NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

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BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: HUMANITIES
WP BRP 14/HUM/2013

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In this paper we address the story of developments in general equilibrium theory (GET) in the USSR during the 1970s through the lens of a single biography. The Soviet advances in mathematical economics, only fragmentarily known in the West, give an occasion to reflect on the extension of the Walrasian paradigm to non-market societies, as well as on the ideological effects of GET and its interpretations in a Soviet context. Our contribution is focused on the development of general equilibrium theorizing in the work of Victor Meerovich Polterovich (b. 1937) who has been one of the leading figures in mathematical economics and general equilibrium theory in the USSR and post-Soviet Russia. His papers on the abstract models of exchange, dynamic general equilibrium and optimal growth theory, excess demand correspondences, monotonicity of demand functions, and disequilibrium theory were for the large part published in English and gained considerable attention within the field. We reconstruct the political and ideological basis of the general equilibrium concept and show how abstract mathematical models reflected the discursive shift from optimal centralized planning to various forms of decentralization. We argue that the Soviet work on general equilibrium was a part of the global development of mathematical economics but was not integrated into it institutionally.

JEL Classification: B16; B21, B23, B31.
Keywords: general equilibrium theory, mathematical economics, history of recent economics, Soviet economics, decentralization, planning.

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[3] This study comprises research findings from the "History of the general equilibrium theory in 1960-1990s in Western Europe and US, and of the Soviet mathematical economics" carried out within The National Research University Higher School of Economics Academic Fund Program in 2012, grant No 12-05-0021.

Introduction

In this paper we tell the fascinating, yet rarely addressed, story of developments in general equilibrium theory (GET) in the USSR during the 1970s through the lens of a single biography. The Soviet advances in mathematical economics, only fragmentarily known in the Western historiography of economics, give an occasion to reflect on the extension of the Walrasian paradigm to non-market societies, as well as on ideological effects of GET and its interpretations in a Soviet context. Our contribution is focused on the development of general equilibrium theorizing in the work of Victor Meerovich Polterovich (b. 1937) who has been one of the leading figures in mathematical economics and general equilibrium theory in the USSR and post-Soviet Russia. His papers on the abstract models of exchange, dynamic general equilibrium and optimal growth theory, excess demand correspondences, monotonicity of demand functions, and disequilibrium economics were published in both Russian and English and gained considerable attention within the field. We argue that Soviet work on general equilibrium was a part of the global development of mathematical economics but was not integrated into it institutionally.

Based on the interviews with Polterovich and other prominent Soviet mathematical economists (Danilov, Makarov and others) conducted in Moscow in 2011 and 2012, as well as on the published work, we reconstruct the factors that influenced Polterovich's intellectual biography and academic trajectory in a broader context of the late Soviet academia. Polterovich's experiences in industry, interactions with colleagues and political outlook clearly affected his theoretical views and modeling ideas.

We also analyze Polterovich's career in the Central Economics and Mathematics Institute of the USSR Academy of Sciences (CEMI), his contacts with the Western colleagues, his first publications in the West and the curious situation of competing with unknown peers without an opportunity to present one's ideas at international conferences. We emphasize the conceptual motivations behind his moving from cybernetics and algorithmic problems of planning to equilibrium theory. However, for a full understanding of the logic behind the conceptual decisions proposed in the works of Polterovich one has to take into account the complex interplay of the theoretical challenges, the influences coming from the Soviet economics (both in its orthodox-Marxist and mathematical guise), and of the political agenda that was (and still is) important for Polterovich.

The success of GET that became a core of neoclassical economics after WWII (Weintraub 1985) might be, to a certain extent, attributed to the Cold war context (Mirowski

2002). The general equilibrium existence proof in a model of a perfectly competitive market as well as the theorems of welfare economics seemed, at least in popular accounts (cf. journalistic interpretations of a Nobel prize for Debreu), to scientifically prove the advantages of the free market economy over socialist planning.

In this context, the interest of Soviet economists in GET might seem puzzling. Notwithstanding the familiar affinity between GET, market socialism, and economic planning, we should still acknowledge that the Walrasian paradigm remained essentially alien to Soviet economic discourse. GE theorizing entered the Soviet intellectual scene relatively late, and its influence on the Soviet academia in the 1970s was not comparable to the West – in fact, it attracted only a minority of theorists employed mainly at four institutions.

