Abstract

The "mean voter theorem" implies that candidates should choose identical policy positions in a two-candidate race if voting is probabilistic. This result is in fact an artifact of the assumption that the candidates maximize expected vote share or probability of win, which is not true for many real-world elections. In this paper I analyze a probabilistic voting model in which the candidates have preferences other than the maximization of the expected number of votes or the probability of win maximization. I derive the comparative statics for two voters and one-dimensional policy space. Each voter cares about both the policy platform and the identity of the candidate. It is shown that an increase in the value of exactly one vote causes each candidate to choose a position closer to that of its partisan voter. Numeric computation of equilibria show that these results can be generalized to three or more voters. The results imply that the nonlinearity and nonsymmetry of payoffs can affect the policy positions of the candidates.

1 Introduction

Few of the existing political systems can be described as purely winner-take-all or proportional representation; the relationship between votes and payoffs to candidates or political parties is usually more complicated.

First, the winner of an election by a wide margin may get a disproportionately large payoff. The parliaments of many countries use the supermajority rule to pass especially important decisions, such as amending the constitution. US Senate is an obvious example, with a 60-vote filibuster-proof majority required to pass legislation on important and sensitive issues. In executive elections, winner's payoffs may increase with his margin of victory because it affects the patterns of political participation, especially in the hybrid or semi-democratic political regimes (Simpser, 2008). Winning with a very large margin discourages potential opponents from becoming rivals, can motivate the supporters to participate more actively, or can depress turnout among opposition voters in the next election. The existence of these additional payoffs is sometimes a cause of overwhelming electoral fraud, intended not to change the identity of the winner, but to increase the winner's margin of victory. Simper finds that, indeed, most electoral fraud in the developing countries was committed on behalf of popular incumbent candidates who would have achieved victory if the elections were clean.

Second, a loser who lost by a narrow margin may receive an additional reward — a "consolation prize". Hojman (2004) describes the existence of such prises in Chilean Senate elections. He argues that those Senate candidates who were fielded by the government and yet received a substantial minority of votes could expect to be rewarded with substantial jobs in the public sector (those value, it is argued, may be even greater than the value of the contested office).

Third, in parliamentary elections there may exist non-linearities in the way that the votes are converted into parliament seats (Lijphart, 1990; Gallagher, 1992). The real-world "proportional representation" electoral systems are, in fact, not fully proportional due to such factors as the existence of regional lists (only Israel and Netherlands have country-wide party lists), floor requirements, and the quotient formula that is used to assign seats. Multi-party parliamentary elections in a majoritarian system (such as the UK) is another example of a system that is neither strictly winner-take-all (as getting 40% of vote in a district may or may not guarantee a seat), nor proportionate.

Finally, in a parliamentary system the payoff to a political party — in the form of political rents — is affected by the likelihood of the party being able to enter a ruling coalition. It is often the case that the largest party forms the government, while the next-largest (even if it loses only by a few seats) is left as an outsider; at the same time, smaller parties may be included in the ruling coalition. There are various approaches to modeling parliamentary coalition formation (Ansolabehere, Snyder, and Ting, 2005; Laver and Shepsle, 1996; Schofield and Sened, 2006, among others); none imply that the probability of entering the cabinet is strictly linear in the number of seats. Moreover, the office rent is further affected by the cabinet's expected longevity, which may depend on various legislative institutional factors (King, Alt, and Laver, 1990; Warwick, 1994).

Suppose that we want to model the decision-making process of candidates (or parties) in an environment where the election outcome is a random variable that depends on the actions of each side. If there are N voters, there are $N+1$ possible election results: Candidate 1 getting 0 votes and Candidate 2 getting N, Candidate 1 getting 1 vote, and Candidate 2 getting $N-1$, and so on. As the outcome of the election is random, will need to define a utility function over the set of outcomes for each candidate. In the general case, the only restriction on these utility functions is that they are increasing in the number of votes. [1] However, extant models assume that the utility function is either linear in the number of votes (meaning that the candidates maximize the expected vote share), or that the function is stepwise (meaning that the candidates maximize the probability of receiving an absolute majority of votes).

