

From Editorial Board. *This issue is devoted to the 80th Anniversary of Prof. Viktor Anatol'evich Solntsev, member of the Editorial Board, prominent scientist in microwave electronics, Doctor of Science (see Comm. Tech. no. 1, 2011).*

In this issue, we present the first scientific work of V.A. Solntsev, which was published in Trudy NII Minradio-proma, no. 1 (21), 1955. This paper was based on the diploma work, which was supervised by A.S. Tager. In spite of the fact that the paper was published long time ago and despite the significant progress in the numerical methods and simulation software, we underline the topicality of the work, since the proposed methods for the approximate calculations of complicated helical slow-wave systems are used at present by experts to estimate the TWT parameters. Note that the work is not familiar for scientists and engineers working in microwave electronics, since the journal was almost inaccessible.

Analysis of the Dispersion of the Helical Slow-Wave System with Dielectric Supports¹

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Abstract—The effect of dielectric supports on the slowing factor and dispersion of the helical line in TWT is considered. A method for the calculation of the slowing down in the helical line with the complicated configuration of the dielectric supports is proposed. A procedure for the experimental study of dispersion in the helical slow-wave system is presented. The calculated results are compared with the experimental data.

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INTRODUCTION

The TWT parameters (dispersion of helix, gain, etc.) are substantially affected by dielectric supporting tubes and rods that fix the helical slow-wave system [1].

The problem is important for the development of new TWTs. Therefore, it is expedient to elaborate a method for the calculation of the helical slow-wave systems with different types of fixing to determine the slowing down and dispersion with the accuracy needed for practical applications. The calculation of the helix that is fixed inside a dielectric tube can be found in [1]. However, the fixing of helix between several dielectric tubes or rods is more convenient for practice.

In this work, we consider the approximate calculation of such systems, specify the results from [1], and propose a method for the experimental study of slowing down and dispersion of helical lines in the centimeter and decimeter wavelength ranges.

The comparison of the calculated and experimental slowing factors and dispersions of the lines shows that the proposed method can be used for the quantitative calculation of the parameters of the helical slow-wave systems.

1. CALCULATION OF THE SLOWING FACTOR OF THE HELIX FOR A COMPLICATED CONFIGURATION OF THE CYLINDRICAL SUPPORTING ELEMENTS

We consider the slowing of the fundamental wave in the helix that is fixed with the aid of one or several dielectric cylindrical elements with arbitrary cross sections and the axes parallel to the axis of helix. In particular, we consider dielectric tubes or rods that hold the helix (Figs. 1b and 1c).

An exact solution of the problem of the wave propagation along such a system is impossible but the effect of insulator on the slowing down of the fundamental wave can be approximately taken into account provided that the problem is reduced to the problem from [1], which involves the propagation of waves along the helix that is fixed inside a dielectric tube (Fig. 1a). We choose the size of the tube in such a way that the effect of the tube on the slowing down of the fundamental wave is equivalent to the effect of the supporting dielectric elements in the system under study.

For this purpose, we assume that the effect of insulator that is placed at a certain point in the vicinity of the helix on the slowing down of the fundamental wave is proportional to the unperturbed electric field strength at the given point. Evidently, such an assumption leads to errors related to the distortions of electric field in the presence of dielectric units. However, the experiments (see section 2) show that the errors are relatively small.

¹ This work is edited in accordance with the journal style.

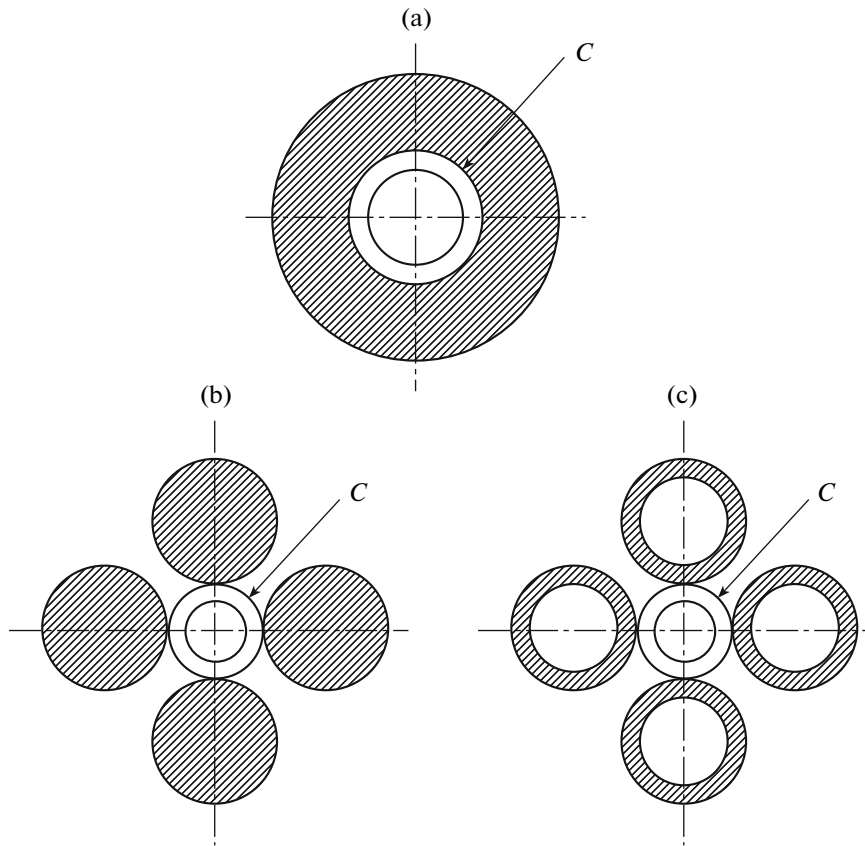


Fig. 1. Methods for the fixing of helix using dielectric rods and tubes (C is helix).