The influence of GET in the USSR might seem insignificant in comparison to the huge institutional and intellectual machines of Marxist political economy and centralized planning later theoretically justified within the so-called economic cybernetics. However, this phenomenon is still worth our attention not only due to its ideological implications for Soviet economic thinking, but in view of grasping the social conditions of production and circulation of the GE theorizing in general. Our study also aims at contributing to the sociology of economic knowledge by exploring 1) social conditions of theoretical work and 2) conceptual specificity of GE models in different national contexts. We assume that a theory (as any other intellectual and institutional phenomenon such as a term or a rule) cannot be "simply" extended to a different intellectual space. This extension requires a work of *interpretation* and *adaptation* to a new context defined by a specific institutional setting, political agenda, perceptions of "economic reality" and other factors internal and external to economic science. We contend that mathematical language, when applied to economics, does not possess an intrinsic "economic" meaning. This meaning has to be created by the scientists and accepted by a broader public of experts and non-specialists (for instance, public administrators or journalists). The interpretations are necessarily related to the social and political context of their production. The intellectual and institutional context of equilibrium theorizing in the Soviet Union will prove essential for the complex interplay of conceptual developments and their social conditions that influenced both the major motivations and "moves" made by economists.

Whereas the American part of the story has been told in various versions (Weintraub 1985, 1991, 2002; Ingrao and Israel 1990; Hands 1985; Mirowski 2002; Weintraub and Düppe forthcoming; and some others), the history of GET in the USSR still remains to be written. In particular, a lot of phenomena observed by the historians of economic thought in the USA and Europe were characteristic for the Soviet case, albeit with some reservations due to the political and ideological milieu.

How did GET become a part of the Soviet science during the Cold War? What were the intellectual factors and institutional conditions of the emergence of this type of theorizing? How was it interpreted in terms of the Soviet economy? What issues did it address? And, finally, what intellectual or ideological influence did it have in the context of the Soviet academia and policy?

In this paper we provide at least partial answers to these broad questions considering the case of Victor Polterovich, one of the most important, and internationally recognized Soviet mathematical economists. His case, if not really typical, is surely an exemplar one. Polterovich is a fellow of the Econometric society (1989), and a member of the Russian Academy of Sciences (2003). His papers were published in *Econometrica* and *Journal of Mathematical Economics* when the appearance of papers by Soviet economists were extremely rare. A relatively less known Russian paper appeared in the authoritative collection on GET (Walker 2000). He has been a member of the editorial boards of several leading professional journals. Similar signs of professional recognition were obtained only by a few Soviet economists whose work had, in general, quite a poor international visibility.

Entering a truly internationalized theoretical field such as GET could be indeed a good strategy to gain international reputation, but it was not successful most of the time. Speaking the same (mathematical) language did not automatically imply being recognized as a part of this larger community.[4] We argue that Soviet developments in GET were intellectually a part of a broader international field. However, Soviet theorists were not institutionally integrated into it, almost never strived to do that (unless they emigrated), and found at times their contributions ignored by foreign peers.

The paper is organized as follows. The first part gives an account of a wider politico-economic, institutional and intellectual context in which the work of Polterovich is embedded. The second part reconstructs the professional and intellectual trajectory of Polterovich which presents singular features as well as some typical characteristics of Soviet mathematical economists. The personal story thus enriches the general perspective on the development of economics in the Soviet Union and contributes to its recent history. In the third part we look more closely at the substantive issues of his ideas and show what conceptual motivations played a role in the equilibrium analysis advanced by Polterovich and why he abandoned optimization, which was a common tool for his colleagues, in favor of equilibrium models. The fourth part deals with the first papers of Polterovich that attracted attention of the international community of mathematical economists and GE theorists. We recount the story of his *Econometrica* publication (Polterovich 1983) that was anything but typical for a Soviet mathematical

[4] For instance, an important contribution by Kantorovich to the development of linear programming could not be acknowledged by a Nobel Prize in 1975 without the strong advocating effort by Koopmans (see a detailed historical account of this episode in: Bockman and Bernstein 2008).

economist at that time, the story of Mitjushin-Polterovich criterion which, unlike the growth model, was never published in English, and give tentative explanations of these unusual facts. The last part concludes by stressing the underlying pragmatic and political concerns of decentralizing economic decisions that turns out to be crucial for the development of equilibrium analysis in need of both conceptual and ideological legitimation.

