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Compact Leaves of Structurally Stable Foliations

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Abstract—We prove that any compact manifold whose fundamental group contains an abelian normal subgroup of positive rank can be represented as a leaf of a structurally stable suspension foliation on a compact manifold. In this case, the role of a transversal manifold can be played by an arbitrary compact manifold. We construct examples of structurally stable foliations that have a compact leaf with infinite solvable fundamental group which is not nilpotent. We also distinguish a class of structurally stable foliations each of whose leaves is compact and locally stable in the sense of Ehresmann and Reeb.

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1. INTRODUCTION

The structural stability of diffeomorphisms and flows on compact manifolds has been one of the most important directions in the qualitative theory of dynamical systems during the last 50 years. The concept of structural stability was introduced by Andronov and Pontryagin in [1]. The development of the structural stability theory is due to the well-known studies of Smale, Anosov, Palis, et al. (see [2–5]). Earlier, this theory concentrated around the proof of the stability conjecture in the C^1 topology, which states that a dynamical system (a diffeomorphism or a flow) is C^1 structurally stable if and only if it satisfies Axiom A and the strong transversality condition.

First, the sufficiency of these conditions was proved for various classes of dynamical systems by Anosov, Palis, Smale, Robbin, de Melo, and Robinson. The necessity of these conditions in the class C^1 was proved by Mañé [6] for diffeomorphisms and later by Hayashi [28] for flows.

As emphasized in [7], the Palis–Smale conjecture stating that Axiom A and the strong transversality condition characterize C^r structurally stable diffeomorphisms remains an open problem for $r \geq 2$.

In the introduction to [8], Katok and Spatzier pointed out that the phenomenon of structural stability was not completely understood.

In the present paper, by *structural stability* (of diffeomorphisms, group representations, and foliations) we mean the C^r structural stability for any fixed $r \geq 1$ unless otherwise stated. All manifolds are assumed to be C^∞-smooth, connected, Hausdorff, and with countable base, and neighborhoods are assumed to be open. In addition, all mappings are assumed to be C^r-smooth and $\mathrm{Diff}^r(T) = \mathrm{Diff}(T)$ unless otherwise stated.

As pointed out by Brunella [9], today there are few examples of structurally stable foliations.

The following question is known to be important (see, for example, [10]): *What manifolds can serve as compact leaves of a structurally stable foliation on a compact manifold*? The goal of the present paper is to study this question.

To prove the existence of a structurally stable foliation with a given compact leaf, we apply suspension foliations, which were introduced by Haefliger. The construction of a suspension foliation generalizes the suspension of a diffeomorphism, which is well known in the theory of dynamical systems.

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As stated by Palis [22] and shown by the author and Chubarov [11], the structural stability of a foliation obtained by a suspension of a homomorphism

$$\rho\colon\ G = \pi_1(B, b_0) \to \mathrm{Diff}(T)$$

is equivalent to the structural stability of the representation ρ. Such a foliation is denoted by $(M, \mathcal{F}) = \mathcal{S}us(T, B, \rho)$. To construct a structurally stable foliation with a given leaf L, it suffices to set $B = L$ and define a structurally stable representation $\rho\colon G = \pi_1(L, b_0) \to \mathrm{Diff}(T)$ such that the group $\Psi := \rho(G)$ has a fixed point. Then the foliation $(M, \mathcal{F}) = \mathcal{S}us(T, L, \rho)$ has a closed leaf diffeomorphic to L (see the proof of Theorem 3).

First, we find sufficient conditions for the structural stability of some representations of groups that have an abelian normal subgroup (Theorem 1 and Corollaries 1 and 2), and then prove the feasibility of these conditions and apply the results obtained in order to construct structurally stable suspension foliations (Theorem 3).

We prove that any compact manifold L whose fundamental group G contains an abelian normal subgroup G_0 of positive rank is a leaf of some structurally stable foliation $(M, \mathcal{F}) = \mathcal{S}us(T, L, \rho)$, where the role of T can be played by an arbitrary closed manifold (Theorem 3). This leaf L can be any compact manifold whose fundamental group has infinite center. We construct examples of structurally stable foliations that contain a compact leaf with infinite solvable fundamental group that is not nilpotent (Example 2).

We establish (Corollary 3) that any compact manifold with abelian fundamental group of positive rank is a leaf of some structurally stable suspension foliation $(M, \mathcal{F})$ on a compact manifold M with the global holonomy group $\Psi := \rho(G)$ isomorphic to the group of integers $\mathbb{Z}$. In this case, every nontrivial transformation in the group Ψ is structurally stable. For such a foliation $(M, \mathcal{F})$ to be structurally stable, it is most essential that the centralizer of a diffeomorphism ψ generating the group Ψ be C^0-discrete (see the proof of Theorem 1).

Grossi Sad [12] constructed a faithful C^3 structurally stable representation

$$\rho_S\colon G = \mathbb{Z} \times \mathbb{Z} \to \mathrm{Diff}(S^m \times S^k)$$

defined on generators by the equalities $\rho_S(1, 0) = (f, \mathrm{Id}_{S^k})$ and $\rho_S(0, 1) = (\mathrm{Id}_{S^m}, g)$, where f and g are Morse–Smale diffeomorphisms of S^m and S^k, respectively, that have precisely two periodic points (a source and a sink) and satisfy some additional conditions. Let $T = S^m \times S^k$ and $L = T^2$ be a two-dimensional torus. Then the foliation $(M, \mathcal{F}) = \mathcal{S}us(T, L, \rho_S)$ is structurally stable and has four compact leaves that are diffeomorphic to the torus T^2.

Let L be a compact leaf of the foliation $(M, \mathcal{F})$. Levine and Shub [13] introduced the concept of stability of a germ of foliations in L. In contrast to the present paper, Levine and Shub considered foliations defined in a neighborhood of an embedded submanifold L in M. They proved that the stability of a germ of foliations in L is equivalent to the stability (also defined by them) of a linear holonomy representation of the fundamental group $\pi_1(L, b_0)$ of the leaf L. Using this fact, Levine and Shub [13] showed the local instability of foliations in a compact leaf diffeomorphic to a torus of dimension higher than 1. They stressed the difference between their notion of local stability of foliations in a compact leaf L and the structural stability of foliations with a compact leaf L when they are considered globally. Our results also illustrate this difference (Corollary 3).

