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1 Reflectionless Acoustic Gravity Waves in the Earth's Atmosphere

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Abstract—The vertical wave propagation in an inhomogeneous compressible atmosphere is studied in the framework of a linear theory. Under specific conditions imposed on atmospheric parameters, solutions can be found in the form of propagating waves with variable amplitudes and wave numbers that do not reflect in the atmosphere in spite of its strong inhomogeneity. Model representations for the sound speed have been found, for which waves can propagate in the atmosphere without reflection. A wave energy flux retains these
1 reflectionless profiles, which confirms that energy can be transferred to high altitudes. The number of these model representations is fairly large, which makes it possible to approximate real vertical distributions of the
3 1 sound speed in the Earth's atmosphere using piecewise reflectionless profiles. The Earth's standard atmo-
1 sphere is shown to be well approximated by four reflectionless profiles with weak jumps in the sound speed gradient. It has been established that the Earth's standard atmosphere is almost completely transparent for the considered vertical acoustic waves in a wide range of frequencies, which is confirmed by observational data and conclusions derived using numerical solutions of original equations.

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1. INTRODUCTION

The existence of acoustic gravity waves in the Earth's atmosphere can be currently considered proven (Golitsyn, 2004; Grigor'ev, 1999; Fritts and Alexander, 2003; Gokhberg and Shalimov, 2008; Durran, 1999). It is important to study acoustic gravity waves, because the energy fluxes transferred by these waves from the lower to the upper atmospheric layers are comparable with those incoming from the solar radiation (Gokhberg and Shalimov, 2008), producing a substantial impact on the energy and dynamic balance in the atmosphere and, as a consequence, on the weather conditions. Acoustic gravity waves affect radiowave propagation in a wide frequency range (Gershman et al., 1984; Huang and Sofko, 1998).

The Earth's atmosphere is strongly inhomogeneous and nonisothermal; it is well-known that waves are as a rule reflected in an inhomogeneous medium (Brekhovskikh, 1973). It is not always apparent from the results of numerical studies which atmospheric layers stimulate wave energy propagation to high altitudes and which layers reflect waves. Thus, for example, the simplest model of the isothermal atmosphere, in which density changes according to an exponential
1 law, allows for the propagation of reflectionless waves, although their amplitudes vary with altitude (Eckart, 2004; Gossard and Hooke, 1978). Acoustic waves can propagate vertically, while gravitational waves only propagate at some angle with respect to a vertical line. This case is considered to be the only example of
1 reflectionless propagation of acoustic gravity waves; therefore, it is commonly believed that acoustic gravity waves cannot propagate to high altitudes in a nonisothermal atmosphere (in particular, with monotonous temperature changes). This conclusion is confirmed by the results of some analytical and numerical calculations (Petrukhin, 1983a,b; 1988; Malins and Erdelye, 2007). This factor undoubtedly affects the energy balance of a medium. In our opinion, it is important to search for conditions under which the wave reflection is minimal or disappears completely.

Meanwhile, it has been recently shown that waves can propagate over large distances without reflection in a strongly inhomogeneous medium with specific inhomogeneity profiles, which is illustrated by examples from the dynamics of long superficial and internal waves in a shallow-water ocean with a variable depth (Didenkulova et al., 2008; Pelinovsky and Didenkulova, 2009; Didenkulova et al., 2009; Didenkulova and Pelinovsky, 2009). The propagation of internal gravity waves deep into the ocean water stratified over density and the flow is considered in (Talipova et al., 2009; Grimshaw et al., 2010; Pelinovsky and Talipova, 2010). It is shown that real profiles of ocean water stratification ensure the weak reflection of waves in the thickness of the ocean (except for a thermocline), which explains the results of numerical simulations and observational data. In this paper, these ideas are used for studying the vertical propagation of acoustic waves in the Earth's strongly inhomogeneous atmosphere. First, we will find all sound speed profiles that ensure the reflectionless propagation of acoustic waves 1
in a compressible atmosphere in the gravitational field. Then, we will consider the application of the developed theory to waves propagating vertically upward in

the so-called Earth's standard atmosphere. It will be shown that a real sound speed profile in the Earth's
1 atmosphere can be approximated using four reflectionless layers with weak jumps in sound speed gradients. This will make it possible to explain the good propagation of acoustic waves into the atmosphere, which is evidenced by observations and numerical simulations.