On the above assumption, the effect of insulator on the phase velocity of the fundamental wave in cylindrical systems is characterized by the integral

$$\Gamma = \int_{(s)} \Psi \rho ds, \tag{1}$$

where Ψ is the function that describes the electric-field distribution over cross section s that is perpendicular to the axis of helix and ρ is the distribution function of the dielectric material over the same cross section. Note that the systems with different shapes, numbers, and positions of dielectric units appear equivalent with respect to the slowing factor of the fundamental wave if integrals (1) are equal to each other.

For the fundamental wave, we have

$$\Psi = \Psi(r). \tag{2}$$

Thus, expression (1) can be represented as

$$\Gamma = \int_0^\infty \Psi P(r) r dr. \tag{3a}$$

Here,

$$P(r) = \int_0^{2\pi} \rho(r, \varphi) d\varphi \tag{3b}$$

is the function that characterizes the distribution of the dielectric material along radius r ($r = 0$ on the axis of helix). The electric field as a function of radius in the unperturbed systems is described with Bessel functions $K_1(\gamma r)$ for components E_r and E_φ and $K_0(\gamma r)$ for component E_z . Here, $\gamma = \sqrt{\beta^2 - k^2}$ is the radial propagation constant of the fundamental wave in the helix, $\beta = \omega/v$ is the propagation constant of the fundamental wave, ω is the circular frequency, v is the phase velocity of the fundamental wave in the slow-wave system, $k = \omega\sqrt{\epsilon_0\mu_0}$ is the wave propagation constant in vacuum, and ϵ_0 and μ_0 are permittivity and permeability of vacuum, respectively.

For the conventional TWTs, we have $\gamma a > 1$ (a is the mean radius of helix) and, hence, $\gamma r > \gamma a > 1$, since $r > a$, in the region of the fixing elements. In this case, we obtain $K_0(\gamma r) \approx K_1(\gamma r)$. On the other hand, we have

$$\left| \frac{E_z}{E_r} \right| \sim \frac{\gamma}{\beta} < 1 \quad \text{and} \quad |E| \sim K_1(\gamma r)$$

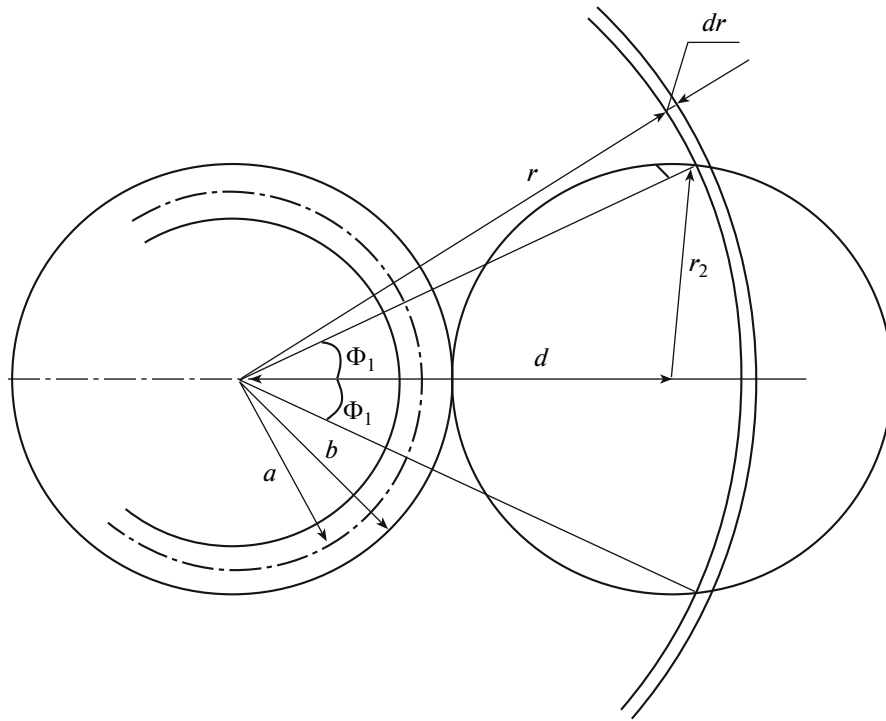


Fig. 2. Schematic drawing for the calculation of function $P(r)$ for the rod.

when $\gamma r \ll 1$ and $\beta \rightarrow k$ [3].

In general, we assume that $\Psi(r) = K_1(\gamma r)$ with the accuracy that is sufficient for practical applications. In this case, integral (3a) is represented as

$$\Gamma = \int_0^{\infty} K_1(\gamma r) P(r) r dr. \tag{4}$$

We can easily calculate function $P(r)$ for real structures. Below, we consider two most frequent systems: (i) the helix that is fixed inside a dielectric tube (Fig. 1a) and (ii) the helix that is fixed between several dielectric rods (Fig. 1b) or tubes (Fig. 1c).

In the first case, we have

$$P_0(r) = \int_0^{2\pi} \rho(r, \varphi) d\varphi = \begin{cases} 2\pi & \text{when } R_1 < r < R_2; \\ 0 & \text{when } r < R_1, r > R_2. \end{cases} \tag{5}$$

Here, R_2 and R_1 are external and internal radii, respectively, of the equivalent tube that encircles the helix.

