The concept of "Translation": history and theory

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ABSTRACT. This article deals with the problem of translations. It covers the history of translation in linguistics and analyzes peculiarities and role of translation in logic. Moreover, the article contains typical examples of embedding operations in terms of different logical theories.

Keywords: logic, translation, embedding, embedment, operation, language, calculi, theory

1 "Theory of translation" in linguistics

The French poet and philosopher, humanist Etienne Dolet [18, p. 6] in the XVI century was one of the first, who was trying to formulate a theory of translation in order to impart scientific justification into this kind of activity. According to his point of view, the true translation must meet the following criteria: perfect understanding of the content of the original text and the author's intentions; mastery of languages, which are involved in the translation process; inappropriateness of literal translation (in order to preserve the authentic atmosphere of the original); preserving the style of the original text, etc.

Later, another researcher T. Sevori [18, pp. 13–14] made a list of requirements for the various authors translations. Outlining of some of his statements will be enough to get his point of view on translation process: 1) the translation must convey the source words, 2) the translation must convey the source ideas, 3) the translation must be read like the original, 4) the translation must be read like a translation, etc. It is important to note that different scientists at different periods of time sometimes demanded from translation totally antithetic requirements. Hereafter, many translators, serious

writers, and, finally, linguists made their own lists of requirements for "translation" and then provide them with the corresponding theoretical justification. Here we are not going to discuss such research, carried out within this framework. Our main goal is to discuss the "linguistic theory of translation", as so far scientific, well substantiated discipline.

The foundations of scientific translation theory were developed in the mid-twentieth century. That was the time of close attention of linguisticians and linguists to this problem. Some philosophers cast doubts on the possibility of translation in general — V. Humboldt, in particular [11]. According to him, every translation is an attempt to solve unsolvable task because of the dependence on the personal characteristics of the individual translator and his attitude to the text. Similar views served as the forerunners of the "theory of untranslatability".

The doubts on the possibility of the translation studies by the methods of linguistics were dispelled when this phenomenon became known as a special kind of verbal activity. This kind of activity was admitted to be the one, in which the units of the target language are selected depending on the specific language units of the source text, but not as a result of individual translation creativity. Here, apparently, there was a final demarcation of this area on the direct field of translation activities and the theory of translation.

J. Vinai, J. Darbelnet attempt to subject different languages to the comparative analysis contributed a lot to the development of the linguistic theory of translation [29]. The analysis helps to detect units from different languages, which can be used in translation interchangeably. These words should a priori carry the same meanings in their two language systems or should come up as equivalents at the end of the translation.

Another linguist R. Jakobson [12] introduced an idea that the theory of translation plays an important role in other sciences, particularly in different branches of linguistics. He offered to define different types of translation: intralingual, interlingual and intersemiotic, in which one system of signs transforms into another. He supposed every transformation which carries the original meaning

to be an adequate translation. This idea of R.Jakobson is closely correlated with the role of translation in logic.

G. Mounin in [21] paid special attention to the semantic structures of languages and claimed to square up to their differences. They occur because semantic discrepancies impose certain translation restrictions, for instance, make it impossible to render the original meaning fully.

It is also important to mention the merits of Russian scientists I.I. Revsin and V.U. Rosentsveig to the linguistic translation theory. In [26] they pointed out that the theory of translation should be not a prescriptive (i.e. the one that a priori formulates translation demands), but a descriptive — the one to describe the objective reality. It is the description that produces standards and regulatory guidelines of translation. They also highlighted the use of the deductive approach, which extends the use of general linguistic concepts in translation process.

I.I. Revsin and V.U. Rosentsvieg defined two methods to transform source text to translated one: a) the direct substitution of the units of the source language into the units of another, b) interpretation. Last was meant to comprehend the reality at first, described by the original language, and then to descript it by the means of the target language.