Intellectual and institutional context

The idea of equilibrium was (re)introduced[5] into the Soviet economic discourse through the theory and methods of optimization based on linear programming that opened the door to Western theory and analytic tools. The theory of optimization was developed in the Soviet Union from the mid-1950s as a part of a broader "mathematical-economic movement". The latter can be considered as a by-product, or a particular field of application, of an interdisciplinary domain, which emerged at the intersection of cybernetics, management science and engineering after WWII. Like in the West, the research in this domain was quite generously supported by the public authorities conscious of its potential for the solution of problems in strategic areas ranging from the military to the national economy. It was also made possible due to the successes of Soviet mathematics (Kantorovich himself was one of its major exponents), in particular functional analysis and topology, optimization, and optimal control theory. Another major factor was an almost overt academic anti-Semitism in the mathematical department of Moscow State University.[6] The Jews who were talented mathematicians fled abroad, but also fled from pure math to more applied fields that were more welcoming. One of the fields that they could both study (and subsequently get an academic position) was "economic cybernetics", or mathematical economics.

From the late 1950s, the "mathematical-economic methods" were introduced into Soviet economic thinking and practice mainly in two forms: input-output tables (intersectoral balances) and optimization methods on the micro-level (enterprises, factories, transportation problems). In 1959, with a delay of more than twenty years, a book by Kantorovich (*The Best Use of Economic Resources*) was published introducing methods of linear programming applied to the national economy. Notably, it suggested the use of prices ("dual values", an economic interpretation of

[5] As early as in the 1920s, which may be considered as the "golden age" of the Russian economic thought, the idea of equilibrium was present in different theoretical contexts such as balances of the national economy (Nemchinov), or a Chayanov's theory of the peasant farm economy. Then in the 1930s it was banished from official economic science, together with mathematical modeling, and remained heterodoxy for at least twenty years, being substituted by the "laws" of the Stalinist political economy.

[6] The evidence of this sad practice is huge. See one of the most recent and illuminating texts by a leading mathematician: Frenkel 2012.

Lagrange multipliers) for the establishment of an optimal plan for the national economy as a whole.

Kantorovich and his followers were harshly criticized by orthodox Soviet economists ("political economists") especially for their idea of "dual values" seemingly contradicting a dogma of the Stalinist political economy. Nevertheless, the idea of optimal planning found its place in the intellectual movement for economic reform in the Soviet Union in the early sixties. The stress on *decentralization* was supported by certain influential intellectuals and by a part of the Party elite. It was especially welcome in the context of the so-called *Kosygin reform* in the 1960s that tried to introduce elements of autonomy at the level of enterprises.

Due to this favorable political agenda – but also in the hope of establishing a scientifically grounded system of optimization and control subject to the aims of the military – a whole institutional infrastructure for research and training in mathematical economics and "economic cybernetics" was created during the 1960s (Ellman 1973; Zauberman 1975).

Nevertheless, the optimization tasks were not univocally and universally interpreted in terms of the decentralization or socialist market in the context of Soviet academia. For instance, Kantorovich himself was trying to reconcile his theory of optimization with the socialist system and the labor theory of value. Most mathematical economists working on the theory of optimization were not advocates of capitalism either, but were trying to propose solutions for improvement of planning tools. By the late 1960s, the theory of optimization had three main interpretations among Soviet mathematical economists (Katsenelinboigen 1980):

1) A mathematical algorithm for the calculation of an optimal plan entirely defined by the central authority;

2) A planning system with a central planner presupposing certain autonomy of economic subjects;

3) A coordination of interests of economic agents (interpreted as regions or economic branches, or both).

Unlike in the West, optimization models were prevailing in the USSR during the sixties, while GE models were virtually unknown until the end of the decade.[7] Intellectual (optimal planning based mainly on linear programming), ideological (socialist framework) and institutional (the relative isolation from the Western academia) reasons may be partly responsible for a belated and limited reception of GET in the Soviet context. The flow of Soviet works on the

[7] Only a few works by Soviet authors on GET appeared during the 1960s. GET as a field of inquiry was, most probably, introduced to USSR in the translations of Karlin's (1959) and Gale's (1960) books on mathematical economics that appeared in 1964 and 1963 respectively. The CEMI economist Volkonsky suggested a potential applicability of the GE models to the Soviet type economy as early as 1967.

subject increased considerably in the 1970s and, especially, in the 1980s, while the interest in GET in the West was in decline.