2. EQUATIONS OF ACOUSTIC GRAVITY WAVES IN AN INHOMOGENEOUS COMPRESSIBLE ATMOSPHERE

We use a system of gas dynamic equations for adiabatic disturbances propagating along a vertical line for analyzing the acoustic wave propagation conditions in a planely stratified atmosphere in a constant gravitational field (Lighthill, 1981):

$$\rho\left(\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial z}\right)+\frac{\partial p}{\partial z}+\rho g=0, \tag{1}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial z}(\rho V)=0, \tag{2}$$

$$\frac{dp}{dt}-c^2\frac{d\rho}{dt}=0, \tag{3}$$

where p is pressure; ρ is the gas density; V is the vertical velocity of particles; g is the gravitational acceleration; and $c=(\gamma p/\rho)^{1/2}$ is the adiabatic speed of sound (γ is the adiabatic constant). The z axis is directed vertically upward. We separate the basic state of the inhomogeneous atmosphere and the wave components

$$\rho(z,t)=\rho_0(z)+\rho'(z,t),\quad p(z,t)=p_0(z)+p'(z,t)$$

and consider that the wave disturbances are small. This makes it possible to rewrite the linearized system of equations (1)–(3) in the form

$$\rho_0(z)\frac{\partial V}{\partial t}+\frac{\partial p'}{\partial z}+g\rho'=0, \tag{4}$$

$$\frac{\partial \rho'}{\partial t}+V\frac{d\rho_0}{dz}+\rho_0(z)\frac{\partial V}{\partial z}=0, \tag{5}$$

$$\frac{\partial p'}{\partial t}-g\rho_0(z)V-c^2(z)\left[\frac{\partial \rho'}{\partial t}+V\frac{d\rho_0}{dz}\right]=0. \tag{6}$$

Here, $c(z)=(\gamma p_0/\rho_0)^{1/2}$ is the unperturbed speed of sound. The equilibrium values of atmospheric pressure and density are determined by vertical temperature distribution $T_0(z)$ in the atmosphere:

$$p_0(z)=p(0)\exp\left[-\int_0^z\frac{dz'}{H(z')}\right];$$
$$\rho_0(z)=\rho(0)\frac{T(0)}{T(z)}\exp\left[-\int_0^z\frac{dz'}{H(z')}\right], \tag{7}$$

where $p(0)$, $\rho(0)$, and $T(0)$ are the pressure, density, and temperature at some fixed level ($z=0$), correspondingly, and $H(z)=c^2(z)/\gamma g$ is the height of the equivalent homogeneous atmosphere at horizon z.

The system of linear equations (4)–(6) is easily reduced to a wave equation for the function $\chi(z,t)=dV/dz$ which is the one-dimensional velocity divergence (Lamb,1993)

$$\frac{\partial^2\chi}{\partial t^2}=c^2(z)\frac{\partial^2\chi}{\partial z^2}+\left[\frac{dc^2(z)}{dz}-\gamma g\right]\frac{\partial\chi}{\partial z}. \tag{8}$$

The coefficients of this equation are determined by the vertical distribution of the sound speed ($c(z)$).

As will be shown below, such sound speed profiles exist, for which solutions of Eq. (8) describe progressive waves with variable amplitudes and phases without reflection in the thickness of the atmosphere.

3. VERTICAL SOUND SPEED DISTRIBUTIONS ADMITTING THE REFLECTIONLESS WAVE 1 PROPAGATION IN THE ATMOSPHERE

Generally, a solution to linear equation (8) describes the transformation of an incident wave into a reflected wave at inhomogeneities of a medium and does not consist of two independent solutions corresponding to progressive waves that propagate in opposite directions. Equations with constant coefficients yield trivial solutions, implying the existence of progressive waves; therefore, we will try to find transforms to reduce Eq. (8) to an equation with constant coefficients.

