

Rogue edge waves in the ocean

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Abstract. Theoretically possible rogue edge wave are studied over cylindrical bottom in the framework of nonlinear shallow water equations in a weakly nonlinear limit. The nonlinear mechanisms (nonlinear dispersion enhancement, modulation instability and multimodal interactions) of possible anomalous edge wave appearance are analyzed.

1 Introduction

The study of anomalously large amplitude surface gravity waves on the sea surface (rogue or freak waves), which can appear suddenly and disappear in the same abrupt way, is very extensive in the recent years (see e.g., book [10] and references there). However, any sudden displacements of water level or changes in flow velocities can also appear in the ocean wave motions of other types, including geophysical large-scale fields. The number of observations of such waves is still very small, they are even almost absent, but the investigations of such possible processes seem to be important for the applications. In the present paper the problem of rogue waves is discussed for **edge waves** in the coastal zone. Such waves belong to the class of topographically trapped waves, which are supposed to play dominant role in the dynamics of oceanic coastal zone. The amplitude of these waves reaches a maximum at the edge, and they are attenuated offshore. Direct visual observations of such waves are difficult, but they have been detected instrumentally in the nearshore wave field many times [2,9]. Edge waves are often considered as the major factor of the long-term evolution of coastal line, forming the rhythmic crescentic bars like shown in Fig. 1 [12].

Edge waves in the last fifty years are the object of numerous studies in geophysical hydrodynamics, ocean and coastal engineering. In the literature, the dynamics of the edge waves was most actively studied and discussed from the point of view of a linear theory, both within the framework of the equations for the shallow-water approximation (2D waves) and for the most simply shaped cylindrical shelves with no longwave approximation (3D waves) [15,22]. This literature provided a rather complete description of the dispersion properties of the edge waves, which were in a good consistency with the observations [21]. Dispersion of the edge wave train is also studied in laboratory tank and numerically [16]. This important feature of the edge wave field is manifested in tsunami problem and leads to the non-uniform distribution of the wave height along the coast, forerunner appearance before intense wave group, significant decrease of wave amplitude in average and increase of tsunami travel time [8,20].

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Fig. 1. Well-defined cusp morphology and swash circulation at the northern end of Pearl Beach, Australia [17].

But as any dispersive wave field, the random superposition of the spectral components moving with different speeds can lead to the forming of high-amplitude short-lived pulses which present the specific *rogue edge waves*. In comparison with focusing of the surface gravity waves this process has some specificity because the edge waves are localized in offshore direction. Such spatial-temporal focusing of the edge waves is studied in [6, 14].

The next step was to study the nonlinear properties of the edge waves [1, 18, 23, 24]. Weakly nonlinear Stokes edge waves of the lowest mode were considered in [23], and the paper [18] shows that the results of shallow-water theory are in a good agreement with the full theory. Properties of nonlinear edge waves were studied in [24] experimentally and theoretically in the framework of the nonlinear Schrodinger equation. In the paper [3], an explicit set of exact solutions of nonlinear equations for describing the propagation of the lowest mode edge waves over a linearly sloping shelf in Lagrange variables was deduced. Such solutions provide a clear illustration to the structure of the edge wave field and suggest a link to explanation of crescent shape of the coastline. Weakly nonlinear edge waves of different modes are investigated in [4, 7, 13]. Possible nonlinear effects in interactions of edge wave triads in a coastal zone are analyzed in [5, 11].

In the present paper we analyze the nonlinear mechanisms of possible anomalous edge wave appearance: nonlinear dispersion enhancement and modulation instability can lead to forming of “freak” edge wave.