J. Catford in [3] presumed that the central problem of the translation theory is to render the meaning of original and translated statements adequately close. In such a way he raised the question of texts equivalence in translation. He supposed that the original language meaning is replaced with the meaning of the target one, therefore the equivalence depends on the accuracy of such replacement.

V.N. Komissarov created an integrated theoretical conception in [17]. There he summarized different aspects of the linguistic analysis of translation, classified the research datum, including some mentioned above. Special attention was eftsoons paid to the problem of equivalence.

The issues stated above are just several fragments of the huge linguistic mosaics of translation theory problematic field. On the assumption of asserted, it is possible to point out two major infer-

ences. These inferences will be the requirements basis for defining the term "translation" and for comparing other definitions with the worked out one later. First of all, any translation should keep the meaning of both the original and target texts. In other words, the problem of invariance should be taking into account while crossing the boundaries of two languages. Secondly, the target text should be theoretically equivalent to the original one. The logical sequence and inter-originating of these two principles can be clearly observed.

2 The "translation" in logic

The area of interest for this research includes particular use of translation method — more specifically, transfers between logical calculi. Hereunder, the concept of translation would require clarification. This concept stays in contrast to the linguistic requirements for translation, which state saving of semantic units as one of the key conditions. It is necessary to preserve the verity for compliance of the translation in logic. That means, logical truths of one language should be translated into logical truths of another.

The consequence of the R. Yakobson's requirement about saving of semantic units is that 1) the true statement translation result is a true statement. As it was already mentioned, the logical verity is important in logic. Considering that, the adaptation in logic angle of the requirement 1) formulated above is the claim that $1)'$ the true logic statement translation result is a true logic statement. Taking into consideration this requirement, the definition of translation will be:

DEFINITION 1 (DF1). The translation of calculus C_1 into the calculus C_2 meant to be such a mapping of φ set of all L_1-formulas of calculus C_1 into the set of all L_2-formulas of calculus C_2, that for every L_1-formula A the following condition holds:

$$if \vdash_{C_1} A, \ then \vdash_{C_2} \varphi(A).$$

However, any calculus will be transferred to any other calculus with this understanding of translation. That leads to questioning the retention of the source statement meaning — plus the fact, what logically true statements are not always equivalent in logical calculi.

Besides, it remains unknown, how the translation result corresponds to the original. According to J. Ketford, it is essential to estimate semantic affinity between the statements in the original and in the translation in order to verify their equivalence. In addition, this definition does not take into consideration the content of the statements. Therefore, it does not lead to source reconstitution by the translation, that is, in principle, impossible as it was indicated by G. Mounin. Then let's introduce another requirement:

DEFINITION 2 (DF2). The translation of calculus C_1 into the calculus C_2 meant to be such a mapping of φ set of all L_1-formulas of calculus C_1 into the set of all L_2-formulas of calculus C_2, that for every L_1-formula A the following condition holds:

$$\vdash_{C_1} A, \text{ if and only if } \vdash_{C_2} \varphi(A).$$

In contrast to the first definition, there is an opportunity to check the logical validity of the first calculus statement, considering the assumption of the logical validity of its image in the second calculus statement. Now it is possible to correlate the translation with the original. But most of the former lacks last even in this definition.

To avoid them, let's add to Df2 the following condition for avoiding those lacks — φ is a recursive function. Then the new definition will be Df3.

But there are a number of drawbacks even in such way. According to R. Epstein [6, p. 291], for example, this definition does not guarantee the safety of the produced formulas structures. That means the impossibility of full source content reproduction on free analogy with Mounin's remark. Then it is necessary to take into account the structure of the formulas, that, in fact, is the requirement of inductive definition? (let's name this definition Df4).

Is it enough to give a good definition of the translation? Apparently, not. The question remains, whether the equivalence of the original and the translation result observes?

It is necessary to work out more strict criteria for what is called "translation". Below such definitions proposed directly by logics will be considered.

3   Definitions of embedding operations

Let's analyze and compare the most important, in our opinion, definitions, suggested by the V.A. Smirnov, R. Wojcicki and R. Epstein.