GET was mainly elaborated in four research institutions. Significantly enough, none of them specialized in economics: the Institute of Mathematics of the Siberian branch of the Academy of Sciences (founded in 1960), the Central Institute for Economics and Mathematics (founded in 1963), the Institute of Control Sciences (founded in 1939, first works on mathematical economics appeared around 1968) and the Chief Computer Center of the Academy of Sciences (department of mathematical economics founded in 1968).[8]

The Central Institute for Economics and Mathematics (CEMI), that was to become the main institution coordinating research in mathematical economics in the USSR, was established in Moscow in 1963. Diverse approaches and uses of analytical methods were developed there under the umbrella of "optimal planning" (for a detailed account, see Katsenelinboigen 1980). A model of optimal planning for national economy (SOFE), considered as a strategic project (informally compared to the atomic one by members of the CEMI), was the principal research focus of the Institute during the sixties and the early seventies. In the 1970s and the early 1980s, the Institute became a target of criticism and subject to several reorganizations allegedly due to the lack of practical results and "ideological unreliability".

In 1969, a leading group of the CEMI researchers formulated an interpretation of optimization for the Institute's research program and submitted a note to the Central Committee of the Communist Party criticizing the existing planning procedures of *Gosplan*, the State planning organ. It proposed new principles for planning based on the economic interpretation of the Dantzig-Wolfe algorithm (hierarchical system, bloc programming), which stressed the ideas of "decentralization", "objectivity of economic processes" and "importance of economic autonomy".[9]

In the shadow of the project on optimal planning, protected by influential senior colleagues sensible to the elegance of mathematical theory and believing in its potential usefulness for economic matters, a few dozens of mathematicians and mathematicians-cum-economists, including Polterovich, found a space relatively free from ideological and pragmatic constraints. In interviews, all of them depict the CEMI as a place with a "special atmosphere" (compared to the rest of the Soviet academia) propitious to autonomous theoretical work. They claim to have chosen the topics and methodology of their studies freely, some of them having a clear preference for purely abstract research without any economic application. The fellows of the Institute had an access to a selection of English-language periodicals in mathematics and

[8] Isolated individuals working on some aspects of GET could be found in some other research and educational centers (e.g., the CEMI filial in Leningrad, MSU Lomonosov, etc.), but research groups were formed only in these four institutions.

[9] This interpretation was proposed in a book by Aron Katzenelinboigen, Juri Ovsienko, and Efim Faerman (1966).

mathematical economics, and to Russian translations of some Western works on the subject.[10] There were also limited opportunities for personal contact with Western scholars, through visits and international conferences (but not for all researchers), and non-Party members, like Polterovich, were, of course, not allowed to go abroad.

To sum up, the intellectual movement (which might be roughly defined as "optimization" ideology) supported by the political agenda allowed the emergence of a whole family of approaches in mathematical economics in the USSR (GET, social choice, input-output analysis, stochastic optimization, optimal growth theory, computable models, simulation etc.). Researchers, in a few specific institutional niches (of which the CEMI is but one important example), engaged in these various domains were taking advantage of a relative intellectual autonomy and were connected to the Western intellectual space. At the same time, their work (the problems posed and the interpretations given to the models' assumptions and implications) had a specificity of their own related to their political and practical concerns, as we will see in the example of Polterovich.

A singular professional trajectory: engineer, mathematician, and economist

Victor Polterovich presents a case of a very successful professional conversion into mathematical economics.

Like their counterparts in the West, most Soviet scholars who became mathematical economists in the 1960s and the 1970s were trained as mathematicians. Indeed, this field, and especially GET, required a level of mathematical skills which prevented most economists and other social scientists from entering. (Soviet training in economic sciences did not include much mathematics before the creation of the departments of "economic cybernetics" during the 1960s.) From the mid-sixties the field of mathematical economics attracted a number of young brilliant scholars who had graduated mainly from mathematical or engineering departments.

Mathematicians employed at the CEMI tried to grow accustomed, with more or less success, to economic problems, but often remained engaged in "pure" mathematics (and publishing in mathematical journals) or went back to their original disciplinary commitment. They rarely present themselves and think of themselves as economists, but rather as purely

[10] To cite a few: Karlin, 1959 (Soviet translation: 1964); Morishima, 1964 (1972); Nikaido, 1969 (1972); and other books which gave a good overview of the Western achievements in the field.

theoretical or applied mathematicians.[11] Polterovich is one of a few Soviet mathematicians-cum-economists of his generation who became effectively engaged and recognized, both nationally and internationally, in this new field.[12]

His successful conversion into mathematical economics might be the result of a rare and fortunate combination of competences as a mathematician, an engineer, and a social scientist. Indeed, each of these competences was necessary to achieve excellence in this field, to transform economic questions into mathematical problems, and to propose economic interpretations of mathematical results.