We distinguish a class of structurally stable foliations all of whose leaves are compact and stable in the sense of Ehresmann and Reeb (Section 6). The foliations of this class are also locally stable in the sense of Levine and Shub in each of their leaves.

In addition, we point out that any compact manifold with finite fundamental group is representable as a leaf of a structurally stable foliation.

2. THE BASIC CONCEPTS AND NOTATIONS

2.1. Structural stability of diffeomorphisms. Since the manifold T is compact, the strong and weak C^r topologies in $\mathrm{Diff}^r(T)$ coincide for every $r \geq 1$. It is well known [14] that the space $\mathrm{Diff}^r(T)$ admits a complete metric, which we denote by σ^r. For $f, h \in \mathrm{Diff}^r(T)$, the distance $\sigma^r(f,h)$ is determined by the r-jets of the diffeomorphisms f and h. A ball of radius $\varepsilon > 0$ with center at $f \in \mathrm{Diff}^r(T)$ is called an ε-neighborhood of f and is denoted by $D_\varepsilon(f)$.

Since T is compact, it is metrizable and admits a complete metric σ_T. The set of homeomorphisms $\mathrm{Homeo}(T)$ of the manifold T can be equipped with the metric

$$\sigma^0(f,h) := \max_{x \in T} \sigma_T(f(x), h(x)),$$

for which the metric topology coincides with the compact-open topology. Denote by $D^0_\varepsilon(f)$ the ball of radius $\varepsilon > 0$ centered at f in the metric space $(\mathrm{Homeo}(T), \sigma^0)$ and call $D^0_\varepsilon(f)$ the ε-neighborhood of f in the space of homeomorphisms $\mathrm{Homeo}(T)$. Any homeomorphism in the ε-neighborhood $D^0_\varepsilon(\mathrm{Id}_T)$ of the identity mapping Id_T is called an *ε-homeomorphism* of the manifold T.

A diffeomorphism $f \in \mathrm{Diff}^r(T)$ is said to be *C^r structurally stable* if, for any ε-neighborhood $D^0_\varepsilon(\mathrm{Id}_T)$ in $\mathrm{Homeo}(T)$, there exists an ε-dependent neighborhood $U = U(f,\varepsilon)$ for f in $\mathrm{Diff}^r(T)$ such that for any $h \in U$ there exists a homeomorphism $d \in D^0_\varepsilon(\mathrm{Id}_T)$ satisfying the equality $d \circ f = h \circ d$. In this case, f is said to be *conjugate to* h by an ε-homeomorphism.

It is well known that the C^1 structural stability of a diffeomorphism f is equivalent to Axiom A and the strong transversality condition.

2.2. Diffeomorphisms with discrete centralizers. The *centralizer* of an element $f \in \mathrm{Diff}^r(T)$ is the subgroup $\mathcal{Z}(f) := \{h \in \mathrm{Diff}^r(T) \mid h \circ f = f \circ h\}$ of the group $\mathrm{Diff}^r(T)$. If $\mathcal{Z}(f) = \{f^k \mid k \in \mathbb{Z}\}$, then f is said to have a *trivial centralizer*.

In [5], Smale lists 18 problems, the 12th of which is called "Centralizers of Diffeomorphisms" and is formulated as follows:

"*Can a diffeomorphism of a compact manifold T onto itself be C^r approximated, for all $r \geq 1$, by a diffeomorphism $f\colon T \to T$ that commutes only with its iterates*?"

Denote by $\mathfrak{A}^r(T)$ the subset in $\mathrm{Diff}^r(T)$ formed by diffeomorphisms satisfying Axiom A and the strong transversality condition.

The results of Kopell [15] and Palis and Yoccoz [16], which are known [17] to be valid in the smoothness class C^r, $r \geq 2$, as well as the recent paper by Bonatti, Crovisier, and Wilkinson [17] for the smoothness class C^1, imply a positive answer to the above question of Smale for the space of C^r diffeomorphisms $\mathfrak{A}^r(T)$ for any fixed $r \geq 1$.

2.3. Metric in the representation space. The terms "representation of a group G in $\mathrm{Diff}^r(T)$" and "C^r action of a group G" are used in this paper as synonyms.

Below, we consider only discrete groups G, with any representation $\rho\colon G \to \mathrm{Diff}(T)$ being a continuous mapping. Denote by $A(G,T)$ the set of all representations of the group G in $\mathrm{Diff}(T)$. Since the group G has a finite number of generators, the space $A(G,T)$ is metrizable as follows.

Let $\xi = \{g_1, \dots, g_m\}$ be the set of generators of the group G. Then the space $A(G,T)$ of representations of the group G in $\mathrm{Diff}^r(T)$ admits a metric $\sigma = \sigma_\xi$ with

$$\sigma(\rho, \rho') := \max_{1 \leq i \leq m} \sigma^r\big(\rho(g_i), \rho'(g_i)\big).$$

We will denote a finitely presented group G with generators $g_1, \dots, g_m$ and defining relations $R_1 = e, \dots, R_k = e$ by $G = \langle g_1, \dots, g_m \mid R_1, \dots, R_k \rangle$.

2.4. Structural stability of representations.

Definition 1. A smooth representation $\rho\colon G \to \mathrm{Diff}(T)$ of a group G in $\mathrm{Diff}(T)$ is said to be *structurally stable* if, for any ε-neighborhood $D^0_\varepsilon(\mathrm{Id}_T)$ of the identity mapping Id_T in the space $\mathrm{Homeo}(T)$, there exists an ε-dependent neighborhood $\mathcal{V} = \mathcal{V}(\rho, \varepsilon)$ of ρ in $A(G,T)$ such that, for any $\rho' \in \mathcal{V}$, there exists a homeomorphism $d \in D^0_\varepsilon(\mathrm{Id}_T)$ that satisfies the equality

$$d \circ \rho(g) = \rho'(g) \circ d$$

for all $g \in G$.

For $r = 1$, this definition coincides with the definition of structural stability of a C^1 action of a 0-dimensional Lie group, as given by Kupka in [18] for $\theta = \mathrm{Id}_G$.