In [10] V.A. Smirnov defined the translations between theories. However, his definition will also be relevant for the calculi with the appropriate modification.

Suppose, that T_1 and T_2 are theories, formulated accordingly in languages L_1 and L_2 with the corresponding logics. Suppose, that φ is a recursive function that matches formulas in language L_1 with formulas in language L_2 for any L_1-formula A. The function meant to be called the translation of theory T_1 to T_2, if the following condition holds: if $A \in \mathrm{T}_1$, then $\varphi(\mathrm{A}) \in \mathrm{T}_2$. If the following additional condition holds: if $\varphi(\mathrm{A}) \in \mathrm{T}_2$, then $A \in \mathrm{T}_1$, then the recursive function φ would be called an embedding operation of the theory T_1 to the theory T_2. Theory T_1 could be embedded to the theory T_2, if and only if there is a recursive function, which embeds T_1 to T_2. Of course, if there is a translation — there is not necessarily the case of embedment. Later we will talk more about the "embedment" since we are deeply interested in cases that satisfy both Smirnov's conditions at once.

V. A. Smirnov's definition actually coincides with the above Df3.

R. Wojcicki offered a different definition [30].

Mapping φ from sentential language L_1 to sentential language L_2, which have the same set of propositional variables, is called the embedment if and only if the following two conditions hold:

1. There is a formula $\phi(p_0)$ from one propositional variable p_0 in L_2, such that for every propositional variable p, $\varphi(p) = \phi(p_0)$.

2. For each logical connective r_i in L_1 there is a formula ϕ_i in L_2, such that for all $\alpha_1, \ldots, \alpha_k$ in L_1, k is the arity r_i,

$$\varphi(r_i(\alpha_1, \ldots, \alpha_k)) = \phi_i(p_1/\varphi(\alpha_1), \ldots, p_k/\varphi(\alpha_k)).$$

Then, the definition of embedment is stated for propositional calculi and theories.

Suppose that $C_1 = (L_1, C_1)$, $C_2 = (L_2, C_2)$ are some propositional calculi, and $T_1 = (L_1, T_1)$, $T_2 = (L_2, X_2)$ are some theories,

then φ is the embedding operation from L_1 to L_2, if for every $T \subseteq L_1$ and for every $\alpha \in L_1$ the following holds:

$$\alpha \in C_1(T) \text{ if and only if } \varphi(\alpha) \in C_2(\varphi(T))(\varphi(T_1) = T_2).$$

Epstein offered a similar definition (ref. [9, pp. 290–291]), but with some differences, which will be defined in detail below.

At first, he formulated the definition of a mapping from one logic to another. This helps to preserve the relation of deductivity:

Mapping of the propositional logic L to the propositional logic M, which preserves the relation of deductivity, is a mapping φ from language L_L to language L_M (L_L и L_M are the languages of logics L and M, respectively), such that for every formula A the following condition holds:

$$\vdash_L A \text{ if and only if } \vdash_M \varphi(A).$$

Mapping is a translation, if for any G and A the following condition holds:

$$G \vdash_L A \text{ if and only if } \varphi(G) \vdash_M \varphi(A), \text{ where } G \text{ is the set of}$$
$$\text{formulas, and } \varphi(G) = \{\varphi(A) : A \in G\}.$$

According to Epstein, such definition of embedment does not cause the preservation of target language structure. This appears to be valid, especially while discussing such examples, as Glivenko's translation of classical propositional logic into the intuitionistic logic [12], which matches the above specified definition.

Epstein then introduced the concept of grammatical mapping:

Mapping φ of propositional language $L_{1 \supset \neg}$ to any propositional language L_2 is called grammatical, if there are schemes λ, φ, ψ in language L_2, such that

$$p^* = \lambda(p),$$
$$(\neg A)^* = \varphi(A^*),$$
$$(A \supset B)^* = \psi(A^*, B^*),$$

whereas it is implied, that in these languages, the set of propositional variables is the same.