2 Governing equations

The basic equations describing long wave motions of the ideal incompressible homogeneous fluid in a basin with a cylindrical shape of bottom with depth profile $h(y)$, are given by famous shallow-water theory

$$\frac{\partial \eta}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = -\frac{\partial(\eta u)}{\partial x} - \frac{\partial(\eta v)}{\partial y} \equiv N^\eta, \quad (1)$$

$$\frac{\partial(hu)}{\partial t} + gh \frac{\partial \eta}{\partial x} = -hu \frac{\partial u}{\partial x} - hv \frac{\partial u}{\partial y} \equiv N^u, \quad (2)$$

$$\frac{\partial(hv)}{\partial t} + gh \frac{\partial \eta}{\partial y} = -hv \frac{\partial v}{\partial y} - hu \frac{\partial v}{\partial x} \equiv N^v, \quad (3)$$

where t is time, x is alongshore coordinate, y is offshore coordinate, $\eta(x, y, t)$ is surface displacement, u and v are alongshore and transverse components of the depth-averaged velocity, and g

is acceleration due to gravity. The boundary conditions (with respect to the transverse coordinate y) for the Eqs. (1)–(3) state that the wave field is finite on the shore line (or the normal component of discharge hv tends to zero) and vanishes at infinity (as we consider trapped and not leaky modes).

In the literature nonlinear version of the shallow-water theory is often used in terms of velocity potential $\Phi(x, y, t)$ as well:

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \left((h(y) + \eta) \frac{\partial \Phi}{\partial x} \right) + \frac{\partial}{\partial y} \left((h(y) + \eta) \frac{\partial \Phi}{\partial y} \right) = 0, \quad (4)$$

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2} \left(\frac{\partial \Phi}{\partial x} \right)^2 + \frac{1}{2} \left(\frac{\partial \Phi}{\partial y} \right)^2 + g\eta = 0. \quad (5)$$

We will use both forms of shallow-water equations for description of the edge waves.

3 Dispersive properties of edge wave field

In the framework of the linear shallow water theory, governing equations reduce to the linear wave equation

$$\frac{\partial^2 \eta}{\partial t^2} - g \operatorname{div}(h \nabla \eta) = 0, \quad (6)$$

where differential operators grad and div act in the horizontal plane. After use of separation of variables in Eq. (6) we obtain a solution presenting monochromatic wave propagating along shore

$$\eta = AF(y) \exp(i(\omega t - kx)), \quad (7)$$

where A is constant (arbitrary) wave amplitude, the offshore structure $F(y)$ of edge waves satisfying the above boundary conditions is described by the eigenvalue problem:

$$\mathbf{L}F \equiv \frac{d}{dy} \left[h(y) \frac{dF}{dy} \right] + \left(\frac{\omega^2}{g} - h(y)k^2 \right) F = 0, \quad \frac{dF}{dy} = 0 \quad \text{at } y = 0, \quad F \rightarrow 0 \quad \text{at } y \rightarrow \infty. \quad (8)$$

The solution of this spectral problem (8) for given bottom profile $h(y)$ and alongshore wave number k is represented by a discrete set of eigenfunctions and eigenvalues $\{F_n(k, y), \omega_n\}$, corresponding to different edge wave modes. General properties of the given eigenvalue problem is described for instance in [15]. We would like to point out also that the phenomenon of minimum frequency at which separate modes of edge waves can exist, and the long-wavelength asymptotic behavior of wave velocity (determined by the velocity at maximum depth) are important characteristics of dispersion curves for edge waves over shelves with constant limit of depth at infinity, such as an exponential shelf, a step shelf etc.

Classical in the theory type of the edge waves (called ‘‘Stokes edge waves’’) is realized for the beach of constant slope, $h(y) = \alpha y$. In this case the offshore structure and dispersion relation of the edge waves are described by the infinite set

$$F_n(k, y) = L_n(2ky)e^{-ky}, \quad \omega_n^2 = (2n + 1)gk\alpha, \quad n = 0, 1, 2, \dots \quad (9)$$

where L_n is the n -th Laguerre polynomial. Offshore structure and dispersion relation of Stokes edge waves are demonstrated in Fig. 2 for beach slope, $\alpha = 10^{-3}$. First of all, it is necessary to point out that dispersion relation for each mode of the edge waves is similar to the dispersion relation of surface gravity waves in deep water. It means that the process of the freak wave formation from the transient unidirectional groups is similar for both classes of waves, and analytical solutions for deep-water surface gravity waves summarized for instance in the book [10] are valid for the single-mode edge waves. Main difference is that the wavenumber k is a scalar for edge waves, and there is a set of edge waves with different modal structure. This will lead to the cardinal difference in the nonlinear mechanisms of the rogue wave formation.