Note that a representation $\rho\colon G \to \mathrm{Diff}(T)$ of a discrete group G with a finite number of generators $\{g_i \mid i = \overline{1,m}\}$ is structurally stable if and only if, for any $\varepsilon > 0$ and every diffeomorphism $\psi_i := \rho(g_i)$, $i = \overline{1,m}$, there exists an ε-dependent neighborhood $U_i = U_i(\psi_i, \varepsilon)$ of ψ_i in $\mathrm{Diff}(T)$ such that for any representation $\rho'\colon G \to \mathrm{Diff}(T)$ in the neighborhood $\mathcal{W} = \mathcal{W}(g_1, \dots, g_m; U_1, \dots, U_m)$, where

$$\mathcal{W}(g_1, \dots, g_m; U_1, \dots, U_m) := \bigl\{\rho' \in A(G,T) \mid \rho'(g_i) \in U_i,\ i = \overline{1,m}\bigr\},$$

there exists an ε-homeomorphism d of the space T that satisfies the equality

$$d \circ \rho(g_i) = \rho'(g_i) \circ d \qquad \forall i = \overline{1,m}.$$

Let $G = \langle g_1, \dots, g_m \mid R_1, \dots, R_k\rangle$ be a finitely presented group. Consider the closed subset

$$N := \bigl\{t = (t_1, \dots, t_m) \in (\mathrm{Diff}(T))^m \mid R_j(t_1, \dots, t_m) = \mathrm{Id}_T\ \ \forall j = \overline{1,k}\bigr\}$$

in $(\mathrm{Diff}(T))^m$. We stress that the points $(\psi'_1, \dots, \psi'_m)$ with $\psi'_i = \rho'(g_i)$ and $\rho' \in \mathcal{W}$ run only through the trace of the neighborhood $U_1 \times \dots \times U_m$ on N, i.e., the set $P = (U_1 \times \dots \times U_m) \cap N$, rather than through the whole neighborhood.

2.5. Structural stability of foliations. We consider the category $\mathcal{F}ol^0$ of C^r-smooth foliations, in which a morphism of two foliations $(M, \mathcal{F})$ and $(M', \mathcal{F}')$ is a continuous mapping $d\colon M \to M'$ that sends the leaves of the foliation $\mathcal{F}$ to the leaves of the foliation $\mathcal{F}'$.

Let M be a C^r-smooth manifold, not necessarily compact. By $\mathrm{Fol}_q(M)$ we denote the set of C^r-smooth codimension q foliations $(M, \mathcal{F})$ with the Hirsch–Epstein topology Θ^r [19]. Let M be compact and Ω^r be the standard C^r topology on the set of C^{r+1} foliations on M identified with the sections of the fibration $f\colon \Gamma \to M$ whose fiber over $x \in M$ is the Grassmannian of all $(n-q)$-dimensional subspaces of the tangent vector space T_xM. Then the inclusions $\Theta^r \subset \Omega^r \subset \Theta^{r+1}$ hold on the set of C^{r+1} foliations on M [19].

Definition 2. A foliation $(M, \mathcal{F})$ is said to be *structurally stable* if, for any neighborhood $U = U(\mathrm{Id}_M)$ in $\mathrm{Homeo}(M)$, there exists a neighborhood $\mathcal{U} = \mathcal{U}(\mathcal{F}, U)$ of the foliation $\mathcal{F}$ in $\mathrm{Fol}_q(M)$ such that, for any foliation $\mathcal{F}' \in \mathcal{U}$, there exists a homeomorphism $d \in U$ that is an isomorphism of the foliations $(M, \mathcal{F})$ and $(M, \mathcal{F}')$ in the category $\mathcal{F}ol^0$.

3. SUFFICIENT CONDITIONS FOR THE STRUCTURAL STABILITY OF SOME REPRESENTATIONS

3.1. Representations of groups with abelian normal subgroups. Since the manifold T is metrizable, $\mathrm{Homeo}(T)$ admits a unique Polish topology with respect to which $\mathrm{Homeo}(T)$ is a topological group [20]. This topology coincides with the compact-open topology, and the inclusion $j\colon \mathrm{Diff}^r(T) \to \mathrm{Homeo}(T)$ is continuous for $r \ge 1$. Recall that a diffeomorphism ψ has a C^0-discrete centralizer if $\mathcal{Z}(\psi)$ is a discrete subgroup in the group $\mathrm{Homeo}(T)$.

Lemma 1. *If a C^r diffeomorphism ψ of a compact C^r manifold, $r \geq 1$, satisfies Axiom* A *and has a trivial centralizer $\mathcal{Z}(\psi)$, then the centralizer $\mathcal{Z}(\psi)$ is C^0-discrete.*

Proof. Suppose the contrary: let there exist an Axiom A diffeomorphism ψ with centralizer $\mathcal{Z}(\psi) = \{\psi^m \mid m \in \mathbb{Z}\}$ such that $\mathcal{Z}(\psi)$ is not a discrete subgroup in $\mathrm{Homeo}(T)$. Then, for any arbitrarily small neighborhood $V = V(\mathrm{Id}_T)$ in $\mathrm{Homeo}(T)$, we necessarily have $\mathcal{Z}(\psi) \cap V \neq \mathrm{Id}_T$. Since $\mathrm{Homeo}(T)$ has a countable base, it follows from the above that there exists a sequence $a_k = \psi^{m_k}$ that converges to Id_T in $\mathrm{Homeo}(T)$, with all m_k being pairwise different. From this sequence, we can extract a converging subsequence, which we denote by ψ^{m_k} as before, where $\{m_k\}$ is a sequence of integers that satisfies one of the following two conditions:

$$0 < m_1 < m_2 < \ldots < m_k < m_{k+1} < \ldots, \tag{1}$$

$$0 < -m_1 < -m_2 < \ldots < -m_k < -m_{k+1} < \ldots. \tag{2}$$

Then

$$\psi^{m_k} \to \mathrm{Id}_T \qquad \text{as} \quad k \to +\infty, \tag{3}$$

as a subsequence of a converging sequence. Hence,

$$\|D\psi_x^{m_k} y\| \to \|y\| \qquad \text{as} \quad k \to +\infty \quad \text{for all} \quad x \in T, \quad y \in T_x T. \tag{4}$$

Suppose that $\{m_k\}$ satisfies (1). Since ψ satisfies Axiom A, ψ has a periodic point $x \in T$ and the orbit $\Psi \cdot x$ of the group $\Psi = \langle \psi \rangle$ is finite. Therefore, there exists a neighborhood $U = U(\mathrm{Id}_T)$ in $\mathrm{Homeo}(T)$ such that $U \cdot x \cap \Psi \cdot x = \{x\}$, where $U \cdot x$ is the orbit of the point x with respect to U. Since $\psi^{m_k} \to \mathrm{Id}_T$, there is a number K such that $\psi^{m_k} \in U$ for all $k > K$; hence, $\psi^{m_k}(x) = x$ for all $k > K$. It follows from Axiom A that

$$\|D\psi_x^{m_k} w\| \to +\infty \qquad \text{as} \quad k \to +\infty$$

for any nonzero vector $w \in E_x^u$. This contradicts (4).