Grammatical mapping is called homophonic if each connective maps into itself. Grammatical embedment is a grammatical mapping, which comes out as an embedment.

R. Wojcicki and R. Epstein's definition meets the one, which was marked above as Df4.

4 Comparison of definitions

As is clear from aforementioned definitions, all of them have specific differences. V.A. Smirnov provides the broadest definition. Glivenko's contribution resulted in translation of the classical propositional logic into the intuitionistic one. His translation meets the definition of V. A. Smirnov, but falls outside the scope of R. Wojcicki and R. Epstein's meaning of embedment.

Let us denote the differences in the definitions of Wojcicki and Epstein. There is a need to reformulate them in a similar style and then compare.

Suppose the languages L' and L'' are given with one and the same set of propositional variables, $\supset$ and $\neg$ are the logical connectives of L' language, φ is the embedding operation from L' language to L'', and $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are some formulas of L'' language.

Then the definition of Wojcicki can be represented in the following form:

- $\varphi(p) = [p_0/p]\boldsymbol{A}$, where $[p_0/p]\boldsymbol{A}$ is the result of substitution p instead of p_0 into the formula $\boldsymbol{A}$, which does not contain occurrences of propositional variables, other than p_0,

- $\varphi(A \supset B) = [p_1/\varphi(A), p_2/\varphi(B)]\boldsymbol{B}$, where $[p_1/\varphi(A), p_2/\varphi(B)]\boldsymbol{B}$ is the result of substitution $\varphi(A)$ instead of p_1 and $\varphi(B)$ instead of p_2 into the formula $\boldsymbol{B}$,

- $\varphi(\neg A) = [p_1/\varphi(A)]\boldsymbol{C}$, where $[p_1/\varphi(A)]\boldsymbol{C}$ is the result of substitution $\varphi(A)$ instead of p_1 into the formula $\boldsymbol{C}$, which does not contain occurrences of propositional variables, other than p_1.

Epstein claims the following:

- $\varphi(p) = [p_0/p]\boldsymbol{A}$, where $[p_0/p]\boldsymbol{A}$ is the result of the substitution p instead of p_0 into the formula $\boldsymbol{A}$,

- $\varphi(A \supset B) = [p_1/\varphi(A), p_2/\varphi(B)]\boldsymbol{B}$, where $[p_1/\varphi(A), p_2/\varphi(B)]\boldsymbol{B}$ is the result of substitution $\varphi(A)$ instead of p_1 into the formula $\boldsymbol{B}$,

- $\varphi(\neg A) = [p_1/\varphi(A)]\boldsymbol{C}$, where $[p_1/\varphi(A)]\boldsymbol{C}$ is the result of substitution $\varphi(A)$ instead of p_1 into the formula $\boldsymbol{C}$, which does not contain occurrences of propositional variables, other than p_1.

So the only varying cases are those, which hold the scope of the embedding operation within propositional variables. Otherwise, R. Wojcicki and P. Epstein's definitions stay matching.

But wherein does the complexity of embedment universal definition as a means for comparison and study of logical systems lie? As referred to by M.N. Rybakov and A.V. Chagrov in [4], it makes sense to impose additional conditions (other than those, that were put forward in the above definition), depending on the purpose of a particular embedment, since it is often necessary to consider the contents of the formulas. Otherwise, it is impossible to adequately represent the formulas of one logic by the means of another logic. In other words, in any embedment could be a list of requirements for embedding operation.

The definition proposed by V.A. Smirnov would be the basis for this research on different embedments of logical calculi. This was motivated by simplicity and convenience of his definition. But our embedding operations are also true within R. Wojcicki and R. Epstein's theories, as it will become clear from their construction.

5 Reasons for embedding operations application in logic. Philosophical and technical aspects

There are several reasons for the use of embedding operations in logic. Here we can highlight technical and philosophical aspects.