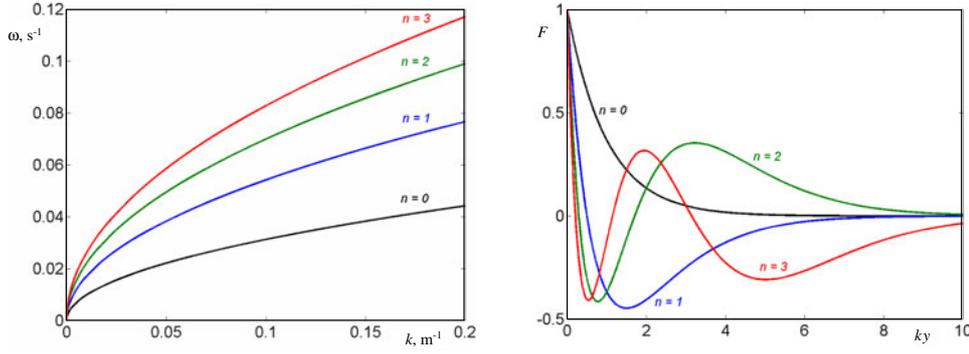


Fig. 2. Offshore structure and dispersion relation for the Stokes edge waves (slope, 10^{-3}).

4 Nonlinear instability of edge wave train

Nonlinear modulation instability of the lowest mode of the Stokes edge waves was investigated in several papers [23, 24] where nonlinear dispersion relation was derived. Such an analysis was extended for the edge waves of any modal number in [4]. Here we briefly reproduce basic results of this analysis and interpret it from the point of view of rogue wave phenomenon.

The solution of the Eqs. (4), (5) is written in the form of the progressive steady waves considering η and Φ as functions of $\theta = \Omega t - kx$ and y . Assuming that amplitude of wave field and potential are small but finite, η and Φ are expanded in the asymptotic series in orders of parameter ka

$$\eta = a\alpha \{ \eta_1(\theta, y) + ka\eta_2(\theta, y) + k^2a^2\eta_3(\theta, y) + \dots \}, \quad (10)$$

$$\Phi = ag\alpha\omega^{-1} \{ \Phi_1(\theta, y) + ka\Phi_2(\theta, y) + k^2a^2\Phi_3(\theta, y) + \dots \}. \quad (11)$$

The wave frequency, Ω should be represented also as series in powers of parameter ka

$$\Omega^2 = \omega^2 \{ 1 + \gamma k^2 a^2 + \dots \}, \quad (12)$$

where ω is linear frequency of any single edge wave mode satisfying to the linear dispersion relation (9). We will not give here the complicated technical details of the calculations of the nonlinear dispersion relation in the third order of the perturbation theory (see [4]). In particular, for the beach of constant slope, coefficients of nonlinear correction in dispersion relation are found for the first 18 edge wave modes: they are always positive and decrease algebraically as the mode number increases and can be approximated by the regression curve $\gamma_n = (1 + 8n)^{-1}$ (Fig. 3). As the nonlinear dispersion relation is known, we can immediately compute the sign of Lighthill's criterion [19] $L = \frac{\partial \Omega}{\partial a^2} \frac{\partial^2 \Omega}{\partial k^2} < 0$, and because it is negative, it means that edge wave packet is modulationally unstable. This result was derived for the lowest mode of Stokes edge waves and remains correct for edge wave of any mode.