Probabilistic voting models evolved as a response to the nonexistence of equilibrium in games of electoral competition with a multi-dimensional policy space. Such models assume that a voter's decision is a random variable that is continuous in the policy position of each candidate. This setting guarantees the existence of a mixed-strategy equilibrium, and, under broad conditions, of a local Nash equilibrium. The conditions for the existence of a global Nash equilibrium are, however, much less transparent, as they require concavity of candidate objective functions (Hinich, Ledyard, and Ordeshook, 1972,1973, Hinich, 1978, Linbeck and Weibull, 1987, Coughlin, 1992, Banks and Duggan, 2005). An additional benefit of a probabilistic model is that it can be integrated with an econometric multinomial choice model that can be used to estimate the individual probability of vote functions from mass survey data (Schofield 2007).

A persistent result in this literature was the "mean voter theorem" — the existence of an equilibrium where both (or all) candidates choose an identical policy position. Under the assumptions that the voters are identical with respect to their nonpolicy preferences toward the candidates, the equilibrium position maximizes the total expected utility of the voters. In a two-candidate setting, it was shown that under much broader conditions the only equilibrium that can exist is the one in which the policy positions of the candidates are identical (Banks and Duggan, 2005). Numerical methods were used to demonstrate that other, nonsymmetric local equilibria may exist in models with several political parties or candidates (Quinn and Martin, 2002, Schofield and Sened, 2006). However, the properties of such divergent equilibria are known only to a limited degree. In particular, Schofield (2007) has shown that in a multi-candidate probabilistic voting game with multi-dimensional policy space, the positions of all candidates are located on one line. [2] Yet it would be safe to say that probabilistic voting models (especially the two-candidate ones) were either unable to produce an equilibrium with nonidentical policy platforms, or were unable to explain the comparative statics of the equilibrium.

The bulk of the probabilistic models assumed political agents that maximized the expected number of votes, which corresponds to a linear function that translates votes into payoffs. The remaining works assumed winner-take-all payoffs. The equivalence of candidate behavior under these two assumptions attracted the attention of several scholars. Hinich (1977), Ledyard (1984) and Duggan (2000) argued in favor of the strategic equivalence of these two assumptions under Eucledian voter preferences and additive uncertainty. However, Patty (2005, 2007) demonstrated that under more general assumptions about the probability of voting functions, the response functions of probability-of-victory maximizers are different from those of expected vote share maximizers, unless some very special conditions on the voting probabilities are met.

In a recent work, Zakharov (2009) has shown that in a two-candidate probabilistic voting model, policy convergence occurs if and only if a strict symmetry condition on the payoff functions are satisfied. Both expected utility maximizers and probability of win maximizers are special cases that satisfy this condition. Most other functions, including all concave functions, do not.

This paper is a continuation of that work. Zakharov (2009) was a (non)existence result; here I derive the comparative statics of the equilibrium. This is done analytically for the simplest two-voter case; the results are augmented by a numeric calculation of Nash equilibria for several voters. The main result of this work is the relationship between the payoff functions of the candidates and group-specific valence [3].

Group-specific valence occurs when, candidate policy platforms being equal, one candidate is preferred to another by some group of voters. It has several sources. The first one is partisanship. Studies of US politics have shown that partisanship is the single most important determinant of vote, even controlling for ideological and policy preferences of the voters (Ansolabehere, Rodden, and Snyder, 2008). The evidence on the dynamics of partisanship is divided, with some arguments in favor of an increase in partisan voting (Bartels, 2000). The second reason for group-specific valence is that a group with a common ethnic, cultural, or religious background may be predisposed to vote for some candidate (possibly, of the same background), or political party. For example, African-American voters on the average favor Democratic candidates, even when controlling for partisan attachment and policy preferences (Adams, Dow, and Merrill, 2006). The third reason is that candidate valence may be multidimensional. Two candidates may be perceived as having different character: one candidate may be perceived as "tough", while another as "intelligent". Those voters who favor toughness may be predisposed to vote for the first candidate, those favoring intelligence — for the second. Issue ownership is a related phenomenon (Petrocik, 1996). Each political party may be seen as having competence in dealing with some special set of issues. Consequently, a voter who feels that some policy issue (such as crime or health care) is important, will (other things being equal) vote for the party that is more competent in dealing with that issue. Whitney et. al. (2005), for example, found that the quantitative effect of voter's preferences over such "valence issues" was stronger than the effect of preferences on left-right spatial issues, such as taxes or foreign policy.