Victor Polterovich was born in Moscow (in 1937), to a family of engineers, and obtained a diploma in engineering from the Moscow Oil and Gas Institute in 1962. This beginning suggested a fully predictable career. In his young years he had an “acute interest in all sorts of humanities, ranging from politics to poetry”[13], that was, admittedly, an important element of the Soviet technocratic engineering culture. A passion for (writing and reading) poetry cultivated in his family cost him a year’s deviation from the standard Soviet educational and professional trajectory. As a student at the Moscow Oil and Gas Institute, he organized an informal poety circle[14] and was, because of this activity, excluded (if only for a short period) from Komsomol and expelled from the Institute. After that, the only way to continue higher education was to get an experience in an industry, in Soviet terms “to reform oneself through labor”. So Polterovich found himself as a worker at a chemical factory for more than a year. During his spare time, he was studying mathematics and philosophy.

After graduation, he was placed as an engineer at the Institute of the Automation of the Oil and Gas Industry (Moscow) where he worked from 1962 to 1966. He was engaged in a project on the automatization of the oil-refining industry. Though it was an applied task having nothing to do with “pure theory”, it gave him, together with his work at the chemical factory, an important experience in dealing with “real economic” problems and thinking about them from an engineer’s point of view. “A factory is not an empty word for me”, Polterovich told us in an interview acknowledging the importance of this experience for his work as an economic theorist.

Parallel to his work at the Institute, Polterovich took a post-graduate degree at the Department of Mechanics and Mathematics of the Moscow State University, which gave the best

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[11] Interviews with M. Levin, V. Danilov, A. Shananin, and others. This particularity is also due to the fact that mathematical economics was developed mostly outside economic departments, in institutions specialized in mathematics, applied mathematics and engineering.

[12] Two other examples of mathematical economists working in the field of GET at the CEMI include Solomon Movshovich, who graduated from a Mathematical Department and then obtained a degree in economics, and thus mastered both domains (he emigrated to the US in the 1970s); and Valery Makarov, the President of the CEMI since 1985, who, on the contrary, obtained a degree in economics and then got a second degree in physics and mathematics.

[13] Here and elsewhere we refer to the two interviews with Polterovich made by the authors 12.07.2011 and 01.08.2011.

[14] It is important to note that the end of the 1950s and the beginning of the 1960s was a period of relative political freedom (so-called “years of the Thaw”) followed by the return of a conservative political agenda.

training in mathematics then available in the Soviet Union. He attended evening classesas according to Soviet rules one could not obtain a second degree as a full-time student. There was another reason for this long and convoluted way to mathematics. Born to Jewish parents, he did not have a good chance to enter the MSU Department of mechanics and mathematics as a full-time student. So the evening classes were an opportunity used by numerous other mathematicians of Jewish origin to access this prestigious institution. These classes, known as "engineers batch", had the advantage of sparing the students the obligation to take courses in the history of Communist party and political economy of socialism concentrating instead on mathematics or physics.

In 1966, Polterovich was invited to the CEMI by Aron Katsenelinboigen, then responsible for the SOFE project and head of the Department of Complex Systems (later transformed into the Department of mathematical economics). Not a mathematician himself, Katsenelinboigen was a fervent proponent of the application of mathematical tools in economics. In the late sixties, the Department employed nearly thirty highly skilled mathematicians. Among them, there were numerous Jews who could not find a place matching their qualifications, in other research or educational institutions (they were forced to work at provincial universities or at low prestige institutions if they preferred to stay in the capital city).[15] A lot of them fled to the US, Israel or other countries in the 1970s, after emigration for Jews was legally allowed by the Soviet authorities (in 1972). Unlike many of his Jewish colleagues, Polterovich was not inclined to leave the country, which could be partly explained by his reformist *Weltanschauung*.

Between 1966 and 1970 Polterovich was working on the problem of the optimization of the Soviet economy, which was the main focus of CEMI and the Department at that time. His later work became more closely concerned with GET, which stayed at the core of his intellectual search until *grosso modo* the collapse of the USSR.