Nonlinearity leads also to asymmetry of the wave profile in time, its trough is sharper than the crest. Figure 4 shows profiles of the first mode ($n = 1$) of Stokes edge wave at the shoreline ($y = 0$) with nonlinear corrections for parameter $ka = 0.14$. On a base of the nonlinear dispersion relation, the nonlinear Schrodinger equation for the complex wave amplitude can be reconstructed, or it can be derived using the asymptotic approach. For independent propagation of any selected mode of Stokes edge wave the nonlinear Schrodinger equation is a classical form

$$i \left[a_t + \frac{\omega(k)}{2k} a_x \right] - \frac{\omega(k)}{8k^2} a_{xx} - \frac{1}{2} \gamma \omega(k) k^2 |a|^2 a = 0, \quad (13)$$

where a is complex wave amplitude, indices denote partial derivatives. Equation (13) can be presented in the dimensionless form:

$$i \left[A_\tau + \frac{1}{2} A_\xi \right] - \frac{1}{8} A_{\xi\xi} - \frac{1}{2} |A|^2 A = 0, \quad (14)$$

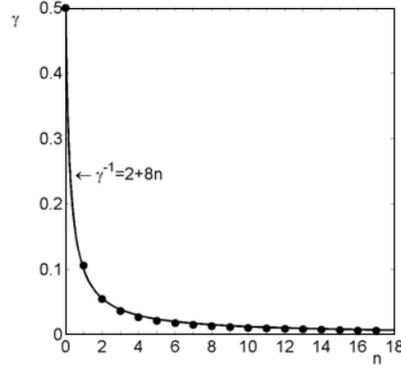


Fig. 3. Coefficient γ for different modes of Stokes edge waves and approximation curve.

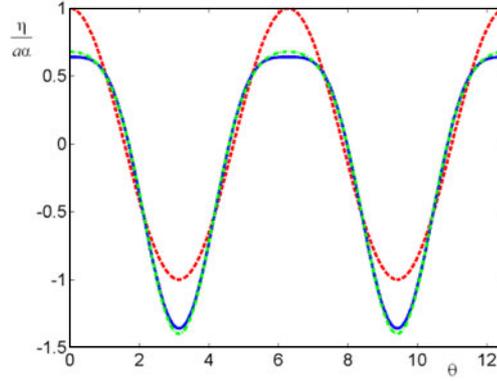


Fig. 4. Profiles of the first mode ($n = 1$) Stokes edge waves at the shoreline ($y = 0$) with nonlinear corrections: η_1 – red, $\eta_1 + k\alpha\eta_2$ – blue, $\eta_1 + k\alpha\eta_2 + k^2a^2\eta_3$ – green.

where $|A| = ka/\sqrt{\gamma}$, $\xi = kx$, and $\tau = \omega t$; k and ω are wave number and frequency of the carrier wave. Earlier this equation has been derived for the lowest mode of the Stokes wave only [1]. Since the nonlinear coefficient decreases as the mode number increases ($\gamma\omega$ behaves as $n^{-1/2}$), it follows that higher-mode edge waves are more linear (dispersive effects grows up, and nonlinear effects reduces) and more stable (instability increment drops down as mode number grows up) if their steepness and wave number are identical. It is principal to mention that nonlinear Schrodinger equation for edge waves is always one-dimensional, because the structure of wave packet in offshore direction is described by the Laguerre polynomial.

Modulational instability in the framework of the nonlinear Schrodinger equation and formation of the rogue waves is well studied for last 10 years and presented in several papers in this special issue of EPJ. We will not discuss it in details.

Real field of the edge waves is random and it contains both amplitude and frequency modulation. We studied joint effect of both kinds of modulations on forming of the rogue waves. Numerically, the initial conditions are given by

$$A = A_0 (1 + m \cos(K(\xi - \xi_0))) \exp(i(\xi - \xi_0)^2/d^2), \quad (15)$$

where A_0 and d determines the amplitude and width of the initial perturbation, ξ_0 is the position of the center of the pulse, m is the modulation coefficient and K is the envelope wavenumber, chosen from the Lighthill's instability criterion $K < K_* = \sqrt{8A_0^2}$. In dimensional variables a domain of unstable wavenumbers converges for higher modes of edge waves as $n^{-1/2}$.

Figure 5 displays the wave focusing of the “pure” amplitude modulation ($m = 0.1$, $d = \infty$) for $A_0 = 0.0235$ and pure frequency modulated wave train ($m = 0$, $d = 5$) for $A_0 = 0.07$. As it can be seen, both mechanisms: modulation instability and nonlinear dispersive focusing - result in appearance of the freak edge waves.

