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September 15–18, 2014

EXTENDED ABSTRACTS

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Karelian Research Centre
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Petrozavodsk 2014

Karelian Research Centre of the Russian Academy of Sciences
Institute of Applied Mathematical Research KarRC RAS

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Foreword

The present volume contains full papers and extended abstracts accepted for the Third Russian Finnish Symposium on Discrete Mathematics held in the Institute of Applied Mathematical Research KarRC RAS, Petrozavodsk, Russia, September 15-18, 2014.

Topics of the symposium include:

- Combinatorics on Words
- Graph Theory
- Automata Theory
- Tilings
- Decidability Problems
- Random Graphs
- Networking Games

and related areas.

Acknowledgements

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On behalf of the Organization Committee

Yuri Matiyasevich
Juhani Karhumäki
Vladimir Mazalov

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On a Scheme of Allocation of Distinguishable Particles into Indistinguishable Cells

Andrei V. Kolchin, Valentin F. Kolchin, and Natalia Yu. Enatskaya

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Abstract

In the framework of a scheme of allocating distinguishable particles into indistinguishable cells, upon defining an easy-to-use representation form of an outcome, we explicitly enumerate the outcomes; solve the direct and reverse problems of enumeration of the outcomes; find the probability distribution for the outcomes; give a recurrence relation for the number of outcomes of the scheme and their probabilities under the condition that the number of nonempty cells is fixed; derive an explicit formula for the total number of the outcomes of the scheme; analyse the distribution of the statistics of empty cells; present a numerical method to find their number; and suggest various methods to simulate the outcomes of the scheme which allow us to carry out an approximate calculation of the number of outcomes of the scheme.

We discuss combinatorial and probabilistic problems in the framework of the scheme of allocating distinguishable particles into indistinguishable cells and similar schemes with certain constraints imposed on the distribution of particles.

By the combinatorial problems are primarily meant those where one has to find the number of outcomes in the basic scheme under consideration and schemes with constraints, as well as to give a visual representation of all outcomes in an effort to simplify the analysis.

By the probabilistic problems we mean to find probability distributions of outcomes or groups of outcomes.

The combinatorial problems are solved in three forms: to find exact analytical formulas for the characteristics of the scheme we are interested in; to construct numerical (algorithmic) methods to evaluate them under given parameters of the scheme; to suggest approximate methods to solve combinatorial problems by means of stochastic simulation.

The basis of the attempts of exact (analytical and numerical alike) calculation of the number of outcomes of a scheme is the visual representation (enumeration) of the outcomes; we suggest two convenient forms to represent the outcomes of the scheme keeping in mind that they are determined by the contents of the cells only with no account for their arrangement because the cells are indistinguishable.

In order to enumerate the outcomes of the scheme we construct a random process of successive one-by-one equiprobable allocation of r particles to n cells in the context of the scheme under consideration. The step of the process consists of allocating the next particle. Upon introducing a certain numeration of the outcomes at each allocation step, at the last rth step we enumerate all outcomes of the scheme. We thus establish a one-to-one correspondence between the outcomes and the labels they get assigned under our numeration; so we obtain an explicit formula for the number of outcomes of the scheme and are able to implement their fast simulation. In order to analyse this random process we draw its state transition graph.

The problem to find the probability distribution of the outcomes of the scheme is numerically solved for any fixed values of the scheme parameters by enumerating all outcomes with the use of the above graph.

We suggest three ways to simulate the outcomes of the scheme:

1. we discard the excessive outcomes while simulating as in [1] the more general scheme of allocation of distinguishable particles into distinguishable cells (with repetitions allowed);
2. with the help of the labelling method described in [1], knowing the probability distribution of the outcomes we randomly draw the outcome label and use the correspondence between the outcomes and their labels;

directly simulate the outcomes by drawing at random the minimum label assigned to the elements in a cell.

Step-by-step algorithms can be given for each of these ways.

The procedure of approximate calculation of the number of outcomes of the scheme is based on rejecting the excessive outcomes in the more general scheme with S_1 outcomes for which a fast and efficient simulation algorithm is known. As a more general scheme we suggest the scheme of allocation of r distinguishable particles to n distinguishable cells with repetitions allowed, where any cell can hold all particles. It is known that the number of outcomes of this scheme is $S_1 = n^r$, while a quite good algorithm to simulate its outcomes is given in [1].

References

1. [illegible]kaya N. Yu. and Khakimullin E. R., *Stochastic Simulation.* Moscow Institute of Electronics and Mathematics, 2012.

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