In the second case, we preliminary calculate function $P_1(r)$ for one rod that is adjacent to the helix (Fig. 2, d is the radius of the position of the rod center). We consider a ring with radius r and width dr . The amount of the dielectric material inside the part of the ring that intersects the rod is proportional to the quantity

$$\int_{-\Phi_1}^{\Phi_1} r dr d\varphi = 2r\Phi_1 dr = P_1 dr. \tag{6}$$

Based on the geometrical data, we find

$$\Phi_1 = \arccos \frac{r^2 + d^2 - r_2^2}{2rd}. \tag{7}$$

Thus, distribution function $P_1(r)$ for the rod is represented as

$$P_1(r) = 2r \arccos \frac{r^2 + d^2 - r_2^2}{2rd}. \tag{8}$$

A similar procedure makes it possible to derive function P_1 for the tube with inner radius r_1 and outer radius r_2 (Figs. 1c and 2):

$$P_1(r) = \begin{cases} 2r \arccos \frac{r^2 + d^2 - r_2^2}{2rd} & \text{when } d - r_2 \leq r \leq d - r_1, d + r_1 \leq r \leq d + r_2; \\ 2r \left[\arccos \frac{r^2 + d^2 - r_2^2}{2rd} - \arccos \frac{r^2 + d^2 - r_1^2}{2rd} \right] & \text{when } d - r_1 \leq r \leq d + r_1. \end{cases} \tag{9}$$

If the helix is fixed in several dielectric rods or tubes, we find distribution function $P_n(r)$ of the system as a sum of functions $P_1(r)$, given by expressions (8)

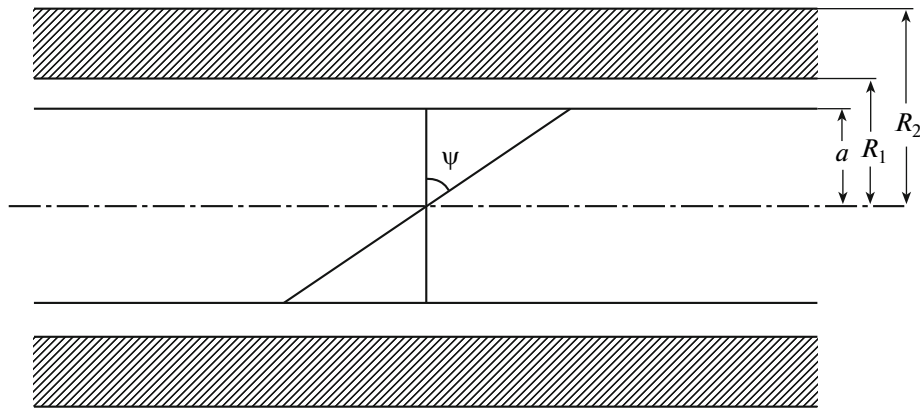


Fig. 3. Schematic drawing of the helix placed inside the dielectric tube.

and (9), for all of the tubes. For n identical rods or tubes, we have

$$P_n(r) = nP_1(r). \tag{10}$$

In accordance with the above approach, we find the size of the dielectric tube that is equivalent to the system of supporting elements in the calculation of the slowing factor using the following equality:

$$\Gamma = 2\pi \int_{R_1}^{R_2} rK_1(\gamma r) dr, \tag{11}$$

where R_1 is the inner radius of the equivalent tube, which can be assumed equal to the outer radius of helix, and R_2 is the outer radius of the tube.

The resulting equation makes it possible to find the thickness of the equivalent tube ($R_2 - R_1$), that provides the slowing down at the given frequency that is identical to the slowing down in the system under study.

For example, the equation for the system consisting of n identical dielectric tubes that hold the helix is written as

$$\int_{\gamma R_1}^{\gamma R_2} xK_1(x) dx = \frac{n}{\pi} \left[\int_{\gamma(d-r_2)}^{\gamma(d+r_2)} xK_1(x) \arccos \frac{x^2 + \gamma^2(d^2 - r_2^2)}{2\gamma xd} dx - \int_{\gamma(d-r_1)}^{\gamma(d+r_1)} xK_1(x) \arccos \frac{x^2 + \gamma^2(d^2 - r_1^2)}{2\gamma xd} dx \right]. \tag{12}$$

The calculations show that the thickness of the equivalent tube decreases with increasing frequency. The physical reason for such a result lies in the fact that an increase in the frequency leads to the compression of the fields around the helix. Thus, the outer

parts of the quartz tubes that hold the helix do not affect the slowing factor (i.e., the effective total amount of insulator decreases).

We conclude that the proposed method makes it possible to reduce the calculation of the slowing down in the helical line with an arbitrary cylindrical system that fixes the helix to the calculation of the slowing down in the helix that is placed inside a dielectric tube. In the calculation of such a system in [1], the helix is changed by an anisotropically conducting cylinder whose radius is equal to the mean radius of the helix. The inner radius of the dielectric tube is also assumed equal to the mean radius of the helix. The thickness of the wire of helix is disregarded, and the calculated slowing factor appears overestimated.

For a more accurate analysis of the slowing down, we must consider a system in which the inner radius of the tube is equal to or slightly greater than the outer radius of the helix (depending on the configuration of the line), so that a gap exists between the helical cylinder and the tube (Fig. 3).

For the calculation of such a system, we can employ the method from [1]. Here, we present the final results, and the details of calculation can be found in [5].