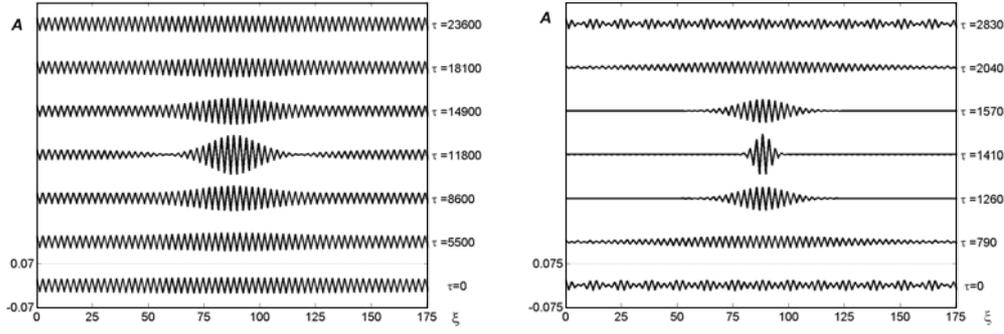


Fig. 5. Illustration of the process of edge rogue wave forming (one-mode edge wave profile on the shoreline is shown schematically): initial amplitude modulation – left panel, and initial frequency modulation – right panel.

5 Nonlinear interactions of edge waves

Due to multimodal character of the edge wave field, different modes can interact between them.

Let us introduce the small parameter ε , describing a weak amplitude of wave motions $\eta = \varepsilon\tilde{\eta}$, $u = \varepsilon\tilde{u}$, $v = \varepsilon\tilde{v}$. Then the initial shallow-water system can be reduced to (tildes are dropped):

$$\frac{\partial^2 \eta}{\partial t^2} - gh \frac{\partial^2 \eta}{\partial x^2} - g \frac{\partial}{\partial y} \left(h \frac{\partial \eta}{\partial y} \right) = \varepsilon N, \quad (16)$$

where $N = \frac{\partial N^{\eta}}{\partial t} - \frac{\partial N^v}{\partial x} - \frac{\partial N^u}{\partial y}$ contains all nonlinear terms. As usual for interaction, slow time and alongshore space scales should be determined: $T = \varepsilon t$, $X = \varepsilon x$ and wave field presented as an asymptotic series

$$\eta = \eta^{(1)} + \varepsilon \eta^{(2)} + \varepsilon^2 \dots, \quad \eta^{(1)} = \sum_r \sum_n \frac{1}{2} A_r^n(T) F_r^n(y) E_r^n + C.C., \quad E_r^n = \exp(i(k_r x - \omega_r^n t)), \quad (17)$$

where $A(T)$ are wave amplitudes slowly varied in time. After substitution of (17) into (16) and collecting terms of the same order of ε , in each approximation we receive inhomogeneous problems for each combination (n, r) . We require the forcing for each component to be orthogonal to the solution of the adjoint of the original eigenvalue problem (8). It is well known that nonlinear interactions are intensive when resonant conditions are realized:

$$\pm k_p \pm k_q - k_r = 0, \quad \pm \omega_p^l \pm \omega_q^m - \omega_r^n = 0. \quad (18)$$

The final evolution equation for each discrete mode in the system has the form:

$$\begin{aligned} \frac{\partial A_r^n}{\partial T} = & i \sum_p \sum_q \sum_l \sum_m \left\{ +T_{pqrl}^{lmn} A_p^l A_q^m \theta(k_p + k_q, k_r) \theta(\omega_p^l + \omega_q^m, \omega_r^n) \right. \\ & + -T_{pqr}^{lmn} A_p^l A_q^{m*} \theta(k_p - k_q, k_r) \theta(\omega_p^l - \omega_q^m, \omega_r^n) \\ & \left. + -T_{qpr}^{mln} A_p^{l*} A_q^{m*} \theta(k_q - k_p, k_r) \theta(\omega_q^m - \omega_p^l, \omega_r^n) \right\}, \quad (19) \end{aligned}$$