If the sequence ψ^{m_k} satisfies condition (2), then, taking a nonzero vector $v \in E_x^s$ instead of $w \in E_x^u$, we arrive at a contradiction in the same way as above. This means that the centralizer $\mathcal{Z}(\psi)$ is discrete in $\mathrm{Homeo}(T)$. $\square$

Theorem 1. *Let G be a finitely generated group with abelian normal subgroup G_0 and $\{g_i \mid i = \overline{0, m}\}$ be a family of generators in G such that $g_0 \in G_0$. Let*

$$\rho\colon G \to \mathrm{Diff}(T)$$

be a representation defined on the generators by the equalities $\rho(g_0) = \psi_0$ and $\rho(g_i) = \mathrm{Id}_T$ for $i \geq 1$. Suppose that ψ_0 is a structurally stable diffeomorphism that has a neighborhood consisting of Axiom A *diffeomorphisms with trivial centralizers. Then the representation ρ is structurally stable.*

Proof. If $G = G_0 = \langle g \rangle$, then the structural stability of the representation $\rho\colon G \to \mathrm{Diff}(T)$ of the group G is equivalent to the structural stability of the diffeomorphism $\psi = \rho(g)$. Therefore, the assertion of Theorem 1 is valid.

Now, let $m \geq 2$. Since the group $\Psi_0 = \rho(G_0)$ is abelian and $\psi_0 \in \Psi_0$, we have $\Psi_0 \subset \mathcal{Z}(\psi_0)$. The fact that the centralizer $\mathcal{Z}(\psi_0)$ is trivial implies that $\Psi_0 = \langle \psi_0 \rangle$. By condition, ψ_0 satisfies Lemma 1; hence, the centralizer $\mathcal{Z}(\psi_0)$ is C^0-discrete. Therefore, there exist neighborhoods $U = U(\psi_0)$ and $W = W(\mathrm{Id}_T)$ in the topological group $\mathrm{Homeo}(T)$ that have the following properties: $\mathcal{Z}(\psi_0) \cap U = \{\psi_0\}$, $\mathcal{Z}(\psi_0) \cap W = \{\mathrm{Id}_T\}$, and $W \circ \psi_0 \circ W^{-1} \subset U$.

In the topological group $\mathrm{Homeo}(T)$, there exists an ε-neighborhood $D = D_\varepsilon^0(\mathrm{Id}_T)$ such that $D \circ D \circ D^{-1} \subset W$.

Since the inclusion $j\colon \mathrm{Diff}(T) \to \mathrm{Homeo}(T)$ is continuous and the diffeomorphism ψ_0 is structurally stable, for a given ε-neighborhood D there exists an ε-dependent neighborhood $U_0 = U_0(\psi_0, \varepsilon)$ of ψ_0 in $\mathrm{Diff}(T)$ such that $U_0 \subset U$ and any diffeomorphism in U_0 is conjugate to ψ_0 by a homeomorphism in the ε-neighborhood D. According to the hypotheses of the theorem, we can assume that the neighborhood U_0 is formed by Axiom A diffeomorphisms with trivial centralizers.

Denote by $V_0 = V_0(\mathrm{Id}_T, \varepsilon)$ an ε-dependent neighborhood of the identity diffeomorphism in $\mathrm{Diff}(T)$ that does not intersect U_0 and is such that $V_0 \subset D$.

Consider an ε-dependent neighborhood of the representation ρ in $A(G,T)$ that is defined as follows:

$$\mathcal{V}(\rho,\varepsilon) := \bigl\{\rho' \in A(G,T) \bigm| \rho'(g_0) \in U_0,\ \rho'(g_i) \in V_0\ \ \forall i = \overline{1,m}\bigr\}.$$

Take an arbitrary representation $\rho' \in \mathcal{V}(\rho,\varepsilon)$. Introduce the notations $\psi'_0 := \rho'(g_0)$ and $\psi'_i := \rho'(g_i)$ for $i = \overline{1,m}$. Let $\Psi'_0 := \rho'(G_0)$.

Since $\psi'_0 \in U_0$, it follows that ψ'_0 is a structurally stable Axiom A diffeomorphism with trivial centralizer $\mathcal{Z}(\psi'_0) = \{\psi_0'^k \mid k \in \mathbb{Z}\}$. Therefore, as for the group Ψ_0 above, we get $\Psi'_0 = \langle \psi'_0 \rangle$.

Since Ψ'_0 is a normal subgroup of the group Ψ, for any ψ'_i there exists an integer $s = s(i)$, depending on i, such that the following equality holds:

$$\psi'_i \circ \psi'_0 = \psi_0'^s \circ \psi'_i. \tag{5}$$

According to the choice of ρ', we have $\psi'_0 \in U_0$; therefore, there exists an ε-homeomorphism d satisfying the equality

$$\psi_0 \circ d = d \circ \psi'_0. \tag{6}$$

Since $d, \psi'_i \in D$, we have $\widehat{d} := d \circ \psi'_i \circ d^{-1} \in W$. Equalities (5) and (6) imply the relation

$$\widehat{d} \circ \psi_0 = \psi_0^s \circ \widehat{d}. \tag{7}$$

Thus, $\psi_0^s \in W \circ \psi_0 \circ W^{-1} \subset U$, and so $\psi_0^s \in \mathcal{Z}(\psi_0) \cap U = \{\psi_0\}$; therefore, $s = 1$. Taking this into account, we find from (5) that $\psi'_i \in \mathcal{Z}(\psi'_0) \cap D$; i.e., $\psi'_i = (\psi'_0)^k = d^{-1} \circ (\psi_0)^k \circ d$ for some $k \in \mathbb{Z}$. Hence, $(\psi_0)^k = d \circ \psi'_i \circ d^{-1} \in \mathcal{Z}(\psi_0) \cap W = \{\mathrm{Id}_T\}$. This implies $\psi'_i = \mathrm{Id}_T$ for any $i = \overline{1,m}$.