The dispersion relation for propagation constant γ is represented as

$$(k \cot \psi)^2 = (\gamma a)^2 \frac{I_0(\gamma a) K_0(\gamma a)}{I_1(\gamma a) K_1(\gamma a)} \Delta_1^2, \tag{13}$$

where ψ is the helix angle (Fig. 3),

$$\Delta_1^2 = \left(1 + \frac{I_0(\gamma a)}{K_0(\gamma a)} \frac{1}{\{3\}} \right) \left(1 - \frac{I_1(\gamma a)}{K_1(\gamma a)} \frac{1}{\{4\}} \right)^{-1}, \tag{14}$$

$$\{3\} = \left[\frac{I_0(\gamma_d R_1) + \{1\} K_0(\gamma_d R_1) \varepsilon_0 \gamma_d}{I_1(\gamma_d R_1) - \{1\} K_1(\gamma_d R_1) \varepsilon_d \gamma} I_1(\gamma R_1) - I_0(\gamma R_1) \right] \quad (15)$$

$$\times \left[\frac{I_0(\gamma_d R_1) + \{1\} K_0(\gamma_d R_1) \varepsilon_0 \gamma_d}{I_1(\gamma_d R_1) - \{1\} K_1(\gamma_d R_1) \varepsilon_d \gamma} K_1(\gamma R_1) + K_0(\gamma R_1) \right]^{-1}, \quad (16)$$

$$\{4\} = \left[\frac{I_0(\gamma_d R_1) + \{2\} K_0(\gamma_d R_1) \gamma_d}{I_1(\gamma_d R_1) - \{2\} K_1(\gamma_d R_1) \gamma} I_1(\gamma R_1) - I_0(\gamma R_1) \right] \times \left[\frac{I_0(\gamma_d R_1) + \{2\} K_0(\gamma_d R_1) \gamma_d}{I_1(\gamma_d R_1) - \{2\} K_1(\gamma_d R_1) \gamma} K_1(\gamma R_1) + K_0(\gamma R_1) \right]^{-1},$$

$$\{1\} = \left[\frac{\varepsilon_d \gamma K_0(\gamma R_2)}{\varepsilon_0 \gamma_d K_1(\gamma R_2)} I_1(\gamma_d R_2) + I_0(\gamma_d R_2) \right] \times \left[\frac{\varepsilon_d \gamma K_0(\gamma R_2)}{\varepsilon_0 \gamma_d K_1(\gamma R_2)} K_1(\gamma_d R_2) - K_0(\gamma_d R_2) \right]^{-1}, \quad (17)$$

$$\{2\} = \left[\frac{\gamma K_0(\gamma R_2)}{\gamma_d K_1(\gamma R_2)} I_1(\gamma_d R_2) + I_0(\gamma_d R_2) \right] \times \left[\frac{\gamma K_0(\gamma R_2)}{\gamma_d K_1(\gamma R_2)} K_1(\gamma_d R_2) - K_0(\gamma_d R_2) \right]^{-1}. \quad (18)$$

Here, $\varepsilon = \varepsilon_d / \varepsilon_0$ is the permittivity ratio of the dielectric medium and vacuum, $\gamma_d = \sqrt{\beta^2 - k_d^2}$ is the radial propagation constant of the fundamental wave in the dielectric material, and $k_d^2 = \varepsilon_d k^2$.

In the absence of the insulator ($\varepsilon_d = \varepsilon_0$ and $\gamma_d = \gamma$), we have $\Delta_1^2 = 1$ and Eq. (13) is transformed into the dispersion relation for the free helix.

The expression for Δ_1^2 can be simplified with allowance for the fact that $\gamma_d = \gamma$ under substantial slowing ($v^2 \ll c^2$). In this case, we have $\{2\} \rightarrow \infty$, $\{4\} \rightarrow -\infty$, and

$$\Delta_1^2 \rightarrow \Delta_{11}^2 = 1 + \frac{I_0(\gamma a)}{\{3\}' K_0(\gamma a)}, \quad (19a)$$

where

$$\{3\}' = \left[\frac{I_1(\gamma R_1) K_1(\gamma R)}{K_1(\gamma R_1) K_0(\gamma R)} \frac{I_0(\gamma R_1) + \{1\}' K_0(\gamma R_1)}{I_1(\gamma R_1) - \{1\}' K_1(\gamma R_1)} - \varepsilon \frac{I_0(\gamma R_1)}{K_0(\gamma R_1)} \right] \left[\frac{K_1(\gamma R_1) I_0(\gamma R_1) + \{1\}' K_0(\gamma R_1)}{K_0(\gamma R_1) I_1(\gamma R_1) - \{1\}' K_1(\gamma R_1)} + \varepsilon \right]^{-1}, \quad (19b)$$

$$\{1\}' = \frac{1}{\varepsilon - 1} \left[\frac{I_0(\gamma R_2)}{K_0(\gamma R_2)} + \varepsilon \frac{I_1(\gamma R_2)}{K_1(\gamma R_2)} \right]. \quad (19c)$$

When $(\gamma a) > 2$, we can additionally simplify Eq. (13) substituting asymptotic expressions for cylindrical functions [1]. Additionally assuming that $\gamma \approx \beta$, we represent expression (13) as

$$\frac{k^2}{\beta^2} \cot^2 \psi = 1 - \exp[-2\beta(R_1 - a)] \times \frac{(\varepsilon^2 - 1) \exp[2\beta(R_2 - R_1)] - 1}{(\varepsilon + 1)^2 \exp[2\beta(R_2 - R_1)] - (\varepsilon - 1)^2}. \quad (20)$$

At a relatively large width of the gap ($R_1 - a$) and a small thickness of the tube ($R_2 - R_1$) (Fig. 3), the insulator insignificantly affects the field of helix and Eq. (20) appears inapplicable, since the correction term that takes into account the effect of insulator becomes less than the error resulting from the substitution of the asymptotic expressions for the cylindrical functions. Figure 4 demonstrates the qualitative dependence of slowing factor c/v on parameter γa .

