Research Article

Leonid V. Zotov and Christian Bizouard

Reconstruction of prograde and retrograde Chandler excitation

Abstract: Observed polar motion consists of uniform circular motions at both positive (prograde) and negative (retrograde) frequencies. Generalized Euler–Liouville equations of Bizouard, taking into account Earth’s triaxiality and asymmetry of the ocean tide, show that the corresponding retrograde and prograde circular excitations are coupled at any frequency. In this work, we reconstructed the polar motion excitation in the Chandler band (prograde and retrograde). Then we compared it with geophysical excitation, filtered out in the same way from the series of the Oceanic Angular Momentum (OAM) and Atmospheric Angular Momentum (AAM) for the period 1960–2000. The agreement was found to be better in the prograde band than in the retrograde one.

Keywords: Earth rotation, Chandler wobble, Panteleev filtering, Atmospheric Angular Momentum (AAM), Oceanic Angular Momentum (OAM)

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1 Introduction and method

Chandler wobble is the main component of the Earth’s polar motion (PM), discovered more than a century ago. With a variable amplitude up to 0.3 arcsec, its excitation is found in Atmospheric Angular Momentum (AAM) and in Oceanic Angular Momentum (OAM) [3, 5]. But the cause of Chandler wobble variability still remains elusive, and in order to study its cause, the inverse problem for the corresponding dynamical system, namely the reconstruction of the excitation function has to be done carefully. In [13, 14], several methods of inversion were applied to this problem, such as singular values truncation [1], Tikhonov regularization [12], and corrective smoothing [15]. They are all more or less equivalent, helping to overcome instability, non-uniqueness of the solution and amplification of observation noises. In this work we applied Panteleev corrective smoothing to reconstruct Chandler excitation at prograde and retrograde Chandler frequency, using new generalised Euler–Liouville equation derived in [2].

Precise interpretation of PM observations requires improvement of the theory, in particular introduction of asymmetric effects into the Euler–Liouville equations and considering the consequences for the ellipticity of the main wobbles, namely annual and Chandler [6].

The Earth’s PM is commonly modelled by the linear Euler–Liouville equation ([8, 9])

\[ \frac{i}{\sigma_c} \frac{dm(t)}{dt} + m(t) = \Psi(t), \]  

(1.1)

where the complex Chandler angular frequency \( \sigma_c = 2\pi f_c (1 + i/2Q) \) depends on the real Chandler frequency \( f_c = 0.8435 \text{ yr}^{-1} \) and the quality factor \( Q \) lies in the interval [40, 200] (in this study we take \( Q = 100 \)). In the dynamical system (1.1) the complex PM trajectory \( m = m_1 + im_2 \) is a filtered response to the effective
input excitation $\Psi = \Psi_1 + i \Psi_2$. The left-hand side is called geodetic excitation, the right-hand side $\Psi(t)$ is the modelled excitation reconstructed form geophysical models and observations.

A new generalised version of the Euler–Liouville equation, derived in [2], has the form

$$\left(1 - U\right)m(t) + \left(1 + eU\right)\frac{i}{\sigma_e} \frac{dm(t)}{dt} = Vm^*(t) + eV \frac{i}{\sigma_e} \frac{dm^*(t)}{dt} = \Psi_{\text{Pure}}(t),$$  \hspace{1cm} (1.2)

where the parameter $U = 0.36556 - i0.353/Q$ depends on rheology, $V = 0.00256 + i0.0051$ characterises the asymmetric response, brought by triaxiality of the Earth and ocean pole tide, $\sigma_e = 2\pi e$ is the Euler frequency, $e = 1/304.46$ is the dynamical ellipticity, and the superscript * means complex conjugation. The geophysical excitation free from rotational effects stands on the right-hand side of (1.2). In order to keep the same form as (1.1), the generalised equation (1.2) is divided by $1 - U$, leading us to define the effective geophysical excitation by $\Psi(t) = \Psi_{\text{Pure}}(t)/(1 - U)$.