After joining the CEMI, where he remains at the time of writing, the professional trajectory of Polterovich has not been rich in shifts or dramatic events. During the Soviet era, for twenty-five years he stayed a "simple" (Senior, then Leading) Research Fellow, in the shade of more ambitious colleagues. It seems that the highest academic recognition as a mathematical economist first came to him from abroad: he became a member of the editorial board of *Journal of Mathematical Economics* (1985-2009), an associate editor of *Econometrica* (1989-1995), and was elected a Fellow of the Econometric Society (1989). In 1991 he was granted the honor of

[15] When Katsenelinboigen came to Nikolai Fedorenko, the head of CEMI telling him that he would like to engage a bright mathematician to work in his department, the boss replied: "Do you want to hire Polterovich? Then do hire two Ivanovs as well!" An "Ivanov" (a Russian "Smith") was indeed hired, but soon left the institution since it was a person patronized by some high-rank authority and in need of an academic job for a time being, without real interest in academic problems. This anecdote, recalled by Polterovich himself, could serve as a nice illustration of a mild anti-Semitism within CEMI, of a paternalistic structure in the Soviet academia, but also of the notion of political correctness typical for that time.

giving the annual Walras-Bowley lecture. The international recognition of his work in GET had thus become an important resource for domestic recognition and career advancement (although among Soviet mathematical economists his authority had been established long before that).

In 1991, after Polterovich obtained a second doctorate degree in economics (habilitation), he became the Head of the Laboratory of Mathematical Economics at the CEMI. In the following decade he continued to cumulate professorships, honorary titles, prizes, editorships and other professional rewards in Russia and abroad.

In what follows, we will take a closer look at his intellectual biography focusing on the contributions to GET. We will consider in detail some exemplary papers by Polterovich that show the directions taken by his analysis and the main theoretical concerns that influenced his thinking in the 1970s.[16]

Theoretical Concerns of a Social Engineer: from Optimization to Equilibrium Analysis

Before joining the CEMI, Polterovich specialized in functional analysis and had some experience in solving optimization problems related to the oil and gas industry. In the late 1960s, at the CEMI there were mathematicians already acquainted with the general equilibrium theory, but Polterovich did not immediately formulate the problems that interested him in these terms.

It is important to note that the path Polterovich took to general equilibrium analysis was through *cybernetics*. Following the concerns of his time, he struggled with the problem of a better control of the (Soviet) economic system. In search of a reasonable scientific foundation he tried out different conceptions popular at that time. The starting point for him was, quite curiously, “The Law of Requisite Variety” advanced by Ashby (1956) whose book was translated into Russian as soon as 1959 (for a detailed history and general contexts of the Soviet cybernetics see Gerovitch 2002). The Law states that, for a system to be stable, the number of states (variety or complexity) of its control mechanism must be greater than or equal to the number of states in the system being controlled. This principle led Polterovich to the idea that for effective control one needed more than just one planning authority having a limited complexity. This “more” was evidently the key factor that motivated him to engage in the study of economic equilibrium. Since the end of the 1960s equilibrium was for him tightly connected to the idea of *decentralization*. To solve the control problem (that in the end of the 1960s was stated mostly as

[16] We would leave Polterovich’s extensive work on disequilibrium economics done in the 1980s for another occasion since it deserves special attention.

an optimization problem) one had to coordinate decisions on the lower level of the system – due to its higher variety (to use Ashby's term) or to the lack of information that can be revealed only when the system is decentralized. These ideas were also important in the context of optimal planning algorithms (of the type of Dantzig-Wolfe decomposition).

The first significant model advanced by Polterovich (in 1970) described the abstract processes of resource allocation. It could not be considered as an equilibrium model *sensu stricto* since it did not use any notion of price system. In this model, an arbitrary allocation of resources is given and the agents start interacting with each other. At each step only a subset of agents with their own utility functions maximizes the sum of their utilities and redistributes their resources accordingly, while the endowments of the others remain unchanged. The question posed was concerned with the existence of a sequence that could guarantee the achievement of an optimal point (maximizing the sum of utilities). Pairwise exchanges, as Polterovich showed, could not always guarantee the convergence of the system to the optimum, especially when fairly general assumptions on the utility functions were made. But *coalitions* that were big enough (roughly, a relation between the number of products and the number of agents in a coalition was established) and that jointly maximized their utilities were shown to necessarily converge to an optimal state. The number of products (that was also a dimension of the space in question) thus characterized the complexity of the system along with the number of agents.

The question behind this model was that of the conditions of achieving the optimal state. Since it is known that perfect decentralization (like in the Arrow-Debreu model) always leads to an optimum, how could partial and successive local exchanges or (to use Polterovich's terminology of the 1960s) reallocations lead to the same result? Since the pairwise exchanges could not always do the job, a sort of a boundary condition of existence for the optimal sequence of states was found. Polterovich also demonstrated that the optimal size of a coalition depended both on the number of resources and on differential properties of utility functions.