This work predicts a relationship between group-specific valence, the payoffs of candidates, and their policy positions. I show that if the candidate utility functions are relatively concave, then they will select policy positions that are closer to the ideal policies of their partisan voters. If the payoffs are convex (for example, when there exist very large payoffs to a candidate who wins by a wide margin) each candidate will pander to the partisans of the opposing candidate. If the level of group-specific valence increases — for example, because of greater partisan attachment — then the magnitude (but not the direction) of this effect increases.

2 Main results

There are 2 candidates who compete in an election by choosing policy platforms $y_1, y_2 \in [0, 1]$. There are two voters, 1 and 2. Let $P_i(y_1, y_2)$ be the probability that voter $i = 1, 2$ votes for Candidate 1, and $1 - P_i(y_1, y_2)$ the probability that she votes for Candidate 2. Suppose that the votes are independent. Assume that the voters behave according to the utility-difference model:

$$P_i(y_1, y_2) = P(u_{i1} - u_{i2}), \tag{1}$$

where u_{ij} is the utility that voter i attributes to Candidate $j = 1, 2$, and $P(\cdot)$ is a continuous, differentiable, strictly increasing function. Let

$$u_{ij} = e_{ij} - \psi(y_j - v_i), \tag{2}$$

where e_{ij} is the nonpolicy preference, or valence, of voter i for Candidate j, $v_i \in [0, 1]$ is the best policy of voter i, and $\psi(\cdot)$ is a twice-differentiable disutility function, with $\psi(d) = \psi(-d)$,

$\psi'(0) = 0$, and $\psi''(d) > 0$ [4]. Let $v_1 = 0$ and $v_1 = 1$. Without loss of generality, let $e_{12} = e_{21} = 0$.

The payoff of each candidate depends on the number of votes he receives. There are 3 possible election results: 2 votes going to Candidate 1, 1 vote for each candidate, and 2 votes for Candidate 2. Assume that each candidate's payoff is a nondecreasing function of the number of votes. Then, without loss of generality, let the utility of 0 votes be 0, the utility of 2 votes be 1, and the utility of 1 vote be $x \in [0, 1]$.

A high x implies that a candidate values winning one half of all votes relatively high compared to winning all votes, or no votes at all. There are several factors that can affect the value of this parameter. For example, suppose that in the event of a 50-50 vote split, the election outcome is decided by a coin toss. If there is a high consolation prize to the losing candidate, x will be higher. On the other hand, if the winner needs a clear majority mandate, x will be lower.

The expected utility functions for both candidates will be

$$U_1 = x((1 - P_1)P_2 + P_1(1 - P_2)) + P_1P_2, \quad (3)$$

$$U_2 = x((1 - P_1)P_2 + P_1(1 - P_2)) + (1 - P_1)(1 - P_2). \quad (4)$$

For $x = \frac{1}{2}$ the utilities are equal to the expected share of the total vote: $U_1 = \frac{1}{2}P_1 + \frac{1}{2}P_2$, $U_2 = 1 - \frac{1}{2}P_1 - \frac{1}{2}P_2$. This special case was analyzed in most of the previous literature.

I now formulate the main analytic results of this work. Local Nash equilibrium (LNE) will be used as the solution concept.

Proposition 1 *Suppose that $e_{11} = e_{22} = e$. Let $P(x) = 1 - P(-x)$. Then there exists a local equilibrium in the electoral competition game with $y = y_1 = 1 - y_2$. The equilibrium is given by*

$$\psi'(y)(P + x - 2Px - 1) + \psi'(1 - y)(x + P - 2Px) = 0. \quad (5)$$

In this setting there are two voters. The value e can be interpreted as a voter's degree of partisanship, or the degree to which a voter supports "her" candidate if the policy positions of the two candidates are identical. Thus, if there are two voters whose probability of vote functions are identical up to the ideal points and the identity of the preferred candidate, then we should expect the candidates to choose policy positions that are symmetric with respect to the voter ideal points. The above result only guarantees the existence of a local equilibrium. Conditions for a global equilibrium will be difficult to interpret even for the two-candidate, symmetric case. A part of the difficulty is that the candidate utility is a complicated function of individual voting probabilities; hence, the concavity of individual probability-of-voting functions P_i does not guarantee the concavity of candidate utilities (as it does in the case when the candidate utility is linear in votes, see Banks and Duggan, 2005).

The comparative statics of the equilibrium are summarized in the following proposition.