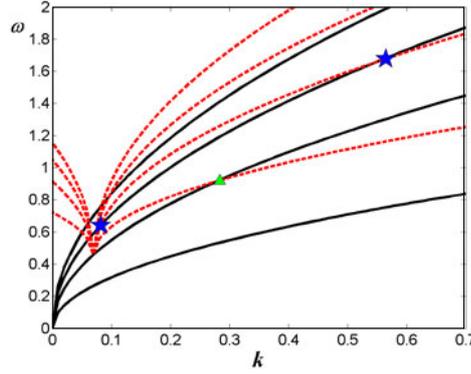


Fig. 6. Geometrical interpretation of synchronism conditions for triads of edge waves propagating the same directions: triangle shows the only point, defining interacting triad with $n_1 = 1$, $n_2 = 0$, $n_3 = 1$; in a triad with $n_1 = 1$, $n_2 = 1$, $n_3 = 2$ two different ways of energy interchanging (asterisk) are possible.

where l, m, n denote number of modes, p, q, r denote number of waves, $\theta(a, b) = 0$ at $a \neq b$, $\theta(a, b) = 1$ at $a = b$, and coefficients of interaction for sum and difference interactions are:

$$\begin{aligned} \pm T_{pqr}^{lmn} = & -\frac{1}{4\omega_r^n \omega_p^l \omega_q^m \int_0^\infty (F_r^n)^2 dy} \int_0^\infty \frac{F_r^n}{h} \left\{ g\omega_p^l \omega_q^m \frac{dh}{dy} \frac{dF_p^l F_q^m}{dy} \mp g^2 \frac{dh^2}{dy} \left[k_p^2 F_p^l \frac{dF_q^m}{dy} + k_q^2 F_q^m \frac{dF_p^l}{dy} \right] \right. \\ & + 2gh F_p^l F_q^m \left[k_p k_q \left((\omega_r^n)^2 \pm \{2ghk_p k_q - \omega_p^l \omega_q^m\} \right) \mp \left((\omega_p^l)^2 k_q^2 + (\omega_q^m)^2 k_p^2 \right) \right] \\ & \left. \mp 2gh \frac{dF_p^l}{dy} \frac{dF_q^m}{dy} \left[(\omega_r^n)^2 \pm (2ghk_p k_q - \omega_p^l \omega_q^m) \right] \pm 2g^2 \frac{dF_p^l}{dy} \frac{dF_q^m}{dy} \left(2 \left(\frac{dh}{dy} \right)^2 - h \frac{d^2 h}{dy^2} \right) \right\} dy. \end{aligned} \quad (20)$$

Let us consider nonlinear three-wave interaction of edge waves propagating over the beach of constant slope in both directions. In the case of positive wave numbers, one or two ways of energy exchange between the modes of the triad is possible, depending on the relationship between the mode numbers (Fig. 6).

We considered interactions in triads composed of the waves of the four lowest modes, because higher modes have higher frequencies and must rapidly attenuate. The calculations have shown the intriguing fact, that the nonlinear interaction coefficients can vanish for certain (but not for all) triads of edge waves, including unidirectional waves (Fig. 7). The coefficients of the resonance three-wave interaction can be expressed in terms of power functions of frequency and slope; for a fixed frequency, they increase as the angle of the slope decreases.

It is interesting to illustrate the process of interaction at least for the simplest example of the single triad propagating over a sloping beach, i.e. when edge wave field is composed of three waves satisfying the synchronism condition (18). For this situation the system of evolution equations for amplitudes of edge wave modes (19) is reduced to

$$\begin{aligned} A_{1T} &= i\mu_1 A_2^* A_3, \\ A_{2T} &= i\mu_2 A_1^* A_3, \\ A_{3T} &= i\mu_3 A_1 A_2, \end{aligned} \quad (21)$$

where A_j – is amplitude of the wave of mode number n_j , index T denotes derivative on T , “*” denotes complex conjugate, i is imaginary unit. Processes described by such systems are actively investigated in nonlinear optics with the field intensity $|A|^2$ used as the measured quantity. In geophysics, the measured quantities are the velocity and displacement fields, which