We will search for a solution to Eq. (8) in a form similar to the wave field representation in the Wentzel-Kramers-Brillouin (WKB) approximation

$$\chi(z,t)=A(z)\Phi(\tau,t),\quad \tau=\tau(z), \tag{9}$$

where all the functions should be determined. Substituting (9) into (8) yields the Klein–Gordon equation with variable coefficients

$$A(z)\left[\frac{\partial^2\Phi}{\partial t^2}-c^2(z)\left(\frac{d\tau}{dz}\right)^2\frac{\partial^2\Phi}{\partial\tau^2}\right]$$
$$-\left[\frac{d}{dz}\left(c^2A\frac{d\tau}{dz}\right)+\left(c^2\frac{dA}{dz}-\gamma gA\right)\frac{d\tau}{dz}\right]\frac{\partial\Phi}{\partial\tau}$$
$$-\frac{d}{dz}\left(c^2\frac{dA}{dz}-\gamma gA\right)\Phi=0.$$

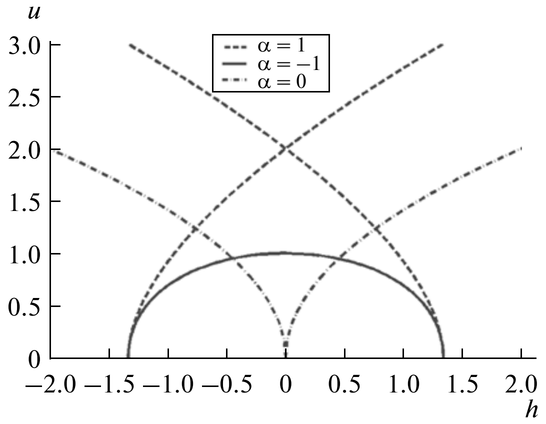
This equation can be transformed into the Klein–Gordon equation with constant coefficients

$$\frac{\partial^2\Phi}{\partial t^2}-\frac{\partial^2\Phi}{\partial\tau^2}=P\Phi, \tag{10}$$

provided that the following conditions are imposed:

$$\tau(z)=\int\frac{dz}{c(z)},\quad A(z)\sim\frac{1}{\sqrt{c(z)}}\exp\left[\int\frac{dz}{2H(z)}\right], \tag{11}$$

$$P=\frac{1}{A}\frac{d}{dz}\left[c^2(z)\frac{dA}{dz}-\gamma gA(z)\right]. \tag{12}$$



1 **Fig. 1.** Reflectionless sound speed profiles ($\beta = 0$).

The physical meaning of the $\tau(z)$ function is obvious: this is the wave propagation time in an inhomogeneous atmosphere. Note that the wave amplitude is determined by the same relationship as in the WKB approach for a smoothly varying medium, although we have an arbitrary inhomogeneity. This is additional evidence for justifying the obtained solutions in the
1 form of reflectionless waves. Thus, the initial equation (8) with variable coefficients is reduced to Eq. (10) with constant coefficients in the course of transformations. The latter has solutions in the form of progressive waves that will be considered in the next section. Now, we analyze the solutions to Eq. (12); with allowance for (11), this equation is a target second-order
1 ordinary differential equation for determining reflectionless velocity profiles. In a dimensionless form, it can be rewritten as

$$\frac{d^2u^2}{dh^2} - \frac{1}{4u^2}\left(\frac{du^2}{dh}\right)^2 + \frac{1}{u^2} = \beta, \tag{13}$$

$$u(x) = c(x)/c_0, \quad h = z/H_0, \quad H_0 = \gamma g/c_0^2,$$
$$\beta = -P/\omega_0^2, \quad \omega_0 = \gamma g/2c_0.$$

Here, c_0 is the value of the sound speed at some height $z = 0$; H_0 is the height of the homogeneous atmosphere for this height; and ω_0 is the cut-off frequency for acoustic waves corresponding to an isothermal atmosphere, in which the speed of sound is c_0. Equation (13)
2 is reduced to the quadratures

$$h = \pm\int\frac{udu}{\sqrt{\beta u^2 + \alpha u + 1}}, \tag{14}$$

where α and β are two arbitrary constants that can vary in wide ranges of values and can change their signs. The integral in (14) is calculated analytically for any values of these coefficients.