Proposition 2 *Suppose that $(y, 1-y)$ is a symmetric equilibrium in the electoral competition game. Then y decreases with x for $x \leq \frac{1}{2}$ and y increases with e for $x < \frac{1}{2}$. Suppose also that*

$$P'(e - \psi(y) + \psi(1-y)) < \frac{\psi'(y)\psi''(1-y) + \psi'(1-y)\psi''(y)}{(\psi'(y) + \psi'(1-y))^3} \tag{6}$$

for all $y < \frac{1}{2}$. Then y decreases with x for all $x \in [0,1]$. Also, y increases with e for $x < \frac{1}{2}$ and decreases with e for $x > \frac{1}{2}$.

It follows that, as the value of x increases, we should expect the candidates to choose policy positions closer to the ideal points of their partisan voters; the magnitude of this effect is increasing in the degree of partisanship. Indeed, suppose that the voters are highly partisan: Voter 1 will vote for her preferred candidate (Candidate 1) unless the policy position of Candidate 2 is much closer to her ideal point. Thus, the utility of both candidates is high if they both locate close to the ideal points of their partisan voters. This result always holds for $x \leq \frac{1}{2}$. If the probability of supporting a candidate does not change too quickly with the utility difference, then condition (6) holds, and the comparative static result holds for $x > \frac{1}{2}$ as well. [5].

Interestingly, the model predicts that if the candidates are risk-loving, each will choose a position that is closer to the ideal policy of the *opposing* candidate's partisan voter. This is true because y is monotonic in x, and because for $x = \frac{1}{2}$ the mean-voter theorem holds, so $y = \frac{1}{2}$. Suppose that both voters are partisan, and the candidates are risk-lovers. In order to maximize the probability of winning both votes, Candidate 1 must maximize the probability of Voter 2 supporting him (given that the probability of Voter 1 supporting Candidate 1 is high enough). So, he will locate near Voter 2.

Example. Let the disutility functions be as follows:

$$u_{ij} = e_{ij} - \beta(v_i - y_j)^2, \tag{7}$$

where v_i is the ideal policy of Voter i, and $\delta \geq 1$. Suppose that

$$P(u_1 - u_2) = \begin{cases} 0, & u_1 - u_2 < -\alpha \\ \frac{\alpha + u_1 - u_2}{2\alpha}, & u_1 - u_2 \in [-\alpha, \alpha] \\ 1, & u_1 - u_2 > \alpha, \end{cases} \tag{8}$$

with $\alpha \geq \beta + e$. Then the equilibrium equation (5) can be rewritten as

$$y = \frac{\alpha + \beta + e - 2x\beta - 2xe}{2\alpha - 2\beta + 4x\beta}. \tag{9}$$

We have, for all e and β and $\frac{\partial y}{\partial x} < 0$ and $\frac{\partial^2 y}{\partial e \partial y} < 0$. It follows that y is increasing in both x and e for $x < \frac{1}{2}$ and decreasing in both variables for $x < \frac{1}{2}$.

Example. Let the disutility be given as in (7). The equilibria for the symmetric two-voter model, as well as for a more general setting with a greater number of voters, can be calculated using numeric methods. The local equilibrium was found using a gradient hill-climbing algorithm implemented in Matlab. The results obtained numerically augment those obtained by analytical methods.

I used the logistic probability of vote function

$$P(u_1 - u_2) = \frac{e^{u_1}}{e^{u_1} + e^{u_2}} = \frac{e^{u_1 - u_2}}{e^{u_1 - u_2} + 1}. \tag{10}$$

Figure 1 shows the calculated equilibrium for different values of x and e.

One can see that, for the chosen probability of vote and disutility functions, y is monotonic with respect to x for all values of x, for all ranges of e and β. Qualitatively, the relationship between e, x and y does not change if one chooses a different δ; in particular, I looked at $1.1 < \delta \leq 4$ in 0.1 increments. I then checked the globality of the calculated equilibria. It appears that the equilibria are global if the value of e is small enough. For $x = 1$, the threshold value is $\bar{e} = 1.96$ (such that all local equilibria for $e < \bar{e}$ are also global, while for $e > \bar{e}$ all local equilibria are not global); for $x = 0.8$, the threshold is $\bar{e} = 2.36$; for $x = 0.2$ we have $\bar{e} = 2.88$.