As it was mentioned above, philosophically, embedment helps to map one theory by another theory terms. The ability of comparing

theories, formulated in different languages, becomes real with such a phenomena, but stands in opposition to Firebrand's idea of "incommensurability of theories". According to this idea terms of one theory cannot be expressed in terms of another one, as they themselves have different meanings. The embedding operations method (if we can call it a method) removes the "problem of understanding". If we embed non-interpreted calculus into calculus, which has some semantic meaning, the interpretation of first embedment formulas becomes possible.

This raises the problem of negation in language: embedment of containing negation language to a different positive language or its own positive part we get an opportunity to speak about the first language facts using only affirmative sentences, i.e. without saying any "no".

Speaking about the technical aspect we should note the problem of decidability. Hereby, embedment of any calculus into a decidable one results into solving the problem of decidability of the first calculus. The problem of languages relative insolvability is also observed in terms of this problem [13]. Embedment of calculi into their own fragments and other calculi fragments decreases the number of connectives, which are necessary to express formulas in different languages. This is again very important for understanding of the original calculi. V.M. Popov got one of the most eloquent and unexpected results in this area, when he embedded classical propositional logic into its implicative fragment and implicative fragment of intuitionistic propositional logic. We think that this idea demands some serious deliberation, as it is not quite clear.

Besides, as it was shown in V.M. Popov's research in [22], use of the embedding operations helps to prove fragment severability in calculi. The characteristics, mentioned here, can be added to much broader list of application areas than discussed here.

6   The history of the concept of "embedment". Specific embedding operations in classical and intuitionistic propositional logics

Besides the above mentioned Smirnov, Wojcicki and Epstein, attempts to determine and organize embedding operations were also

taken by N.A. Shanin [27], who, according to the V.A. Smirnov (ref. [28, p. 120]), had first coined the term, D. Prawitz and P. Mamnos [25], W.A. Carnielli and M.L. D'Ottaviano [2]. Based upon the last work of Carnielli and D'Ottaviano, A.S. Karpenko concluded that the application of embedding operations is a key tendency in the development of contemporary logic [13].

The term of embedding operation was first time introduced, according to V.A. Smirnov [28, p. 120], by A. N. Kolomogorov in 1925 [16] while embedding classical logic into intuitionistic. Particular attention is worth paying to the problem of relation between classical and intuitionistic logics towards the embedding operations after many leading scientists who made it a point.

For that let's specify after [15] the calculi PC (classical propositional calculus) and Int (intuitionistic propositional calculus).

Language $L_{\wedge\vee\supset\neg}$ of these calculi is the conventionally determined propositional language with a set of propositional variables $\{p_1, p_2, p_3, \dots\}$.

Calculi PC and Int are the calculi of Hilbert type with the conventionally determined concept of proof. The set of deduction rules for each of these calculi has the only rule: $A, A \supset B/B$. Therefore, it is sufficient to define the set of its axioms in order to specify any of these calculi.

Calculus PC. The set of all axioms of calculus PC is the set of all formulas, each of which is stated in at least one of the following types:

1. $A \supset (B \supset A)$,

2. $A \supset (B \supset C) \supset ((A \supset B) \supset (A \supset C))$,

3. $(A \wedge B) \supset A$,

4. $(A \wedge B) \supset B$,

5. $A \supset (B \supset (A \wedge B))$,

6. $A \supset (A \vee B)$,

7. $B \supset (A \vee B)$,

8. $(A \supset C) \supset ((B \supset C) \supset ((A \vee B) \supset C))$,

9. $(A \supset B) \supset ((A \supset (\neg B)) \supset (\neg A))$,

10. $(\neg(\neg A)) \supset A$.

Calculus *Int*. The set of all axioms of Int is the union of the set of all formulas, each of which is a formula of at least one of the above mentioned 1–8 types with the set of all formulas of the type:

9′ $(\neg A) \supset (A \supset B)$.