Thus, the equality

$$\psi_j \circ d = d \circ \psi'_j \tag{8}$$

holds for all $j = \overline{0,m}$, which implies the structural stability of the representation ρ. $\square$

Remark 1. If the diffeomorphism ψ_0 in Theorem 1 is C^1 structurally stable, then Axiom A certainly holds for some neighborhood of ψ_0 in $\mathrm{Diff}^1(T)$.

Example 1. Let H be an arbitrary group with a finite number of generators and G_0 be an arbitrary abelian finitely generated group. Then the semidirect product of groups $G = H \ltimes G_0$ provides an example of a finitely generated group with the abelian normal subgroup G_0.

3.2. Representations of solvable groups. Let G be an arbitrary group and $A, B \subset G$. The commutator of A and B is denoted by $[A,B]$; i.e., $[A,B] := \{a^{-1}b^{-1}ab \mid a \in A,\ b \in B\}$. If the derived series of the group G,

$$G^{(0)} = G,\ G^{(1)} = \bigl[G^{(0)}, G^{(0)}\bigr],\ G^{(2)} = \bigl[G^{(1)}, G^{(1)}\bigr],\ \dots,\ G^{(i)} = \bigl[G^{(i-1)}, G^{(i-1)}\bigr],\ \dots,$$

terminates, i.e., if there exists a positive integer n such that $G^{(n)} = e$, where e is the identity of the group G, then the group G is said to be *solvable*. We stress that $G^{(k)}$ is a normal subgroup of the group G.

Solvable groups contain all nilpotent groups. The latter include all abelian groups.

The following two statements are corollaries of Theorem 1.

Corollary 1. *If a finitely generated group G is solvable, its derived series terminates at $G^{(n)} = e$, and $G_0 = G^{(n-1)}$, then a representation $\rho\colon G \to \mathrm{Diff}(T)$ satisfying the hypotheses of Theorem 1 is structurally stable.*

Corollary 2. *For a representation $\rho\colon G \to \mathrm{Diff}(T)$ of an arbitrary finitely generated abelian group G of positive rank to be structurally stable, it suffices that there exist a system of generators $\{g_i \mid i = \overline{1,m}\}$ in G such that $\rho(g_1) = \psi$, where ψ is a structurally stable diffeomorphism that has a neighborhood consisting of Axiom* A *diffeomorphisms with trivial centralizers, and $\rho(g_i) = \mathrm{Id}_T$ for $i \geq 2$.*

4. APPLICATION OF SUSPENSION FOLIATIONS TO STRUCTURAL STABILITY

The construction of a suspension foliation consists in the following. Let B and T be smooth connected manifolds and $\rho\colon G := \pi_1(B, b) \to \mathrm{Diff}(T)$ be a group homomorphism. Let $\Psi := \rho(G)$ and $\widehat{p}\colon \widehat{B} \to B$ be a universal covering map. We define a right action of G on the product of manifolds $\widehat{B} \times T$:

$$\Theta\colon\ \widehat{B} \times T \times G \to \widehat{B} \times T\colon \quad (x, t, g) \mapsto (x \cdot g, \rho(g^{-1})(t)), \tag{9}$$

where $\widehat{B} \to \widehat{B}\colon x \to x \cdot g$ is a covering transformation induced by an element $g \in G$.

The mapping $p\colon M := (\widehat{B} \times T)/G \to B = \widehat{B}/G$ defines a locally trivial fibration over B with standard fiber T; this fibration is associated with the principal bundle $\widehat{p}\colon \widehat{B} \to B$ with the structure group G. The quotient mapping $f_0\colon \widehat{B} \times T \to (\widehat{B} \times T)/G = M$ induces a smooth foliation $\mathcal{F} = \{f_0(\widehat{B} \times t) \mid t \in T\}$ on M that is transversal to the fibration $p\colon M \to B$. A pair $(M, \mathcal{F})$ is called a *foliation obtained by a suspension of the homomorphism* ρ or a *suspension foliation* and is denoted by $\mathcal{S}us(T, B, \rho)$. The fibration $p\colon M \to B$ is said to be *transversal*, and T is called a *transversal manifold* for this foliation. The diffeomorphism group $\Psi := \rho(G)$ of the manifold T is called the *global holonomy group* of the suspension foliation $(M, \mathcal{F})$.

Everywhere below we assume that the manifold T is compact and the fundamental group $G = \pi_1(B, b_0)$ is finitely generated. Under these assumptions, the author and Chubarov proved [11, Theorem 2] that the foliation $(M, \mathcal{F}) = \mathcal{S}us(T, B, \rho)$ is structurally stable if and only if the representation ρ is structurally stable. Here the foliated manifold M is, generally speaking, noncompact.

Thus, applying Theorem 1 and [11, Theorem 2], we obtain the following statement.

Theorem 2. *Let $(M, \mathcal{F})$ be a foliation obtained by a suspension of the homomorphism*

$$\rho\colon\ G = \pi_1(B, b_0) \to \mathrm{Diff}(T)$$

and the following three conditions hold:

(1) *the group G has an abelian normal subgroup G_0;*

(2) *G has a system of generators $\{g_i \mid i = \overline{0,m}\}$ such that $g_0 \in G_0$, $\rho(g_0) = \psi_0$, and $\rho(g_i) = \mathrm{Id}_T$ for $i \geq 1$;*

(3) *the diffeomorphism ψ_0 is structurally stable and has a neighborhood consisting of Axiom* A *diffeomorphisms with trivial centralizers.*

Then the foliation $(M, \mathcal{F})$ is structurally stable.

Owing to Corollaries 1 and 2, Theorem 2 implies the structural stability results for foliations obtained by suspensions of homomorphisms $\rho\colon G = \pi_1(B, b_0) \to \mathrm{Diff}(T)$ of solvable (in particular, nilpotent and abelian) groups G.