Example. Again consider the power loss function (7). Let the number of voters be $n > 0$. Denote by P_i the probability that voter i supports Candidate 1. Let V_j be the number of votes received by Candidate $j = 1, 2$. The probability that Candidate 1 receives exactly l votes is

$$p(V_1 = l) = \sum_{S \subseteq N, |S| = l} \left(\prod_{i \in S} p_i \prod_{i \notin S} (1 - p_i) \right), \tag{11}$$

where $N = \{1, \ldots, n\}$.

I assume that the candidates have the Cobb-Douglas utility function over the number of votes:

$$U_j = V_j^{\gamma_j}, \tag{12}$$

where V_j is the number of votes in favor of Candidate j, and $\gamma_j \geq 0$ is the parameter that determines the risk preference of the candidate. If $\gamma_j \in (0,1)$, then the candidate are risk-averse; if $\gamma_j = 1$, he is risk-neutral; finally, if $\gamma_j > 1$, then candidate j is a risk-lover.

In order to obtain equilibrium comparative statics with respect to γ_j and other parameters, I calculated equilibrium policy positions for various values of model parameters. I assumed that there are two groups of voters of sizes $N_1 + N_2 = N$. For voter j in Group 1, took $v_j = 0$, $e_{1j} = e$, and $e_{2j} = 0$. For voter j in Group 2, I had $v_j = 1$, $e_{1j} = 0$ and $e_{2j} = e$. I fixed $\beta = 1$.

The comparative statics results are similar to the two-voter analytic results. Figure 2(a) shows equilibrium positions of the two candidates for the case when there are three voters, with $N_1 = 2$. The candidates had identical utility functions: $\gamma_1 = \gamma_2 = \gamma$.

Figure 2(b) shows the effect of the number of partisan voters on the equilibrium. As the number of partisans for Candidate 1 (the size of Group 1) decreases, the equilibrium positions of both candidates move closer to the ideal policy of Candidate 2's partisans. In all cases, increases in γ cause candidates to adopt policy positions that are further away from the ideal policy position of their partisan voters; for $\gamma = 1$ the candidates choose identical policy positions, as predicted by theory (Banks and Duggan, 2005).

As the number of voters increases, the policy positions of the candidates coincide when both candidates are risk-neutral. Table 1 shows the calculated equilibria for several scenarios when the number of voters N and number of Candidate 1 partisans are multiples of 3 and 2, respectively.

Table 2 shows similar results for voter groups of equal size, and candidates with different utility functions. It still appears that the equilibrium policy positions converge to those for risk-neutral candidates: $y_1 = y_2 = 0.5$.

The intuition behind this convergence of candidate policy positions is straightforward. If the voting is probabilistic, then the vote share of each candidate is a random variable. If the votes are not correlated, then this random variable converges to the candidate's expected vote share as the number of voters increases. Hence the candidates act as expected vote share maximizers if the number of voters is large.

The following proposition investigates the equilibrium comparative statics with respect to the properties of the voter disutility functions.

Proposition 3 *Let $\psi_k(\cdot)$ be a family of twice continuously differentiable disutility functions indexed by $k = 1, 2, \ldots$ such that for all k we have $\psi_k(d) = \psi_k(-d)$, $\psi'(d) > 0$ for $d > 0$, $\psi_k'(0) = 0$, and for some $\eta > 0$ we have*

$$\lim_{k \to \infty} \psi_k'(d) = \eta \tag{13}$$

for all $d > 0$. Then, if $x > \frac{1}{2}$, there exists a sequence of equilibria $(y_k, 1 - y_k)$ such that

$$\lim_{k \to \infty} y_k = 0. \tag{14}$$

If $e > \eta$ and $x < \frac{1}{2}$, then there exists a sequence of equilibria $(y_k, 1 - y_k)$ such that

$$\lim_{k \to \infty} y_k = 1. \tag{15}$$

It follows that if $x > \frac{1}{2}$, then, as the disutility functions become more and more linear, each candidate in equilibrium chooses a policy position closer and closer to that of his partisan voters. For $x < \frac{1}{2}$, the candidates converge to the positions of the *opposing* partisan voters, if the probability of a voter supporting her partisan candidate is always greater than $\frac{1}{2}$. The latter requirement is expressed in the condition $e > \eta$.