First of all, let us consider solutions to (14) for $\beta = 0$, when $P = 0$ and Eq. (10) is reduced to a wave equation. These solutions will be referred to as "nondispersion" profiles. There exist two profiles: for $\alpha = 0$,

$$u = \sqrt{2|h|}, \tag{15}$$

and for $\alpha \neq 0$

$$h = \pm\frac{2}{3\alpha^2}\sqrt{1+\alpha u}(\alpha u - 2). \tag{16}$$

The reflectionless profiles described by (15) and (16) 1
for $\beta = 0$ are shown in Fig. 1. Therefore, there are only two different profiles for the speed of sound when $P = 0$: they begin at zero and approach finite or infinite values with increasing height.

For a nonzero P value, there are three different forms of solutions to Eq. (14), depending on the sign of β:

(1) For $\alpha = 0$,

$$h^2 - \beta u^2 = 1, \tag{17}$$

and the $u(h)$ function is a hyperbola or an ellipse, depending on the sign of β.

(2) For $\beta > 0$, the reflectionless sound speed pro- 1
files are described by the relationship

$$\pm h = \frac{1}{\beta}\sqrt{\beta u^2 + \alpha u + 1} - \frac{\alpha}{2\beta^{3/2}}\ln\left[2\sqrt{\beta\left(\beta u^2 + \alpha u + 1\right)} + 2\beta u + \alpha\right], \tag{18}$$

(3) For $\beta < 0$,

$$\pm h = -\frac{1}{|\beta|}\sqrt{-|\beta|u^2 + \alpha u + 1} + \frac{\alpha}{2|\beta|^{3/2}}\arcsin\left[\frac{-2|\beta|u + \alpha}{\sqrt{\alpha^2 + 4|\beta|}}\right]. \tag{19}$$

One such profile is shown in Fig. 2.

4. WAVE FIELD IN A REFLECTIONLESS ATMOSPHERE 1

As was already stated, the Klein–Gordon equation (10) coincides with the wave equation when $P = 0$. Its solutions describing progressive waves are

$$\chi(t, z) = GA(z)\Phi\left[t - \int\frac{dz}{c(z)}\right],$$

where $\Phi(t)$ describes the wave field at an emitter and G is an arbitrary constant. For $P \neq 0$, the solution to Eq. (10) is searched for the monochromatic wave

$$\chi(t, z) = GA(z)\exp\left[i\left(\omega t - K\int\frac{dz}{c(z)}\right)\right] \tag{20}$$

with the dispersion relationship

$$K = \pm\sqrt{\omega^2 + P}.$$

If $P > 0$, waves of all frequencies propagate upward in the atmosphere; if $P < 0$, only high-frequency waves with frequencies

$$\omega > \sqrt{\beta}\omega_0 = \frac{\sqrt{\beta}\gamma g}{2c_0}.$$

propagate in the atmosphere. The cut-off frequency for waves in a reflectionless atmosphere can be both higher and lower than the cut-off frequency for acoustic gravity waves in an equivalent isothermal atmosphere.

All components of the wave field can be found using relationship (20). Thus, particle velocity V and the wave part of pressure are (Lamb, 1993)

$$V = -\frac{1}{\omega^2}\left[c(z)^2\frac{\partial\chi}{\partial z} + \gamma g\chi\right],$$
$$p' = \frac{i\rho_0}{\omega^3}\left[gc^2(z)\frac{\partial\chi}{\partial z} + \left(\gamma g^2 - \omega^2 c^2\right)\chi\right]. \quad (21)$$

It is also easy to calculate the energy flux density along a vertical line (Lighthill, 1981):

$$\Pi = \frac{1}{2}[p'V^* + Vp'^*],$$

where (*) means complex conjugation. By substituting (21) into the last equation, with allowance for (7), we obtain

$$\Pi = -\frac{\gamma|G|^2 p(0)}{2\omega^3}, \quad (22)$$

which suggests that the energy flux does not depend on z and remains unchanged in spite of the strong atmospheric inhomogeneity. As a result, a monochromatic wave can propagate to high altitudes without energy losses. This conclusion is valid for waves in all of the considered reflectionless profiles, regardless of the value or the sign of the P parameter.