Here, curve 1 characterizes the dispersion of the helix in the absence of the insulator (see, for example, [2–4]). Curve 2 corresponds to scenario from [1]: $R_1 = a$ (the helix made of a very thin wire). Curve 3 shows the dispersion relation given by expression (13) at $R_1 \neq a$. A distinctive feature of this curve is the maximum slowing down at $\gamma a = (\gamma a)_{\text{opt}}$ and the anomalous dispersion (an increase in the phase velocity of the wave with increasing frequency) at $\gamma a > (\gamma a)_{\text{opt}}$. The physical interpretation of this phenomenon is evident: at relatively high frequencies, the electromagnetic field is compressed around the helix and is concentrated in the gap between the insulator and helix, so that the effect of the insulator on the slowing factor becomes weaker and the phase velocity of the fundamental wave increases and tends to $c \cot \psi$.

Note that, in the vicinity of the maximum, the phase velocity of the wave slightly depends on the frequency, so that the presence of the dielectric tube can lead to an increase in the dispersion at extremely low and high frequencies or to a decrease in the dispersion in the range $\gamma a = (\gamma a)_{\text{opt}} \approx 1-3$.

2. DISPERSION IN THE HELICAL LINE: METHOD FOR THE STUDY AND EXPERIMENTAL RESULTS

We investigate the slowing down and dispersion in the helical line using the probe method to measure waveforms of standing waves and employ the results to determine the wavelength of slow wave in the system λ_g . Figure 5 demonstrates the block diagram of the setup. The high-power signal from an LMS-551 oscillator (1) is fed via the matching unit (2) to the helical

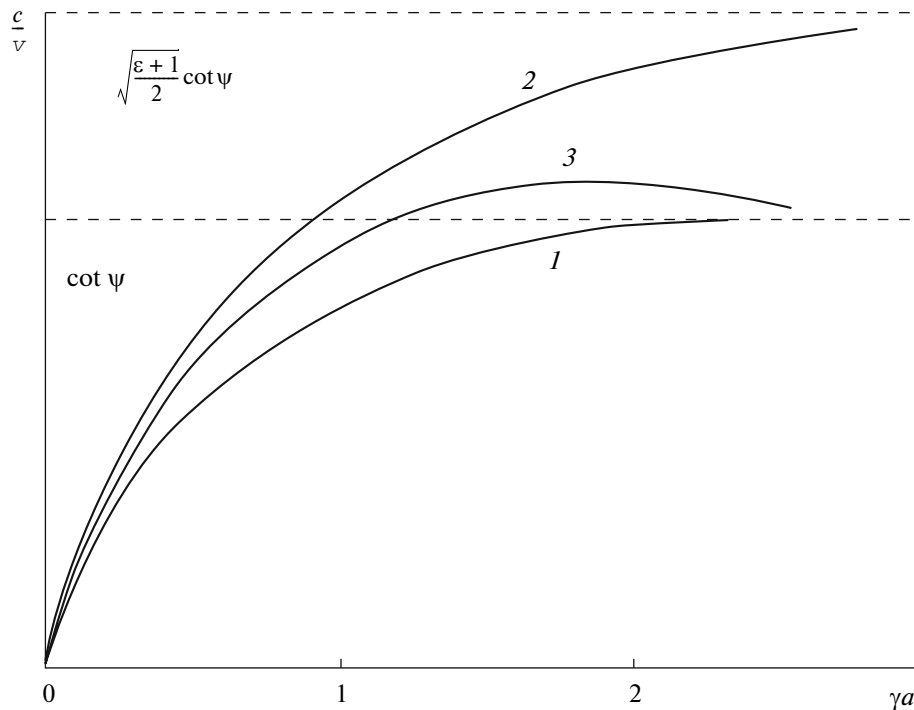


Fig. 4. Qualitative dispersion curves for (1) free helix, (2) the helix placed inside the dielectric tube whose inner radius is equal to the mean radius of helix, and (3) the helix placed inside the dielectric tube whose inner radius is greater than the mean radius of helix.

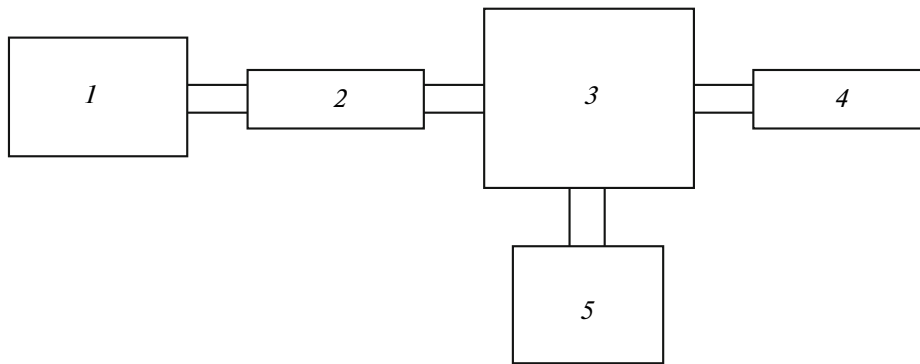


Fig. 5. Block diagram of the setup for the dispersion measurements of the helical line: 1 signal generator, 2 matching unit, 3 measurement line with the helix, 4 equivalent of antenna, and 5 microammeter.

line that represents a helix fixed in dielectric tubes and surrounded with a metal cylinder. The diameter of the external cylinder is small enough to prevent the propagation of higher modes. The helical line is fixed on holder (3) with a translation stage that is equipped with a probe head. The probe is introduced through a slit that is cut along the external cylinder. The position of the probe is determined with an accuracy of 0.05 mm.

