can – respective narrow strips will be spattered evenly with high concentration color.

Representation of lateral parts $Lp_{left}^{(i)}$ and $Lp_{right}^{(i)}$ of intervals that state categories is similar to the previous case. However, we'll obtain quite rough mirror images of widening stripes on the left and on the right sides of prototype regions $Cr_i$, $i = \overline{1, M}$. If the i-th interval as a whole has boundary values $a_i$ and $b_i$, then these values are considered as centers of F-points ($a_i \equiv c_{fp}^{left}$ and $b_i \equiv c_{fp}^{right}$) having diameters $\varphi_i^{left} > \varphi_i$ and $\varphi_i^{right} > \varphi_i$. All points that lie outside $Cr_i$ regions on their left and right are attributed to various degrees of confidence within ranges

$\left[\varphi_i^{left}, \varphi_i\right[$ and $\left]\varphi_i, \varphi_i^{right}\right]$, correspondingly. Both left and right components of entire EL-representation of each category can also be drawn using diminutive spray-can; filling of considered widening stripes will be characterized by pronounced inhomogeneous color concentration (Fig.1 – trapezoids shown should be regarded as a kind of outline (regular form) circumscribing actual «clouds» and swirls drawn with spray-can). If $\widehat{\Psi}_q$ finds itself either in $Lp_{left}^{(i)}$ or $Lp_{right}^{(i)}$ (case of intersection of lateral parts of adjacent categories), then we obtain levels of confidence that are inversely proportional to the diameters of two imaginary disks, for which $\widehat{\Psi}_q = c_{fp}$ (Fig.2). Latterly, $\widehat{\Psi}_q$ can be verbalized by experts in virtue of simple comparison of confidence levels calculated for both alternatives. The same procedure is repeated for all $\widehat{\Psi}_q$ components of the vector $\Psi_{MOD}(K) = \left(\widehat{\Psi}_N, \widehat{\Psi}_{N-1}, ..., \widehat{\Psi}_0\right)$, $q = \dim(K), ..., 1, 0$.

## 6. Conclusion

With regard to the problem of obtaining numeric and verbal estimates of structural complexity (connectivity) of simplicial complex K at each dimensional level q of its analysis (as provided by Q-analysis procedure), the paper described the way to employ extended lines in representation of category's prototypes and categories in a whole. In fact, interval form of categories visualization set up peculiar «boundaries», within which we are using spray-can to draw f-lines. As a result of applying such *f-transformations* we depart from classical objects of Euclidean geometry towards f-lines (*extended lines*) characterized by both constant and variable thickness. In author's opinion, given form of proposed representation is complete enough, easy-to-use and adequate as it reflects close to the actual perception by expert group of available information concerning simplicial complex and its structural traits.

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