The next paper (Polterovich 1973, English version: Polterovich 1973a) should be read in the context of the preceding work by Edward Eisenberg and David Gale (Eisenberg and Gale 1959; Gale 1960; Eisenberg 1961). The underlying idea for this body of work was to characterize equilibrium as a solution of a convex optimization problem. They considered linear utility functions, but Eisenberg (1961) generalized this approach to include concave homogenous utility functions as well. One then proceeded by deriving an aggregate utility function and thus representing equilibrium as a solution of one "big" optimization problem.

Polterovich saw this problem as a normative interpretation of an equilibrium idea. The coordination mechanism inherent in general equilibrium models *was not normative enough* to be understandable and meaningful (primarily for the decision-making authorities in the USSR) in comparison to the optimization models where the choice criterion is explicitly formulated. Note that Eisenberg (1961), for example, referred to the same problem as an integrability issue for aggregate demand asking the question: "Can this [total] demand be thought of as expressing the behavior of a single (fictitious) consumer acting according to a well-defined (aggregate) utility function?" (337).

The idea of reducing the whole coordination issue to the analysis of one optimization problem was, of course, very attractive and still evokes interest in some fields of operations research and algorithmic game theory (see e.g. Codenotti and Varadarajan 2007). But, as Polterovich stated in 1973, representing an equilibrium as a solution of an optimization problem involves finding the weighting coefficients for the "global" utility function to be maximized – a task that could become as difficult as, indeed, finding the equilibrium itself.

A more important message Polterovich wanted to convey in this 1973 paper concerned the importance of equilibrium as an *allocation mechanism*. In particular, he introduced four axioms of "distribution" and a rule (a set-valued mapping) that maps sets of utility functions onto subsets of feasible allocations. As Gamp (2012) notes, this rule of distribution is similar to the familiar concept of the "social choice rule" (a formal difference is that Polterovich defines the alternatives on the set of utility functions, and not preferences, as is normally done in the mechanism-design literature). Polterovich then proceeds as follows. First he defines the solution of the distribution problem as a mapping that satisfies certain axioms. They were clearly inspired by the axioms used by Nash (1950) in his bargaining solution. We will not delve into details and leave this topic for the future, the only thing to note here is that, as Polterovich himself emphasizes, the axioms describing his solution *do not use the notion of price*. Hence, we are still dealing with the abstract model of allocation (not exchange) and it is consumper goods that are distributed. But then, after showing that the solution exists, Polterovich proves that it coincides with the competitive solution in a general equilibrium model. Equilibrium thus serves as a "principle of distribution" and the purpose in the paper is to defend equilibrium analysis on the ground that it yields solutions equivalent to "reasonable" (though abstract) allocation problems. Every reasonable allocation (i.e. allocation that is derived when the axioms hold) is Pareto-efficient. He thus also derives a subset of Pareto-efficient allocations.[17]

Then Polterovich turned his interest towards the integration of equilibrium theory and the theory of optimal economic growth. The intellectual context of this work was defined by the large literature on optimal growth and turnpike theorems in the West, but also by the results obtained by his colleagues Makarov and Rubinov, summarized in their 1973 book *Mathematical*

[17] The motivation behind this issue was to get a narrower characterization of "reasonable" Pareto-efficient allocations and to show that they constitute an equilibrium in a Walrasian model.

Theory of Economic Dynamics and Equilibria. This book was an exposition of von Neumann-type theory of optimal growth, and it also contained a new and elegant mathematical formulation of the dynamic problem (Gale 1978).

This theory was interesting for the Soviet context since it provided a good dynamic interpretation of balanced planning compatible with some exogenously given criteria. The optimal growth would be described by a trajectory that would correspond to those criteria. Turnpike theory was also important since it showed the long-run planning perspective, disregarding the initial state and concentrating instead on the final one.

In Makarov & Rubinov's 1973 book that served as an important source for Polterovich there was a chapter on equilibrium theory restating the results of Arrow and Debreu and providing the game-theoretic treatment of those results. But this chapter was not explicitly linked to the dynamic problems discussed in the rest of the book. Hence, the idea of linking equilibrium and optimal growth theories was something that could come to mind of a sophisticated reader.

The main interest of Polterovich was to formulate the theory of *equilibrium growth* so that the theory of optimal growth becomes a special case. In particular, in describing equilibrium growth one would want to get asymptotic results similar to those obtained in the von Neumann–Gale theory and to see what would happen if the horizon of planning were longer.