The detected signal from the probe head is fed to a galvanometer or a high-sensitivity microammeter

(5). Note that the penetration depth of the probe in the measurements of the field in the helix must be significantly greater than the penetration depth in the measurements of the field in the conventional coaxial line, since a decrease in the field strength in the helical line with an increase in the distance from the inner conductor is significantly stronger than such a decrease in the conventional coaxial line. Evidently, an increase in the probe length in the helical lines does not enhance the shunting effect, since the ratio of the power trans-

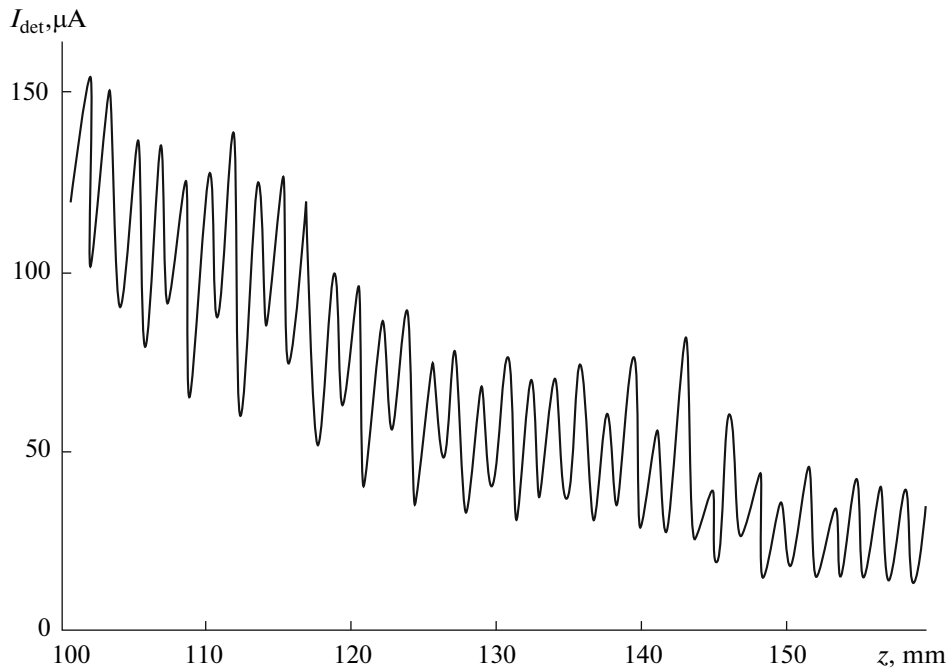


Fig. 6. Standing-wave pattern along the helical slow-wave structure.

ferred to the probe to the power in the line remains relatively small.

Figure 6 demonstrates a typical pattern of the field distribution along the helix that is measured using a square probe. It is seen that standing waves exist along the entire helix and the presence of such waves makes it possible to determine the wavelength in the system. The existence of standing waves along the entire helix is unexpected.

We cannot interpret the effect using the reflections at the ends, since the helices made of constantan wire exhibit a relatively high attenuation (0.6–1.0 dB/cm) at a total length of the helix of 22–30 cm. Apparently, the presence of the reflected waves is related to insignificant inhomogeneities over the length of helix (spread of pitch, variations in the thickness and resistance of the wire, inhomogeneities in the dielectric tubes, etc.). The problems of the local reflections in the slow-wave lines and the effect of reflections on the TWT parameters are important for the theoretical analysis and practical applications. Such parameters as the measured attenuation of the slow-wave line and the intrinsic feedback in the TWT may depend on the local reflections.

A relatively high sensitivity of the probe method can be used to determine the reflections from inhomogeneities that are artificially introduced into the slow-wave structure, for example, the reflection from absorbing substances that are deposited on a small fragment of the line for the creation of the local attenuation.

Figure 7 illustrates the efficiency of the method and shows the pattern of standing waves along the helix for the poor matching of the fragment that is covered with aquadag and the working part of the helical line.

The wavelength measurement in the helical line is impeded by the superposition of the fundamental slow wave in the helix and the aperiodic local waves that emerge in the vicinity of inhomogeneity.

Figure 8a demonstrates the pattern of the field in the measurement line in the presence of one slow wave. Figure 8b shows the field distribution for the superposition of the slow wave and the local waves that are generated in the vicinity of inhomogeneities. It is seen that the field distribution in the last case is significantly more complicated and the distance between the neighboring minima can be λ_g or intermediate value rather than $\lambda_g/2$, (see also Fig. 7).

To avoid errors in the wavelength measurements, we must, first, measure the field pattern at relatively large distances from sharp inhomogeneities (e.g., the ends of helix) and, second, measure the wavelengths at the fragment that contains no less than 8–10 waves. In this case the measurement error of λ_g can be less than 2%.

Figures 9–11 demonstrate the results of the measurements of dispersion in three helical slow-wave systems. For comparison, we also present the theoretical curves that are calculated using the above formulas and the formulas from [3], in which the dielectric units were not taken into account.