5. REFLECTIONLESS WAVE PROPAGATION THROUGH THE EARTH'S ATMOSPHERE

Generally, the study of the propagation of wave disturbances in the Earth's atmosphere is a fairly complicated problem, since parameters change with height. The wave equation for nonisothermal atmosphere models with close-to-real temperature distributions cannot be generally solved analytically. An exception is the study of short acoustic waves, for which the WKB approximation is valid (Gossard and Hooke, 1978). An analytic solution for internal gravity waves in a noncompressible layer with a monotonously changing temperature was obtained in (Savina, 1996), which was essentially a solution for one reflectionless profile for an internal wave in a noncompressible inhomogeneous liquid. In most cases, the propagation of wave disturbances in the Earth's atmosphere has been studied numerically. The Earth's atmosphere has been

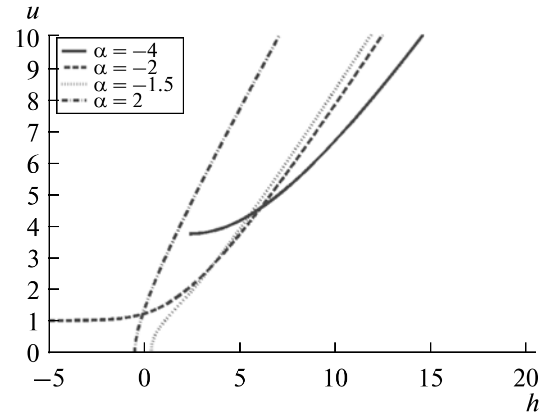


Fig. 2. Reflectionless sound speed profiles ($\alpha \neq 0$, $\beta = 1$).

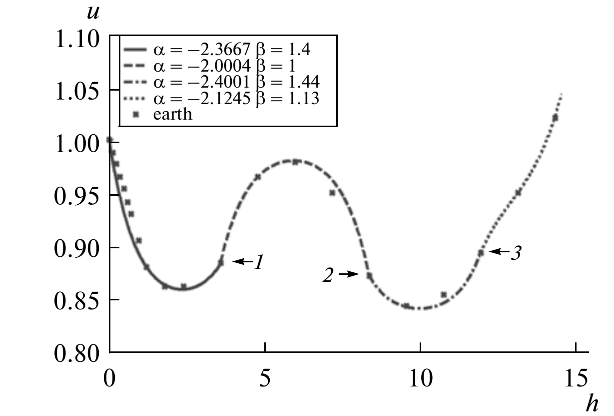
represented by a set of plane-parallel layers, in each of which the atmosphere has been considered to be isothermal (Yeh and Liu, 1975; Fransis, 1974). Calculations of the acoustic gravity wave propagation in the Earth's atmosphere are currently carried out using direct numerical integration of original hydrodynamic equations (Akhmedov and Kunitsyn, 2003; 2004). The results of all of these papers indicate that acoustic gravity waves pass through the Earth's atmosphere relatively freely. However, these papers do not answer the question why a weak wave reflection can occur in the atmosphere with essentially inhomogeneous parameters and substantial temperature gradients.

In our opinion, the answer is associated with the possibility of approximating the real distributions of atmospheric parameters by reflectionless profiles. The sound speed distribution in the so-called Earth's standard atmosphere (GOST, 1981) is illustrated in Fig. 3. Here, the height is normalized to the height of the isothermal atmosphere ($H_0 = 8.4$ km), and the speed of sound is normalized to $c(0) = 330$ m/s (both parameters correspond to the Earth's surface). The observed sound speed distribution in the Earth's atmosphere is very well approximated by four reflectionless profiles (18) with different values of parameters α and β (Fig. 3). The jumps in the sound speed gradient are visible at join points *1* and *2*, while the jump in the second derivative occurs at point *3*. Low jumps in sound speed gradients at the boundaries of reflectionless layers imply little reflections of wave energy, indicating the effective wave propagation to the upper atmosphere.

Let us consider the wave transformation at the join boundary between reflectionless profiles in more detail. The boundary conditions express the continuity of the vertical velocity of gas motion and pressure

$$[V_z]|_-^+ = 0, \quad [p]|_-^+ = 0, \quad (23)$$

where the square brackets [] mean the difference between the values on both sides of the jump. By using



1 **Fig. 3.** Approximation of a sound profile in the Earth's standard atmosphere by four reflectionless profiles.