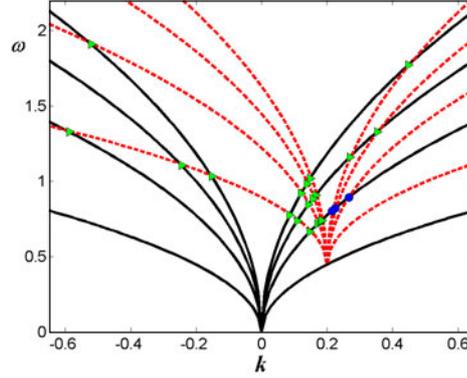


Fig. 7. Geometrical interpretation of synchronism conditions of edge wave triads for four lowest modes with $n_1 = 0$ $n_1 = 0$. Markers show points corresponding to synchronism conditions: circles mark triads with zero interaction coefficients, and triangles mark triads with nonzero interaction coefficients.

depend in the case under consideration on two horizontal coordinates. Therefore, a complex interference structure of the wave field appears even if a unique resonance triad is present. Consider, for example, the triad "001" with multiple wave numbers; for this triad $\mu_1 = 8\mu_2$, $\mu_3 = 9\mu_2$, $\mu_2 = \frac{K}{8}\sqrt{\frac{gK}{\alpha}}$, $k_1 = 4K$, $k_2 = -K$, $k_3 = 3K$ (here K is an arbitrary multiple which can be used to nondimensionalize wavenumbers, frequencies and interaction coefficients). Stable balance state in system (21) is reached for triad with constant moduli of complex amplitudes:

$$\begin{aligned}
 |A_1| = |A_2| = A, \quad |A_3| &= A\sqrt{\frac{\mu_3}{\mu_1 + \mu_2}}, \quad A = const, \\
 \arg A_{1,2} = \varphi_{1,2} &= \mu_{1,2}A\sqrt{\frac{\mu_3}{\mu_1 + \mu_2}}t + \varphi_{01,02}, \\
 \arg A_3 = \varphi_3 &= A\sqrt{\mu_3(\mu_1 + \mu_2)}t + \varphi_{01} + \varphi_{02}, \quad \varphi_{01,02} = const.
 \end{aligned} \tag{22}$$

For the specified triad "001" edge wave field normalized on the value of A , in non-dimensional coordinates $\tilde{x} = Kx$, $\tilde{y} = Ky$, $\tau = \sqrt{\alpha g K T}$ has the form

$$\begin{aligned}
 \tilde{\eta}(\tilde{x}, \tilde{y}, \tau) &= F_1^0(4\tilde{x}) \cos\left(4\tilde{y} - 2\tau + \frac{KA}{\alpha}\tau\right) + F_2^0(\tilde{x}) \cos\left(\tilde{y} + \tau + \frac{KA}{8\alpha}\tau\right) \\
 &+ F_3^1(3\tilde{x}) \cos\left(3\tilde{y} - 3\tau + \frac{9KA}{8\alpha}\tau\right).
 \end{aligned} \tag{23}$$

Figure 8 shows that the interacting edge wave field has a complex interference structure even in the case of a single triad presented.

6 Discussion

As it is demonstrated, non-stationary and nonlinear dynamics of the edge waves induce the appearance of the short-living large-amplitude localized pulses which can be called by the freak edge waves. They can appear as a result of the action of two physical mechanisms: dispersive focusing and nonlinear modulation instability. Possible nonlinear effects in interactions of edge wave triads in coastal zone are analyzed. Edge waves of different modes can exchange a large amount of energy. In comparison with wind waves where the modulation instability and dispersive focusing are the major sources of the freak waves, the nonlinear interaction of different