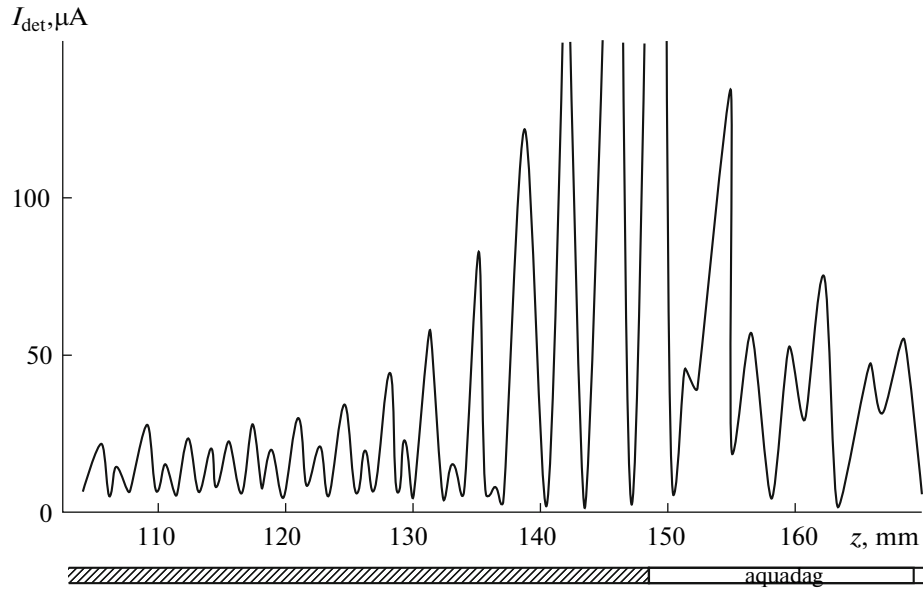


Fig. 7. Standing-wave pattern along the helix whose small fragment is covered with aquadag.

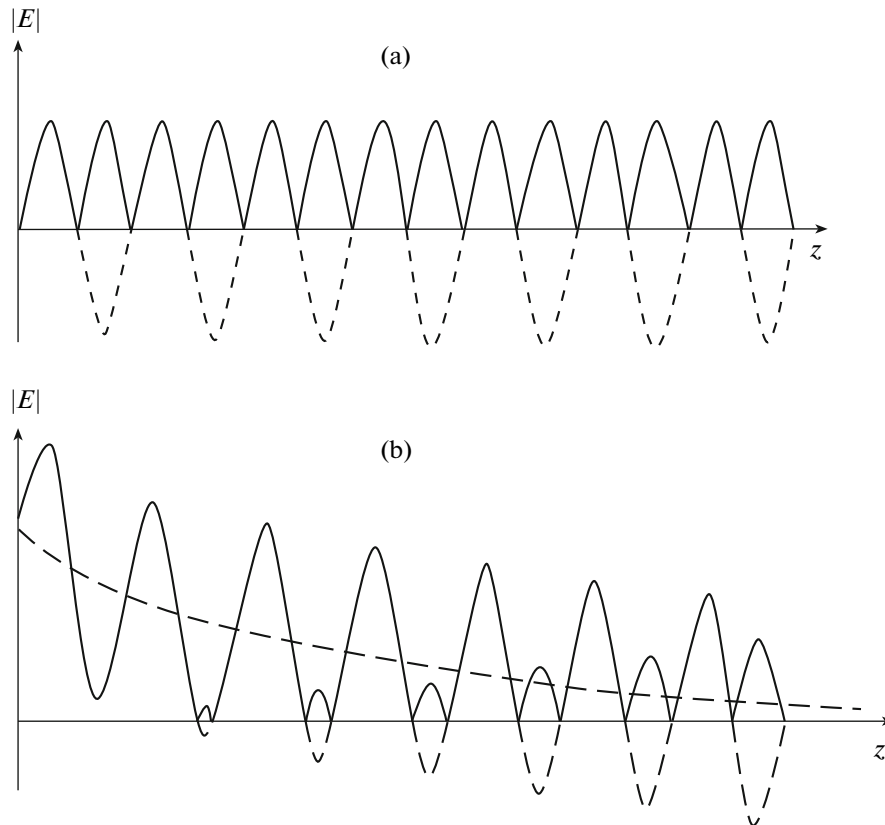


Fig. 8. Schematic plot of the absolute value of electric field $|E|$ vs. length of helix z (a) in the presence of a single slow wave and (b) with additional local waves.

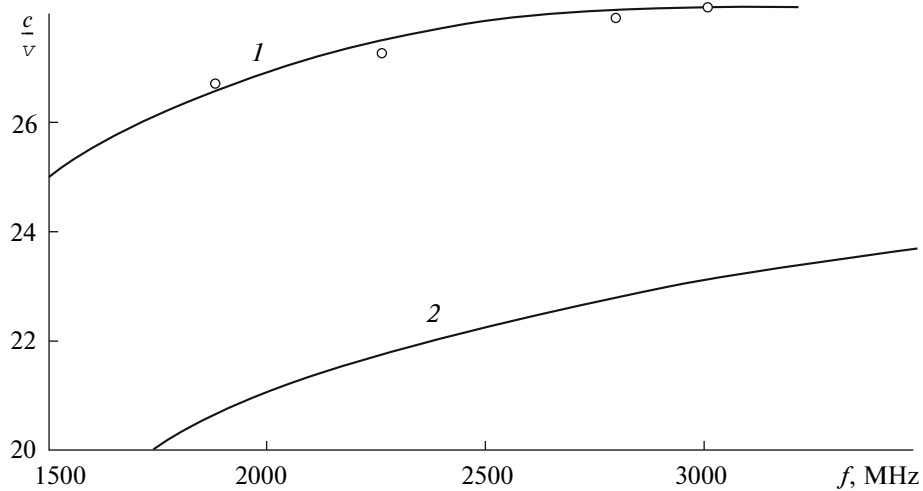


Fig. 9. (1) Dispersion curve calculated using formulas (13)–(20) and (dots) experimental slowing factors for the helix placed inside the dielectric tube and (2) dispersion curve for free helix.