Introducing the inverse symmetric transfer function

$$L_{\text{sym}}^{-1}(i\omega) = 1 + \frac{(1 + eU)}{1 - U} \frac{i}{\sigma_e} (i\omega) = 1 + \frac{i}{\sigma_e} (i\omega),$$  \hspace{1cm} (1.3)

which coincides with the inverted transfer function of the common equation (1.1), and the asymmetric inverse transfer function

$$L_{\text{asym}}^{-1}(i\omega) = \frac{eV \frac{i}{\sigma_e} (i\omega) - V}{1 - U},$$  \hspace{1cm} (1.4)

equation (1.2) can be rewritten in the frequency domain as

$$L_{\text{sym}}^{-1}(i\omega)\tilde{m}(\omega) + L_{\text{asym}}^{-1}(i\omega)\tilde{m}^*(-\omega) = \Psi_{\text{sym}}(\omega) + \Psi_{\text{asym}}(\omega) = \hat{\Psi}(\omega),$$  \hspace{1cm} (1.5)

where $\hat{\cdot}$ is the Fourier transform and the rule $\tilde{m}(\omega) = \hat{m}^*(-\omega)$ was applied. According to this rule, the asymmetric operator (1.4) acts on the conjugated PM spectrum with opposed frequency $\hat{m}^*(-\omega)$. Let us introduce the operator $s^*$, making such a transformation: $s^* \hat{m}(\omega) = \hat{m}^*(-\omega)$.

In the linear equation (1.1) the input at a particular frequency produces an output at the same frequency. In equation (1.2) the presence of both direct and conjugated variables $m(t)$ makes input at one frequency producing an output at both prograde and retrograde frequencies. Taking $\hat{m}(\omega)$ at a particular frequency, we can reconstruct symmetric and asymmetric excitations for it (both of these excitations have prograde and retrograde components), using operators (1.3) and (1.4), whose amplitude responses as a function of $\omega = 2\pi f$ (angular frequency) are shown in Figure 1.

It is interesting to note that the direct problem of $\hat{m}$ estimating from a given excitation $\hat{\Psi}(\omega)$ can be formulated from equation (1.5) as

$$\hat{m}(\omega) = \frac{1}{L_{\text{sym}}^{-1}(i\omega) + L_{\text{asym}}^{-1}(i\omega)s^*} \hat{\Psi}(\omega).$$  \hspace{1cm} (1.6)

This dynamical system differs from the standard linear case, where response is at the same frequency as input [4]. Presence of the exotic spectral conjugation operator $s^*$ (introduced earlier) in the denominator for the asymmetric part makes the transfer function not easy to resolve. To obtain the response $\hat{m}(\omega_0)$ at a particular frequency $\omega_0$, we need to know excitation at both prograde and retrograde frequencies $\hat{\Psi}(\omega_0)$, $\hat{\Psi}(-\omega_0)$. The operator $s^*$ cannot be represented in a simple analytic form, but it can be interpreted as an image (Fourier transform) of the complex conjugation operator. It is easy to avoid introduction of $s^*$ by using the two-channel matrix dynamical system (separate equations for real and imaginary parts) instead of the complex equation (1.5) as done in [2].

Equation (1.5) is used for solving the inverse problem at Chandler frequency. In this framework the output ($m$) at Chandler frequency is produced by the input ($\Psi$) at prograde and retrograde frequencies. We will isolate the prograde and retrograde Chandler frequencies with the narrow-band Panteleev filters developed in [14]. Panteleev corrective filtering was initially proposed for marine gravimetry [10]; it allows to avoid amplification of noises in the regions, where the inverse operator module is large. It can be considered as an alternative to Tikhonov regularization or singular values truncation for practical linear problems, where the
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Figure 1. Amplitude responses of the inverse operators $|L_{\text{sym}}^{-1}(\omega)|$, $|L_{\text{asym}}^{-1}(\omega)|$, prograde and retrograde Panteleev filters, and PM spectrum.