Glivenko in 1929 suggested embedding operation for classical and intuitionistic logics, which lies within the language L — the conventionally determined propositional language with a set of logical connectives $\{\wedge, \vee, \supset, \neg\}$. This embedding operation associates each L-formula A with L-formula $\neg(\neg A)$. Precisely, Glivenko proved the following theorem, which let's call T1 for convenience of reference.

THEOREM 1 (T1). *$G \vdash_{PC} A$ if and only if $\neg(\neg G) \vdash_{PC} \neg(\neg A)$.*

V. Popov noted an interesting fact about the Glivenko's embedding operation to fail for the predicative versions of intuitionistic and classical logics.

In 1933 Gödel showed [10] that *Int* could be considered as the extension of classical propositional logic, formulated in the language $L_{\wedge\neg}$ within the meaning of the following theorem (let's call it T2).

THEOREM 2 (T2). *For $L_{\wedge\neg}$-formula A it is true that $\vdash_{PC_{\wedge\neg}} A$ if and only if $\vdash_{INT} A$.*

PROOF. The proof of this theorem right to left is obvious, because the set of all *Int* theorems is included into the set of all *PC* theorems. Let's prove that if $\vdash_{PC_{\wedge\neg}} A$, then $\vdash_{INT} A$.

The proof is carried out by induction on the structure of $L_{\wedge\neg}$-formula A.

Now there are three options: *1)* A is a propositional variable p_i, *2)* A is $\neg B$, *3)* A is $B_1 \wedge B_2$.

Let's consider *1)*. Here the following statement requires provement — if $\vdash_{PC_{\wedge\neg}} p_i$, then $\vdash_{INT} p_i$. But none of propositional variables can be the theorem of *PC* calculi. Therefore, the theorem

would be true here, taking into consideration the characteristics of the classical implication.

Let's consider *2)*. Here the following statement requires provement — if $\vdash_{PC_{\wedge\neg}} \neg B$, then $\vdash_{INT} \neg B$. But after Glivenko's result [9] that $\vdash_{PC} \neg A$ if and only if $\vdash_{INT} \neg A$ the previous statement would be true.

Let's consider *3)*. Here the following statement requires provement — if $\vdash_{PC_{\wedge\neg}} B_1 \wedge B_2$, then $\vdash_{INT} B_1 \wedge B_2$. Taking into consideration the characteristics of $PC_{\wedge\neg}$ it is true that *a)* if $\vdash_{PC_{\wedge\neg}} B_1 \wedge B_2$, then $\vdash_{PC_{\wedge\neg}} B_1$ and $\vdash_{PC_{\wedge\neg}} B_2$.

Using the inductive assumption, we have: *b)* if $\vdash_{PC_{\wedge\neg}} B_1$, then $\vdash_{INT} B_1$ and $\vdash_{PC_{\wedge\neg}} B_2$, then $\vdash_{INT} B_2$.

In *Int* the following formula is provable: *c)* $B_1 \supset (B_2 \supset (B_1 \wedge B_2))$. From *a)*, *b)* and *c)* by the definition of *Int* proof, we obtain that $\vdash_{INT} B_1 \wedge B_2$.

Thus, the theorem T2 is proved. 　　　　　　　　　　Q.E.D.

Now let's introduce the result of Łukasiewicz [19] on embedment of *PC* into *Int*, when the binary identical relations of classical logic were used for connectives $\supset$ and $\vee$. This was initiated for representing the *PC* formulas, stated in the $L_{\wedge\vee\supset\neg}$ language, into the *Int* formulas. There has been constructed the following embedding operation (here and below we will use the original symbols for embedding operations):

- $p^* = p$

- $(A \wedge B)^* = A^* \wedge B^*$

- $(\neg A)^* = \neg(A^*)$

- $(A \vee B)^* = \neg((\neg A^*) \wedge (\neg B^*))$

- $(A \supset B)^* = \neg(A^* \wedge (\neg B^*))$,

THEOREM 3 (T3). $\vdash_{PC} A$ *if and only if* $\vdash_{INT} A^*$.