3 Conclusion

One contribution of this work is methodological. The probabilistic voting model is a very promising tool, as it can be used to integrate game-theoretic modeling of political actors with empirical analysis of mass survey data. This work finally demonstrates that in the probabilistic voting setting, one can obtain transparent comparative statics that explain why candidates select different policy platforms.

As a topic for future research, one can consider several competing parties/candidates. It is well known that if the payoffs of all candidates are linear in their vote share, then the mean voter theorem can be generalized to more than two candidates (Lin, Enelow, and Dorussen, 1999; Schofield, 2007; McKelvey and Patty, 2005). Most likely, this is also an artifact, and the result will not hold of the payoff linearity assumption is relaxed. The results will be even more interesting, as for multi-party parliamentary elections the shape of the function that links party vote shares and party seat shares can be tied to observable electoral institutions, such as the number of party lists, floor requirement, or the quotient formula.

The numerical results suggest that, regardless of their preferences, the candidates will act as expected vote share maximizers if the number of voters is large. Establishing this fact formally will involve will need to address several issues, technical as well as substantive. One will have to start with a metrization of the space of candidate payoff functions for different sizes of electorates, then look at the limiting case we the number of voters tends to infinity. The relevant question is whether any such metrizatioon will, in the limiting case, produce an equilibrium that is equivalent to the trivial risk-neutral candidate payoffs.

Proofs

Proof of Proposition 1. The first-order conditions for a Nash equilibrium are

$$\frac{\partial U_1}{\partial y_1} = x(P_{11} + P_{21}) + (1-2x)(P_{11}P_2 + P_{21}P_1) = 0, \tag{16}$$

$$\frac{\partial U_2}{\partial y_2} = (x-1)(P_{12} + P_{22}) + (1-2x)(P_{12}P_2 + P_{22}P_1) = 0. \tag{17}$$

Take $y_1 = y$, $y_2 = 1 - y$. Denote $p = P_1 = 1 - P_2$ and $p' = P'(e - \psi(y) + \psi(1-y)) =$

$P'(-e+\psi(y)-\psi(1-y))$. We can rewrite the first first-order condition:

$$\frac{\partial U_1}{\partial y_1} = p'\left(\psi'(y)(p+x-2px-1)+\psi'(1-y)(x+p-2px)\right)=0. \tag{18}$$

As we assume that $\psi'(0)=0$ it follows that for $y=0$ we have $\frac{\partial U_1}{\partial y_1}>0$ and for $y=1$ we have $\frac{\partial U_1}{\partial y_1}<0$. As $\frac{\partial U_1}{\partial y_1}$ is continuous in y, for some $y_1=y$, $y_2=1-y$ we must have a local maximum of U_1. Likewise, $1-y$ will be a local maximum for U_2. The proof is complete.

Proof of Proposition 2. Take derivatives of (5):

$$\begin{aligned}
\frac{\partial H}{\partial y} &= -(1-2x)(\psi'(y)+\psi'(1-y))^2 p' + \\
&\quad +\psi''(y)(p+x-2px-1)-\psi''(1-y)(x+p-2px) \qquad (19)\\
\frac{\partial H}{\partial x} &= (\psi'(1-y)+\psi'(y))(1-2p), \qquad (20)\\
\frac{\partial H}{\partial e} &= (1-2x)p'(\psi'(y)+\psi'(1-y)), \qquad (21)
\end{aligned}$$

where $p'=P'(e-\psi(y)+\psi(1-y))$. We have $p>\frac{1}{2}$ for $y\le\frac{1}{2}$ by definition. For $y>\frac{1}{2}$ we also have $p>\frac{1}{2}$ because of (5). Hence, $\frac{\partial H}{\partial x}<0$ and

$$\frac{\partial y}{\partial x} = -\frac{\frac{\partial H}{\partial x}}{\frac{\partial H}{\partial y}} < 0 \tag{22}$$

whenever $\frac{\partial H}{\partial y}<0$.

The sign of $\frac{\partial H}{\partial e}$ is equal to the sign of $1-2x$. Hence

$$\frac{\partial y}{\partial e} = -\frac{\frac{\partial H}{\partial e}}{\frac{\partial H}{\partial y}} \tag{23}$$

is positive if $x<\frac{1}{2}$ and negative if $x>\frac{1}{2}$, given $\frac{\partial H}{\partial y}<0$.