noise frequency band can be isolated, see [7, 11, 15]. The transfer function of the Panteleev filter centered at the prograde/retrograde Chandler frequency $\pm f_c$ is

$$L_h(f) = \frac{f_0^5}{(f \mp f_c)^4 + f_0^5}.$$ 

(1.7)

The plots of these filters are given in Figure 1. The filter parameter (defining its width) was selected to be $f_0 = 0.04 \text{ yr}^{-1}$. For the selected $f_0$ and $f_c$, the filter (1.7) has narrow band-pass and does not change the phase of the signal. The time-window of more than twenty years extent corresponds to it. As the filtered signal undergoes edge effects, the result is not reliable for the first and last ten years of the considered time interval. The trustworthy region is depicted by the red rectangle in the plots.

2 Analysis and comparison of results

Firstly, taking IERS C01 pole coordinates spanning the period 1830–2010, the Chandler wobble was isolated with the Panteleev filter (1.7) for both prograde and retrograde bands. The results are presented in Figure 2, left for the $X$ component. The $Y$ component has similar amplitude changes, but the carrying (Chandler) oscillation is out-of-phased by $\pi/2$.

Then the corresponding geodetic excitation was obtained through multiplication by the symmetric (1.3) and asymmetric (1.4) inverse operators in frequency domain. The common Chandler excitation, associated with prograde Chandler wobble and pure symmetric inverse operator, is up to 4 mas, as evidenced in Figure 2, right. As it was noted in [14], it has an amplitude modulation almost synchronous with the Lunar 18.6-year tide.
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**Figure 2.** Filtered prograde and retrograde Chandler wobble (left) and prograde Chandler excitation obtained through common inversion with \( L_{\text{sym}}^{-1}(\omega) \) (right). Lunar 18.6-year tide is shown along the abscissa with grid lines at maxima.

**Figure 3.** The symmetric \( \Psi_{\text{sym}} \) (left) and asymmetric \( \Psi_{\text{asym}} \) (right) components of the geodetic excitation at the prograde and retrograde Chandler frequencies.

Total symmetric and asymmetric parts of the geodetic excitation are shown in Figure 3. For each of them, we distinguish their prograde and retrograde components.

The asymmetric part is mostly retrograde and has an amplitude up to 1 mas, repeating the shape of the prograde Chandler wobble (Figure 2, left) for it was obtained by multiplying this wobble by the complex value \( L_{\text{asym}}^{-1}(\omega_c) \) (Figure 1). Its order of magnitude is almost the same as the one of the common Chandler wobble excitation (up to 4 mas, Figure 2, right), and is significant in light of the contemporary level of observational precision (0.05 mas for pole coordinates).

The symmetric part (Figure 3, left) is also dominated by a retrograde component, especially large before 1900 (up to 40 mas). Before the 1960s it could be caused by the observational noise amplification at retrograde Chandler frequency by the inverse operator \( L_{\text{sym}}^{-1}(\omega) \) (Figure 1), where its amplitude response is much larger than at prograde Chandler frequency. For instance, taking a maximal uncertainty of 10 mas for the retrograde Chandler wobble in the 1900s, corresponding retrograde excitation has an uncertainty of
Figure 4. Comparison of geodetic excitation (sum of symmetric and asymmetric parts) in prograde (left) and retrograde (right) Chandler frequency bands with the geophysical excitation, related to AAM (top), OAM (middle), AAM+OAM (bottom).
about $2 + 10 = 20$ mas, as large as the observed variations. Since the 1960s, retrograde Chandler wobble is determined with an uncertainty smaller than 1 mas, it contributes to the corresponding excitation at a level smaller than 2 mas, so that the obtained variations become significant.