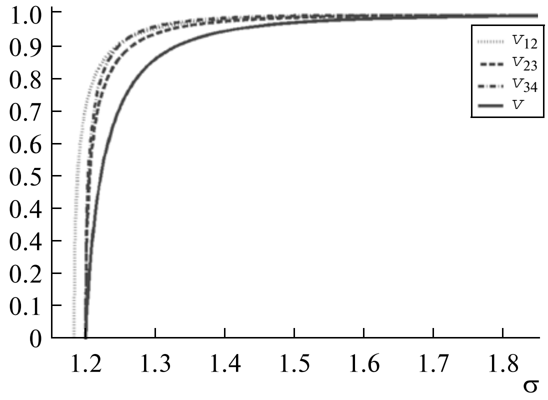


Fig. 4. Coefficients of wave propagation through the Earth's atmosphere.

(21), conditions (23) can be reduced to the following conditions:

$$[\chi]_{-}^{+} = 0, \quad \left[\frac{\partial \chi}{\partial z}\right]_{-}^{+} = 0. \qquad (24)$$

Since the $u(h)$ sound speed remains continuous at joining points, while its derivative du/dh is discontinuous (as well as parameters α and β), the complex coefficients of reflection R and propagation T for a monochromatic wave passing through the boundary between two layers j and s can be obtained from (24) (wave variable χ is considered):

$$R_{js} = \frac{2i(l_j - l_s) - (u'_j - u'_s)}{(u'_j - u'_s) + 2i(l_j + l_s)},$$

$$T_{js} = \frac{4il_j}{(u'_j - u'_s) + 2i(l_j + l_s)}.$$

Here,

$$l = \sqrt{\sigma^2 - \beta}, \quad l = K/\omega_0, \quad \sigma = \omega/\omega_0, \quad u' = du/dh.$$

Let us estimate the coefficient of wave propagation through the Earth's atmosphere ($0 < z < 130$ km) using the above model. Acoustic waves are mainly observed in the layer bounded at these heights, while the impact of electromagnetic factors can be neglected in this case (Gokhberg and Shalimov, 2008). As was already mentioned, energy losses can only occur at points *1*, *2*, and
1 *3* (Fig. 3) and not at the reflectionless layers themselves. The energy parameter of wave propagation through the js boundary will be determined by the coefficient

$$V_{js} = \frac{\Pi_s}{\Pi_j} = \frac{l_s |T_{js}|^2}{l_j}, \qquad (25)$$

where Π_j is the energy flux of the wave in the j layer determined by (22).

It is necessary to multiply values (25) in order to estimate the energy flux passing through all three layer boundaries of reflectionless profiles: 1

$$V = V_{12}V_{23}V_{34}. \qquad (26)$$

Formula (26) is valid for the case when the reflection from boundaries is weak and multiple wave reflections from boundaries can be neglected. Figure 4 shows V_{js} and V as functions of dimensionless frequency σ. For all waves with $\sigma > 1.5$ or periods less than 200 s, it is not necessary to take secondary reflections into account; the atmosphere is almost transparent for these waves (Fig.4). Note that acoustic gravity waves with these periods (not only vertical but also inclined towards the horizon) have been observed in the ionosphere directly above earthquake epicenters (Gokhberg and Shalimov, 2008), as well as after spacecraft and rocket launches and powerful explosions (Karlov et al., 1980; Nagorskii, 1998; Adushkin et al., 2000). Therefore, we conclude that the Earth's atmosphere has almost reflectionless parameters in the 1
considered approximation. Note that energy fluxes in reflectionless profiles remain here unchanged for all 1
frequencies and not only for high frequencies as it generally follows from the WKB method.

6. CONCLUSIONS

The vertical propagation of waves in an inhomogeneous compressible atmosphere is studied using the linear theory of ideal hydrodynamics. We have found a family of sound speed profiles, for which the wave field can be represented by a progressive wave that is not reflected in the atmosphere despite the fact that it is strongly inhomogeneous. A flux of wave energy in these reflectionless profiles remains unchanged, 1
which confirms that energy can be transferred to high altitudes. The number of these profiles is fairly large, which makes it possible to approximate real vertical distributions of the sound speed in the Earth's atmosphere by piecewise reflectionless profiles. It is shown 3 1

that the Earth's standard atmosphere is well approxi-
1 mated by four reflectionless profiles and this model of the atmosphere is almost fully transparent for vertical acoustic waves in a wide range of frequencies.

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SPELL: 1. reflectionless, 2. quadratures, 3. piecewise