According to Epstein (ref. [6, p. 213]), neither the embedment in the sense of T2, nor the embedment in the sense of T3 preserves the relations of consequences. That happens because these theorems

are false in the wording of $G \vdash_{PC} A$ if and only if $G \vdash_{INT} A$. For example, if we have $\neg\neg p \vdash_{PC} p$, then we also have $\neg\neg p \vdash_{INT} p$, and hence we obtain by the theorem of deduction the false statement $\vdash_{INT} \neg\neg p \supset p$.

Proposed by Gentzen in 1936 [8], his embedment preserves the relation of consequences. This statement consists of the language $L(\neg, \supset, \wedge, \vee)$ and the following embedding operation:

- $p^\circ = \neg\neg p$

- $(A \wedge B)^\circ = A^\circ \wedge B^\circ$

- $(\neg A)^\circ = \neg(A^\circ)$

- $(A \vee B)^\circ = \neg((\neg A^\circ) \wedge (\neg B^\circ))$

- $(A \supset B)^\circ = A^\circ \supset B^\circ,$

THEOREM 4 (T4). *$G \vdash_{PC} A$ if and only if $G^\circ \vdash_{INT} A^\circ$.*

Here Gentzen managed to preserve the relation of consequences while embedment process exactly because of non-standard mapping of propositional variable (through double negation).

It also makes sense to specify here the aforementioned significant result of V.M. Popov [23] in order to close the review on the history of embedment of classical propositional logic into intuitionistic.

Classical propositional logic is axiomatized by calculus $Cl_{\supset f}$. Its axioms are those and only those $L_{\supset f}$-formulas, each of which is given as $A \supset (B \supset A)$ or $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$ or $((A \supset f) \supset f) \supset A$. The inference rule here: $A, A \supset B/B$.

Implicative fragment of intuitionistic propositional logic is axiomatized by calculus $Int_{\supset}$. Its axioms are those and only those $L_{\supset}$-formulas, each of which is given as $A \supset (B \supset A)$ or $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$. The inference rule here: $A, A \supset B/B$.

The following operations are offered here: Sd (first introduced by V.M. Popov in [24]), and T. Sd is meant to be a mapping of the set of all $L_{\supset f}$-formulas into the set of all $L_{\supset}$-formulas, and T is a mapping of the set of all $L_{\supset}$-formulas into the set of all $L_{\supset}$-formulas.

- $Sd(f) = p_1$,

- $Sd(p_i) = p_{i+1}$ *(where $i \in \{1, 2, 3, \dots\}$)*,

- $Sd(A \supset B) = Sd(A) \supset Sd(B)$.

- $\mathrm{T}(p_1) = p_1$,

- $T(p_i) = (p_i \supset p_1) \supset p_1$ *(where $i \in \{1, 2, 3, \dots\}$)*,

- $T(A \supset B) = T(A) \supset T(B)$.

Further the theorem provement takes place (T5 in our notation).

THEOREM 5 (T5). $\vdash_{Sd \supset f} A$ *if and only if* $\vdash_{Int \supset} \mathrm{T}(Sd(A))$.

V.A. Bocharov, M. Zaharyashev, V.I. Markin, A.V. Chagrov, L.L. Esakia should be also mentioned as the contemporary Russian scientists who gave their tribute to the study of logical systems through embedding operations. V.A. Bocharov in [1] constructs embedment of Boolean algebra into syllogistics, among other works on this subject. V.I. Markin in his book [20, pp. 35–43] embedded the systems of clear positive Aristotelian syllogistics into the predicate calculus. A translation from the calculus RM to the positive fragment of RM is constructed in [14]. Work of M. Zaharyasheva and A.V. Chagrova [5] is dedicated to embedment of intuitionistic logic and its extensions to different normal modal logics. L.L. Esakia in [7] considers the new aspects of Gödel's embedment of intuitionistic logic into the modal logic *S4*.

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