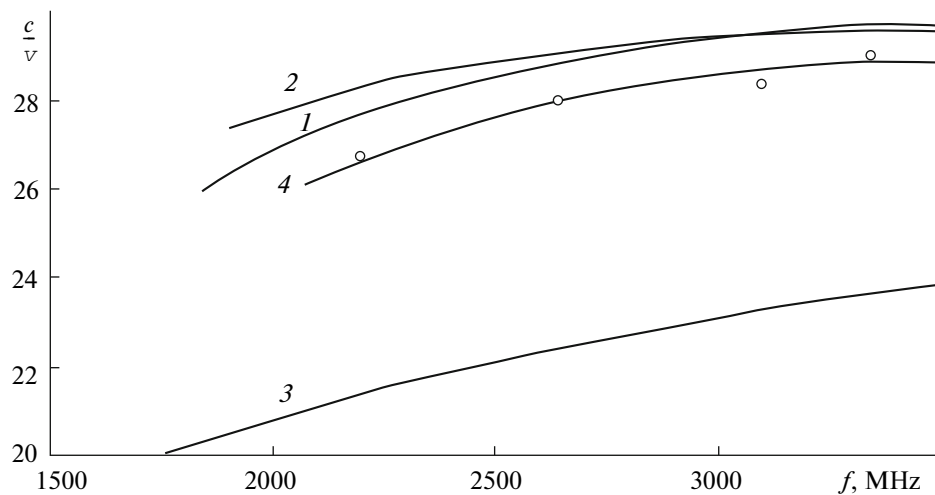


Fig. 10. Dispersion curves calculated using formulas (12), (13), and (19a) (1) with neglect of and (2) with allowance for the frequency dependence of the thickness of equivalent tube and (4) experimental data for the helix fixed between four dielectric tubes and (3) dispersion curve for free helix.

Figure 9 shows the dispersion characteristics of the helix that is placed inside a dielectric tube. Curve 1 is plotted using formula (13) in which factor Δ is given by expression (19a). Curve 2 is plotted with the aid of formula (B22) from [3]. The dots correspond to experimental data.

Figure 10 demonstrates the dispersion characteristics of the same helix that is fixed between four quartz tubes. For the calculation of curve 1, we reduce four tubes to one tube at a frequency of 3000 MHz using formula (12) and calculate the slowing factor using formulas (13) and (19a). Curve 2 is also calculated with the aid of formulas (12), (13) and (19a) but with

allowance for the frequency dependence of the thickness of equivalent tube. Curve 3 shows the dispersion characteristic of the helix in the absence of insulator. Curve 4 is plotted using experimental data.

Figure 11 shows the dispersion characteristics of the helix with different geometrical parameters that is fixed in four quartz tubes. Curve 1 is calculated using formulas (12), (13), and (19a). Curve 2 is calculated with neglect of insulator. Curve 3 is plotted using experimental data.

The comparison of the theoretical curves and the experimental data shows that, in spite of several assumptions in the derivation of the formulas for cal-

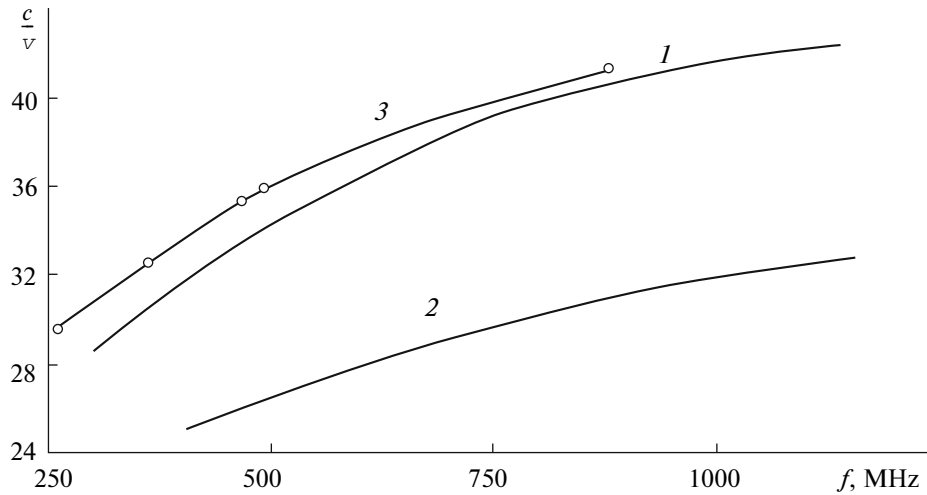


Fig. 11. (1) Dispersion curve calculated using formulas (12), (13), and (19a) and (3) experimental data for the helix with different (with respect to the helix in Fig. 10) geometrical parameters that is fixed between four dielectric tubes and (2) dispersion curve for free helix.

culations, the proposed method allows the calculation of the helical slow-wave structures with reasonable accuracy.

Note that the frequency dependence of the thickness of equivalent tube can be neglected in the calculations of the dispersion curve in a relatively small frequency interval. This circumstance substantially simplifies the practical calculations.

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