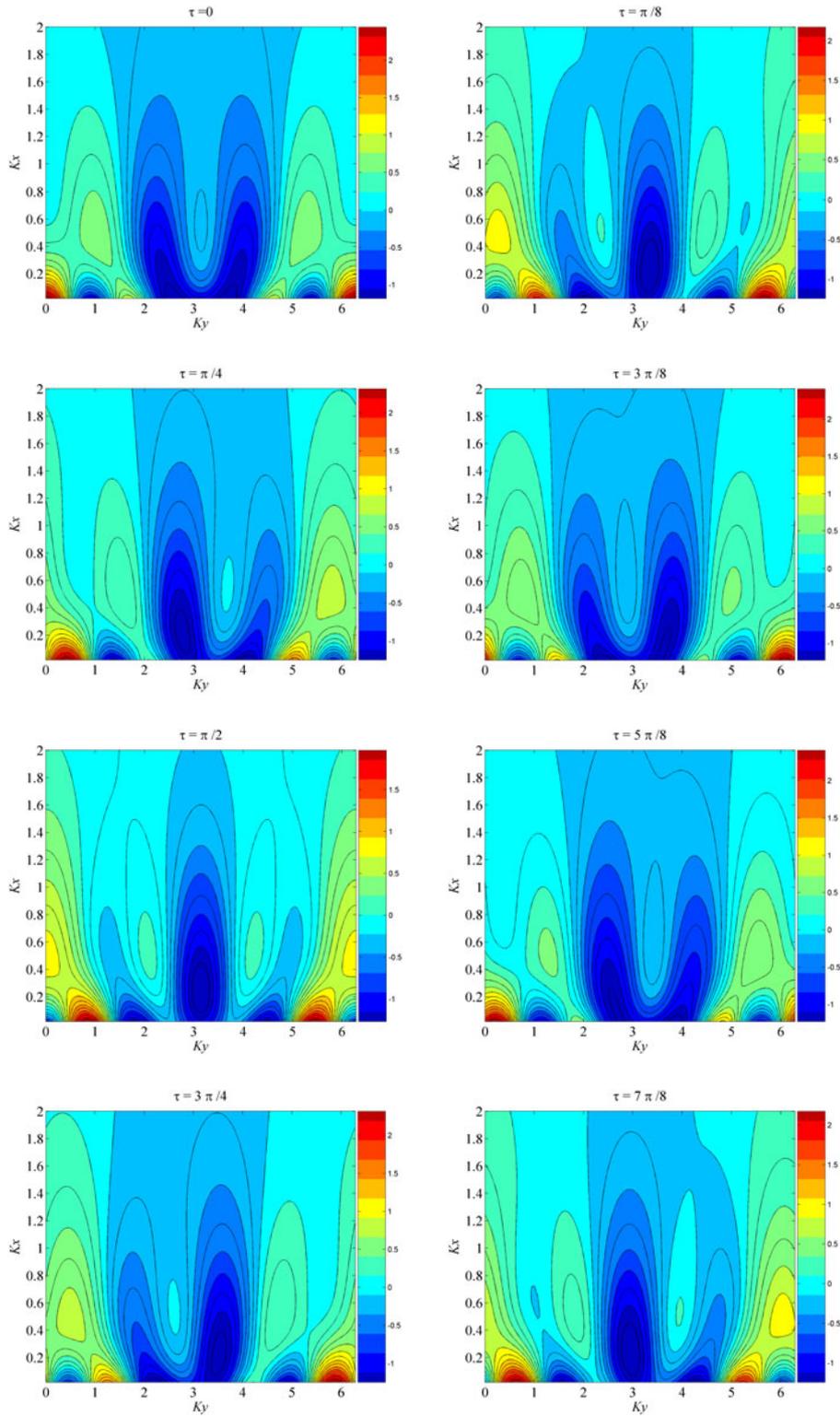


Fig. 8. Edge wave field (23) corresponding to the stationary-amplitude solution of system (21) for triad “001” ($KA = 8\alpha$) for different dimensionless times $\tau = \sqrt{\alpha g K T}$.

modes can contribute to the forming of freak waves, and this mechanism is new in the rogue wave phenomenon.

Edge waves in the ocean are the large-scale waves and their existence depends from the bathymetry of the coastal zone. Any irregularities of the bottom relief and coastal line lead to scattering of the wave energy and effective dissipation. External wind wave field as well as unstable alongshore current generating the edge waves also lead to the randomization of the edge wave field. Due to lack of experimental observations, the effectiveness of considered mechanisms of the edge freak wave formation in the nature could not be now evaluated, and this is an open problem.

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