We have $\frac{\partial H}{\partial y}<0$ if $x\le\frac{1}{2}$. If $x>\frac{1}{2}$, the sufficient condition that guarantees $\frac{\partial H}{\partial y}<0$ is

$$p'(\psi'(y)+\psi'(1-y))^3 < \psi'(y)\psi''(1-y)+\psi'(1-y)\psi''(y) \tag{24}$$

for all $y<\frac{1}{2}$. The proof is complete.

Proof of Proposition 3. Let $\bar{\psi}(d) = \eta|d|$. Take $y_1 = y$, $y_2 = 1-y$. Let $\bar{U}_1$ be the utility of Candidate 1 with $\psi(\cdot) = \bar{\psi}(\cdot)$. Put

$$\bar{\Delta}(y) = \frac{\partial \bar{U}_1}{\partial y_1}_{y_1=y, y_2=1-y} = 1 - \frac{\partial \bar{U}_2}{\partial y_2}_{y_1=y, y_2=1-y} = \eta P'(e+\eta(1-2y))(1-2x)(2P(e+\eta(1-2y))-1). \tag{25}$$

Let $\psi_k(\cdot)$ be a family of disutility functions with the properties as in the statement of this theorem. Let U_{k1} be the utility of Candidate 1 with $\psi(\cdot) = \psi_k(\cdot)$, with U_{k2} defined similarly. Take

$$\Delta_k(y) = \left.\frac{\partial U_{k1}}{\partial y_1}\right|_{y_1=y, y_2=1-y} = 1 - \left.\frac{\partial U_{k2}}{\partial y_2}\right|_{y_1=y, y_2=1-y}. \tag{26}$$

For any $\epsilon > 0$, we have $|\bar{\Delta}(y) - \Delta_k(y)| < \epsilon$ for large enough k. It follows then that for $x > \frac{1}{2}$ and $y < \frac{1}{2}$ we have $\Delta_k(y) < 0$ for large enough k. As $\Delta_k(y)$ is continuous in y and $\Delta_k(0) > 0$ since $\psi_k'(0) = 0$ for all k, we the following statement: for any $0 < y < \frac{1}{2}$, there exists $k > 0$ such that there exists a local equilibrium with $y_1 = 1 - y_2$, $y_1 < y$. This is the first part of the theorem's statement.

Now let $\eta < e$. Then for any $y \geq 0$ we have $P > \frac{1}{2}$ for a large enough k. Then for any $\delta > 0$, we have $\bar{\Delta}(1-\delta) > 0$ and $\Delta_k(1-\delta) > 0$ for k large enough. It follows that for any $y < 1$ there exists k such that there exists a local equilibrium with $y_1 = 1 - y_2$, $y_1 > y$. This is the second part of the theorem's statement.

The proof is complete.

Notes

1. And, without loss of generality, that the utility of zero votes is zero, and the utility of N votes is one.

2. The same work gave the conditions under which the convergent equilibrium would unravel due to failure of second-order conditions.

3. Valence, since Stokes (1962), refers to the set of candidate or party characteristics, other than policy platforms, about which voters are concerned. A candidate with a higher valence may attract voters who, *ceteris paribus*, prefer the opponent's policy platform. The most commonly named source of exogenous valence is incumbency. Groseclose (2001) gives a review of topic.

4. This implies $\psi'(d) > 0$ for $d > 0$.

5. Suppose that y_1, y_2 are in equilibrium, and that each candidate chooses a policy position closer to one's partisan voters: $y_1' = y_1 - \delta$, $y_2' = y_2 + \delta$. The condition (6) implies that Candidate 1's utility will be higher at y_1 than at y_1', and Candidate 2' utility will be higher at y_2.