Remark 2. The solvability of the fundamental group $\pi_1(M, x_0)$ of the manifold M implies the solvability of the group $G = \pi_1(B, b_0)$ for any suspension foliation $(M, \mathcal{F})$, because the construction of a suspension foliation implies that G is isomorphic to a normal subgroup of the group $\pi_1(M, x_0)$.

Remark 3. Anderson [21] was the first to apply (Morse–Smale) diffeomorphisms with C^0-discrete centralizers to the structural stability of representations of abelian groups and of the corresponding suspension foliations. The next study in this direction was Palis' article [22].

5. EXISTENCE OF STRUCTURALLY STABLE SUSPENSION FOLIATIONS

Lemma 2. *Let G be a finitely generated group with abelian normal subgroup G_0 of rank $p > 0$, $G \neq G_0$. Then the group G has a system of generators $\{g_i \mid i = \overline{0, m}\}$ such that $G = \langle g_0, \dots, g_m \mid g_p^{n_1}, \dots, g_{p+q-1}^{n_q}, R_\alpha(g_{p+q}, \dots, g_m), \alpha \in A\rangle$, where $\{g_0, \dots, g_{p+q-1}\}$ are generators of G_0.*

Proof. The finitely generated abelian group G_0 of positive rank p is equal to the direct product $\mathbb{Z}^p \times \mathbb{Z}_{n_1} \times \ldots \times \mathbb{Z}_{n_q}$ of the free abelian group $\mathbb{Z}^p$ and the finite abelian group $\mathbb{Z}_{n_1} \times \ldots \times \mathbb{Z}_{n_q}$. Let $\{g_0, \dots, g_{p+q-1}\}$ be generators of G_0 such that $\{g_0, \dots, g_{p-1}\}$ are generators of the group $\mathbb{Z}^p$ and g_{p+j-1} is a generator of the group $\mathbb{Z}_{n_j}$, $j = \overline{1, q}$. Then $G_0 = \langle g_0, \dots, g_{p+q-1} \mid g_p^{n_1}, \dots, g_{p+q-1}^{n_q}\rangle$.

Denote by $f\colon G \to G/G_0$ the canonical epimorphism of the group G onto the quotient group G/G_0. Since G is finitely generated, the quotient group G/G_0 is also finitely generated. Let $G/G_0 = \langle k_1, \dots, k_s \mid R_\alpha(k_1, \dots, k_s), \alpha \in A\rangle$. Then there exist elements $h_1, \dots, h_s$ in G that do not belong to G_0 and are such that $k_j = f(h_j)$, $j = \overline{1, s}$. One can easily verify that $\{g_0, \dots, g_{p+q-1}, h_1, \dots, h_s\}$ is a sought family of generators of the group G. $\square$

Definition 3. If G is a finitely generated group with abelian normal subgroup G_0 of positive rank, then a family of generators of G that satisfies Lemma 2 is said to be G_0-*adapted*.

Theorem 3. *Let T and L be compact C^∞ manifolds. Suppose that the fundamental group $G := \pi_1(L, b_0)$ has an abelian normal subgroup G_0 of positive rank. Then there exists a representation $\rho\colon G \to \mathrm{Diff}^\infty(T)$ such that $(M, \mathcal{F}) = \mathcal{S}us(T, L, \rho)$ is a C^∞ structurally stable foliation on a compact manifold M with a compact leaf L and transversal manifold T.*

Proof. Denote by $\mathfrak{A}^\infty(T)$ the subset of $\mathrm{Diff}^\infty(T)$ formed by Axiom A diffeomorphisms satisfying the strong transversality condition, and denote by $\mathfrak{A}_{(1)}(T)$ the subset of diffeomorphisms in $\mathfrak{A}^\infty(T)$ that have a fixed source or a fixed sink. Note that $\mathfrak{A}_{(1)}(T)$ is a nonempty open subset in $\mathfrak{A}^\infty(T)$ that contains Morse–Smale diffeomorphisms having a source or a sink (of period 1). Denote by $\mathfrak{A}_{(1),0}(T)$ the subset of $\mathfrak{A}_{(1)}(T)$ formed by diffeomorphisms with trivial centralizers. It follows from the results of Kopell [15] and Palis and Yoccoz [16, Theorem 2] that the set $\mathfrak{A}_{(1),0}(T)$ is open and dense in $\mathfrak{A}_{(1)}(T)$. Thus, any element $\psi_0 \in \mathfrak{A}_{(1),0}(T)$ is a structurally stable C^∞ diffeomorphism of T that has a fixed point (a source or a sink) and satisfies condition (3) of Theorem 2.

Set $B := L$. By Lemma 2, there exists a G_0-adapted family of generators $g_0, \dots, g_m$ of the group $G := \pi_1(B, b_0)$. Here $g_0 \in G_0$ is an element of infinite order, and the equalities $\rho(g_0) := \psi_0$ and $\rho(g_k) := \mathrm{Id}_T$ for all $k = \overline{1, m}$ define a homomorphism $\rho\colon G \to \mathrm{Diff}(T)$. Thus, the group G and the representation $\rho\colon G \to \mathrm{Diff}(T)$ satisfy Theorem 2, according to which the foliation $(M, \mathcal{F}) := \mathcal{S}us(T, B, \rho)$ is structurally stable.

It follows from the construction of $(M, \mathcal{F})$ that T is a transversal manifold for this foliation. As above, $\Psi := \rho(G)$; then $\Psi = \langle \psi_0 \rangle \cong \mathbb{Z}$. Let $L_0 := f_0(\widehat{B} \times \{z_0\})$ be a leaf of the foliation $(M, \mathcal{F})$ that corresponds to a fixed point $z_0 \in T$ of the diffeomorphism ψ_0, where we used the notation introduced at the beginning of Section 4. Since the stationary subgroup Ψ_{z_0} of the point z_0 coincides with the whole group Ψ, it follows from the definitions of the suspension foliation $(M, \mathcal{F})$ and of the fibration $p\colon M \to B$ that the restriction $p|_{L_0}\colon L_0 \to B = L$ is a diffeomorphism.