Finally, the symmetric and asymmetric parts of the geodetic excitation and their sum were compared to the geophysical excitation. The atmospheric contribution was obtained by filtering of NCEP/NCAR AAM combination of pressure (IB) and wind terms using the Panteleev filter (1.7) with parameters chosen above. The oceanic part of the excitation was obtained in the same way from ECCO OAM time series, sum of bottom pressure and current terms. In Figure 4 we plot the geodetic and geophysical excitations in prograde (left) and retrograde (right) Chandler bands for AAM (top), OAM (middle) and their sum (bottom). Despite the initial time series span is 1949–2010, the filtered excitations are compared only over the period 1960–2000 (depicted with a red rectangle) because of the edge effect. The agreement is good in the prograde band, while in the retrograde band OAM+AAM sum does not totally explain the geodetic excitation.

Table 1 presents the correlation coefficients between the geodetic excitation (symmetric part, asymmetric part, and total) and OAM, AAM, and OAM+AAM geophysical excitations in prograde and retrograde Chandler bands (the values are almost equal for the $X$ and $Y$ components). The time interval is 1960–2000. Notice that the standard errors for correlation values between two time series can be estimated as $\sqrt{2/(N - 3)}$ where $N$ is the number of independent points in the series, namely $N \sim 800$, giving a standard error of about 0.05. The correlation with OAM is much better than with AAM, and adding AAM to OAM slightly improves the correlation.

<table>
<thead>
<tr>
<th>Geodetic excitation</th>
<th>AAM</th>
<th>OAM</th>
<th>AAM+OAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric prograde</td>
<td>0.64</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>Symmetric retrograde</td>
<td>-0.02</td>
<td>0.43</td>
<td>0.36</td>
</tr>
<tr>
<td>Asymmetric prograde</td>
<td>0.22</td>
<td>0.63</td>
<td>0.51</td>
</tr>
<tr>
<td>Asymmetric retrograde</td>
<td>0.58</td>
<td>-0.51</td>
<td>0.04</td>
</tr>
<tr>
<td>Total prograde</td>
<td>0.64</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>Total retrograde</td>
<td>0.10</td>
<td>0.36</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 1. Correlation coefficients between geodetic and geophysical (AAM, OAM, their sum) excitations for symmetric, asymmetric excitations, and their sum.

In the prograde Chandler band, the common symmetric dynamical system, equation (1.1), and the generalised case, equation (1.2), obtained by adding asymmetric part, give the same correlation. In the retrograde band, the common dynamical system gives a zero correlation with AAM, but the generalised system yields a larger correlation with AAM+OAM (0.40 instead of 0.36). The low correlation at the retrograde frequency could have several explanations. Firstly, it could be the result of observational noise amplifications during inversion. The noise can remain in the narrow filter band for early periods of astrometric observations. Secondly, some other factor can excite the retrograde wobble. Finally, some defect could remain in the transfer function of the dynamical equation, causing an overestimation of the inverse amplitude response at this frequency. In any case, the new equation (1.2) introduces an asymmetric part much smaller than the symmetric one, bringing the results presented in Figure 4 and Table 1 in close agreement to what is obtained with common symmetric equation (1.1).

3 Conclusion

We derive the geodetic excitation in the prograde and retrograde Chandler band in the framework of the generalised Euler–Liouville equation (1.2), taking into account asymmetry brought by ocean pole tide and triaxiality. Geodetic excitation was obtained by applying Panteleev’s corrective filtering for solving inverse problems, and compared with the atmospheric and oceanic excitation. A new formalism introduces an
asymmetric input at the level of 1 mas, which is significant at the contemporary level of observational precision; however, it does not significantly improve the atmospheric and oceanic budget of PM excitation. More puzzling, excitation at the retrograde Chandler frequency was found to be larger than at the prograde one (up to 6 mas since the 1960s) but cannot be explained by the corresponding atmospheric and oceanic forcing.

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References