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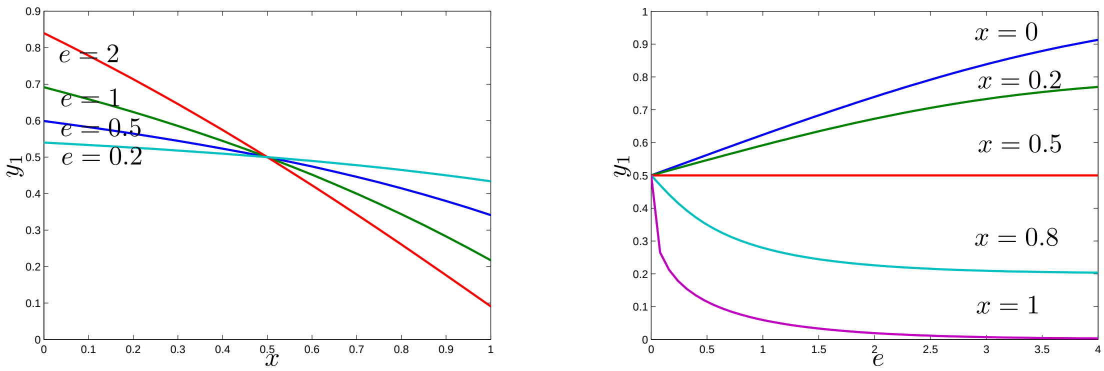
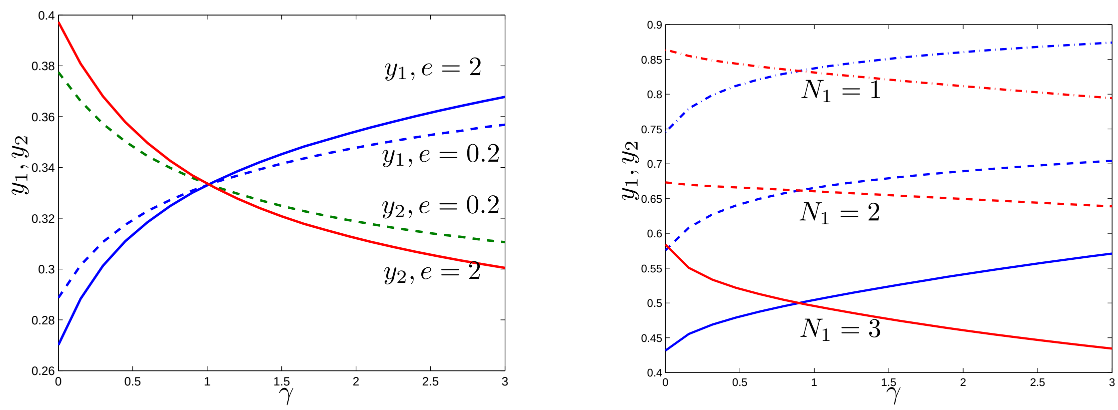


Figure 1: The equilibrium position of Candidate 1 for different values of x and e, with $\beta = 0.5$.



(a) Equilibrium policy positions, three voters

(b) Equilibrium policy positions, six voters, $e = 1$

Figure 2: Equilibrium policy positions for three and six voters.

		$N=3$ $N_1=2$	$N=6$ $N_1=4$	$N=9$ $N_1=6$	$N=12$ $N_1=8$	$N=15$ $N_1=10$
$\gamma_1=0.2$	y_1	0.4652	0.4157	0.3714	0.3540	0.3476
$\gamma_2=0.2$	y_2	0.2151	0.2903	0.3116	0.3186	0.3221
$\gamma_1=0.5$		0.4031	0.3658	0.3500	0.3443	0.3415
$\gamma_2=0.5$		0.2762	0.3108	0.3200	0.3237	0.3264
$\gamma_1=1$		0.3333	0.3333	0.3333	0.3333	0.3333
$\gamma_2=1$		0.3333	0.3333	0.3333	0.3333	0.3333
$\gamma_1=2$		0.2665	0.2992	0.3102	0.3160	0.3194
$\gamma_2=2$		0.4056	0.3686	0.3567	0.3508	0.3472

Table 1: Equilibrium policy positions.

		$N=4$ $N_1=2$	$N=6$ $N_1=3$	$N=8$ $N_1=4$	$N=10$ $N_1=5$	$N=12$ $N_1=6$
$\gamma_1=0.5$	y_1	0.5519	0.5286	0.5183	0.5135	0.5107
$\gamma_2=1$	y_2	0.5006	0.5002	0.5001	0.5000	0.5000
$\gamma_1=0.5$		0.5502	0.5276	0.5179	0.5132	0.5105
$\gamma_2=2$		0.5577	0.5385	0.5289	0.5231	0.5193
$\gamma_1=1$		0.4992	0.4996	0.4998	0.4999	0.4999
$\gamma_2=2$		0.5583	0.5387	0.5290	0.5232	0.5194

Table 2: Equilibrium policy positions for groups of voters of equal size.