According to Theorem 2, the representation ρ is C^∞ structurally stable. Hence, the foliation $(M,\mathcal{F}) := \mathcal{S}us(T,B,\rho)$ is also C^∞ structurally stable and has a closed leaf L_0 diffeomorphic to the base L. Since the manifold M is a fiber space over a compact base B with compact standard fiber T, we see that M is also compact. $\square$

Remark 4. In [23], Palis and Yoccoz proved that for the q-dimensional torus T^q, $q \geq 2$, the set of Anosov diffeomorphisms with trivial centralizers $\mathfrak{A}_0^A(T^q)$ is open and dense in the set of all Anosov diffeomorphisms of T^q. Therefore, we can take any diffeomorphism $\psi_0 \in \mathfrak{A}_0^A(T^q)$ with a fixed point to construct a structurally stable suspension foliation of codimension q that has the global holonomy group $\Psi = \langle\psi_0\rangle$ and satisfies Theorem 3.

Palis and Yoccoz [16] proved that for any closed two-dimensional surface T, the set of diffeomorphisms with trivial centralizers is open and dense in the space $\mathfrak{A}^\infty(T)$. Fisher [24] showed that a similar statement holds in any smoothness class C^r, $2 \leq r \leq \infty$. Therefore, if the manifold T in the hypotheses of Theorem 3 is two-dimensional, then the assertion of this theorem holds in every smoothness class C^r, $2 \leq r \leq \infty$.

Example 2. Denote by $\mathcal{K}$ the fundamental group of the Klein bottle K^2. Suppose that the group G is an extension

$$1 \to A \to G \to \mathbb{Z} \to 1,$$

where the group A is either $\mathbb{Z}\times\mathbb{Z}$ or $\mathcal{K}$; i.e., one of the following two cases holds:

(1) $G = (\mathbb{Z}\times\mathbb{Z}) \ltimes \mathbb{Z}$,

(2) $G = \mathcal{K} \ltimes \mathbb{Z}$,

where $\ltimes$ stands for the semidirect product of groups. In both cases, G is a solvable group without center. Since any nilpotent group has nontrivial center, G is not nilpotent.

It is well known [25] that the group G is representable in both cases as the fundamental group of a compact 3-manifold L that is a Seifert fiber space over the circle S^1, with the standard fiber given in case (1) by the torus T^2 and in case (2) by the Klein bottle K^2.

In both cases, the group G has a system of generators g_0, g_1, g_2 such that g_0 is a generator of the normal subgroup $G_0 \cong \mathbb{Z}$ and $\{g_1, g_2\}$ are generators of the subgroup $\mathbb{Z}\times\mathbb{Z}$ in case (1) and of the subgroup $\mathcal{K}$ in case (2). Just as in the proof of Theorem 3, let T be an arbitrary compact manifold and ψ_0 an arbitrary element in $\mathfrak{A}_{1,0}(T)$. Setting $\rho(g_0) = \psi_0$ and $\rho(g_1) = \rho(g_2) = \mathrm{Id}_T$, we define a homomorphism $\rho\colon \pi(L, b_0) \to \mathrm{Diff}(T)$. Then, according to Theorem 2, we obtain a C^∞ structurally stable foliation $(M,\mathcal{F}) := \mathcal{S}us(T,L,\rho)$ with a compact leaf L, solvable fundamental group G, and transversal manifold T.

Corollary 3. *For an arbitrary positive integer q, any compact smooth manifold L with abelian fundamental group of rank $k \geq 1$ is a leaf of some structurally stable suspension foliation of codimension q on a compact manifold. In particular, the role of L can be played by the torus of arbitrary dimension $k \geq 1$.*

6. STRUCTURALLY STABLE FOLIATIONS ALL OF WHOSE LEAVES ARE STABLE IN THE SENSE OF EHRESMANN AND REEB

The notion of stability of leaves of foliations was introduced by Ehresmann and his student Reeb, the founders of the theory of foliations. Recall that a subset of a foliated manifold is said to be *saturated* if it can be represented as a union of some leaves of the foliation. A foliation is said to be *compact* if all of its leaves are compact.

Definition 4. A leaf L of a foliation $(M,\mathcal{F})$ of codimension q is said to be *locally stable in the sense of Ehresmann and Reeb* if there exists a family of its saturated neighborhoods W_β, $\beta \in \mathcal{B}$, with the following properties:

(1) there exists a locally trivial fibration $f_\beta\colon W_\beta \to L$, $\beta \in \mathcal{B}$, with a q-dimensional disk D^q as a standard fiber, whose fibers are transversal to the leaves of the foliation $(W_\beta, \mathcal{F}_{W_\beta})$;

(2) for some $\gamma \in \mathcal{B}$, the traces of these neighborhoods form a base of the topology of a fiber of the fibration $f_\gamma\colon W_\gamma \to L$ over x at the point x.

According to the well-known theorem of Reeb, any compact leaf of a foliation with finite holonomy group is locally stable. Recall that a leaf of a foliation is said to be closed if it is a closed subset of the foliated manifold.

Throughout this section, we assume the smoothness of class C^r, $r \geq 2$.

Theorem 4. *Let B be an arbitrary manifold with finite fundamental group $G = \pi_1(B, b_0)$ and $\rho\colon G \to \mathrm{Diff}(T)$ be an arbitrary representation of the group G. Then*

(1) *the suspension foliation $(M, \mathcal{F}) := \mathcal{S}us(T, B, \rho)$ is structurally stable*;

(2) *every leaf of the foliation $(M, \mathcal{F})$ is closed, has a finite holonomy group, and is locally stable in the sense of Ehresmann and Reeb, and the leaf space $M/\mathcal{F} = T/\rho(G)$ is a smooth orbifold.*

Proof. The author and Chubarov proved [11, Theorem 2] that the structural stability of the suspension foliation $(M, \mathcal{F}) := \mathcal{S}us(T, B, \rho)$ is equivalent to the structural stability of the representation ρ.

It is well known (see, for example, [13, 26]) that any differentiable action of a finite group G on a compact manifold T is structurally stable. Therefore, the representation ρ and the foliation $(M, \mathcal{F}) = \mathcal{S}us(T, B, \rho)$ are structurally stable.

Since the group $G = \pi_1(B, b_0)$ is finite, the global holonomy group $\Psi := \rho(G)$ of the suspension foliation $(M, \mathcal{F}) := \mathcal{S}us(T, B, \rho)$ is also finite. Let $\Psi = \{\psi_\alpha \mid \alpha = \overline{1, s}\}$. Consider some Riemannian metric g_0 on T. Set $g_T := \sum_{\alpha=1}^{\alpha=s} \psi_\alpha^* g_0$, where ψ_α^* is the codifferential of the diffeomorphism ψ_α. Then g_T is a complete Riemannian metric on the manifold T, and Ψ is a group of isometries with respect to g_T.

We will use the notation of Section 4. Since the orbit $\Psi \cdot z$ of any point $z \in T$ is closed in T, each leaf of the foliation $(M, \mathcal{F})$ is closed.

According to Nomizu and Ozeki's theorem [29], on every manifold with countable base there exists a complete Riemannian metric. Let g_B be a complete Riemannian metric on the manifold B and $\widehat{p}\colon \widehat{B} \to B$ be a universal covering map. Then $\widehat{p}^* g_B$ is a complete Riemannian metric on $\widehat{B}$. Moreover, the Riemannian metric of the product $(\widehat{B} \times T, g_0)$ of the Riemannian manifolds $(\widehat{B}, \widehat{p}^* g_B)$ and (T, g_T) is also complete. Note that the group G acts on the Riemannian product $(\widehat{B} \times T, g_0)$ by Θ according to (9) as an isometry group. Therefore, the quotient mapping $f_0\colon \widehat{B} \times T \to M = (\widehat{B} \times T)/G$ induces a complete Riemannian metric g_M on M, and f_0 is a Riemannian covering with respect to g_M.

Since g_M is locally projectable with respect to both the foliation $(M, \mathcal{F})$ and the foliation formed by the fibers of the submersion $p\colon M \to B$, it follows that $(M, \mathcal{F})$ is a parallel foliation of the complete Riemannian manifold (M, g_M) and all of its leaves are closed. Since a parallel foliation is a Riemannian foliation, we can apply Reinhart's results [27], which imply that each leaf of this foliation is locally stable in the sense of Ehresmann and Reeb and the leaf space $M/\mathcal{F}$ is a smooth orbifold. It remains to be noted that $M/\mathcal{F} = T/\Psi$. $\square$

Corollary 4. *If B is a compact manifold with finite fundamental group $G = \pi_1(B, b_0)$ and $\rho\colon G \to \mathrm{Diff}(T)$ is any representation of the group G in the diffeomorphism group of an arbitrary compact manifold T, then $(M, \mathcal{F}) := \mathcal{S}us(T, B, \rho)$ is a structurally stable compact foliation on a compact manifold M and all of its leaves are stable in the sense of Ehresmann and Reeb.*

Proof. It follows from the definition of a suspension foliation that M is the space of a locally trivial fibration with standard fiber T and base B. The compactness of T and B implies the compactness of M. Other assertions follow from Theorem 4. $\square$

Corollary 5. *Let B be a manifold with finite fundamental group and T be any compact manifold. Then the trivial foliation $(B \times T, \mathcal{F})$, where $\mathcal{F} = \{B \times \{t\} \mid t \in T\}$, is structurally stable.*

Remark 5. The situation described in Corollary 5 also occurs in the classical theory of dynamical systems. An example is given by a structurally stable flow on the two-dimensional cylinder $M = \mathbb{R}^1 \times S^1$ whose integral curves form the trivial foliation $\mathcal{F} = \{\mathbb{R}^1 \times \{t\} \mid t \in S^1\}$.

Example 3. Let $k \geq 2$ be a positive integer and $(k_1, \dots, k_m)$ be a set of positive integers that are coprime to k. We will regard the odd-dimensional sphere S^{2m+1} as

$$S^m_{\mathbb{C}} = \big\{(z_1, \dots, z_{m+1}) \bigm| |z_1|^2 + \ldots + |z_{m+1}|^2 = 1\big\}$$

in the complex space $\mathbb{C}^{m+1}$. To define an action of the group $\mathbb{Z}_k$ on the sphere $S^m_{\mathbb{C}}$, we specify it on the generator $1 \in \mathbb{Z}_k$ by the equality

$$1 \cdot (z_1, \dots, z_{m+1}) = \big(z_1 e^{2\pi i/k}, z_2 e^{2\pi i k_1/k}, \dots, z_{m+1} e^{2\pi i k_m/k}\big). \tag{10}$$

Since k_i, $i = \overline{1,m}$, and k are coprime, the group $\mathbb{Z}_k$ acts freely on the sphere $S^m_{\mathbb{C}}$; hence, the smooth quotient manifold $L(k, k_1, \dots, k_m) := S^m_{\mathbb{C}}/\mathbb{Z}_k$ is well defined, which is called a *generalized lens space*. For $m = 1$, the closed 3-manifold $L(k, k_1)$ is a lens space. The fundamental group $\pi_1(L(k, k_1, \dots, k_m), b_0)$ is isomorphic to $\mathbb{Z}_k$.

Consider the even-dimensional sphere S^{2q} with $q \geq 1$ as the submanifold

$$\big\{((z_1, \dots, z_q), x) \in \mathbb{C}^q \times \mathbb{R}^1 \bigm| |z_1|^2 + \ldots + |z_q|^2 + |x|^2 = 1\big\}$$

in $\mathbb{C}^q \times \mathbb{R}^1$. Define a faithful representation $\rho\colon \mathbb{Z}_k \to \mathrm{Diff}(S^{2q})$ by the equality

$$\rho(1)((z_1, \dots, z_q), x) = \big((z_1 e^{2\pi i/k}, \dots, z_q e^{2\pi i/k}), x\big). \tag{11}$$

The group $\Psi = \rho(\mathbb{Z}_k)$ has two fixed points $N = (0, \dots, 0, 1)$ and $S = (0, \dots, 0, -1)$ on the sphere S^{2q}.

Set $B = L(k, k_1, \dots, k_m)$ and $T = S^{2q}$. Let ρ be defined by (11). This defines a foliation $(M, \mathcal{F}) := \mathcal{S}us(T, B, \rho)$ of codimension $2q$ on a closed manifold M of dimension $2m + 2q + 1$. Two leaves L_N and L_S of this foliation that correspond to the fixed points N and S, respectively, are diffeomorphic to the generalized lens space $L(k, k_1, \dots, k_m)$; the other leaves are simply connected and diffeomorphic to the sphere S^{2m+1}. According to Corollary 4, $(M, \mathcal{F})$ is a structurally stable foliation on the compact manifold M and all its leaves are compact and stable in the sense of Ehresmann and Reeb.

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