Optimal and equilibrium growth relate to each other roughly in the same way as optimization problems and equilibrium models. Optimal growth theories have a single criterion and a planning perspective in their background, whereas equilibrium growth means that at each point of time we have to balance supply and demand and coordinate the actions of heterogeneous agents. Polterovich wanted to pose the question that had been answered for the theories of optimal growth: to what extent does the result of development depend on initial conditions? But now the question was posed in a dynamic equilibrium framework. That amounted to making equilibrium theory dynamic and rendering the asymptotic analysis more realistic, because it now dealt with various objectives. But this move was marked by the same idea that governed the whole work of Polterovich at that time: to show the connections between well-known (and politically correct!) optimization results, on the one hand, and equilibrium theory – on the other.

The first paper on this topic written in 1975 was published in Russian in 1976 and in English translation in 1977.[18] It defines an equilibrium trajectory as a sequence of prices, consumption, input and output vectors that satisfies standard balance conditions and maximizes profits over the whole path. Utilities are not analyzed explicitly, but it is assumed that the consumption vectors of the equilibrium trajectory are taken from a domain of demand correspondence $C(p)$ where p is the price vector. The stationary state is further defined as a

[18] It was later republished in an authoritative anthology edited by David Walker (2000).

quadruple (p^*, c^*, x^*, y^*)[19] that maximizes profits. It is further shown that under some specific conditions the equilibrium trajectory is close to the stationary state. The most important of those conditions is monotonicity of demand correspondence. This condition was given an elegant economic interpretation by linking monotonicity with the weak axiom of revealed preference (in 1976 Polterovich constructed a two-period vector-function of consumption for which WARP was equivalent to the monotonicity of the demand correspondence).

The next paper on the topic (Polterovich 1978) generalized these results for cases of the changing technology sets and demand mappings. It also analyzed the asymptotics of infinite trajectories. This paper turned out to be the one that became his first important publication in the West.

Breaking Through the Iron Curtain: The First Western Publications

The works by Polterovich discussed in the previous section were done in relative isolation from the international scientific community. Of course, the isolation was not complete since Polterovich had the opportunity to read international journals, to be a part of the Soviet mathematical-economic community, to publish his results and even to get them translated into English. (Since 1964, the journal *Matecon* published selected translations of the most important Soviet papers in mathematical economics originally published in Russian in order to make them available to a broader audience. However, as citation patterns show, the journal was not widely read by American and Western European economists.)

More importantly, Polterovich, like many other Soviet mathematical economists, was unable to participate in international conferences, which appear to be the most important way to disseminate one's work and to establish one's scientific reputation inside the "international community". An economist had to travel in physical space to get a better position in the social one. Participation in conferences and in other forms of academic dialogue was also important in order to stay aware of the latest developments in economics of the 1970s and 1980s, since preprints were difficult to obtain (for example, some of them were not available even in European libraries, not to mention the Soviet ones).

Without these opportunities for travelling and discussing his results Polterovich's work was virtually non-existent for the community of neoclassical economists throughout the 1970s.

[19] *x* and *y* denoting input and output vectors, respectively. In Polterovich (1983) the same object is referred to as a static equilibrium and – in a different context – as a turnpike.

The major breakthrough happened in the 1980s due to a combination of scientific (the relevance of results for mainstream theoretical discussion) as well as social (personal acquaintances, international networking) factors. In order to illustrate this point, let us consider two cases in which the models advanced by Polterovich in the 1970s became later known in the West.

Econometrica: silent acceptance

In the beginning of the 1980s mathematical economists Mohamed A. El-Hodiri and Iosif Aronovich Krass who were aware of Polterovich's work published in Russian decided to translate and submit his 1978 paper integrating GE and optimal growth theories to *Econometrica.*

El-Hodiri was of Egyptian origin and as a part of an exchange programme was a graduate student at Moscow State University in 1958-1959. There he got acquainted with the Soviet mathematics that at the time was beginning to blossom. Soon after his short stay in the USSR he went to the U.S. (following the instable course of Egypt-USSR diplomatic relations) and began his studies at the University of Minnesota. Among his advisors was one of the founding fathers of the contemporary GET Lionel McKenzie. In the beginning of the 1960s, prior to his appointment at the University of Kansas, El-Hodiri was Leonid Hurwicz's assistant at Minnesota. Hurwicz encouraged him to keep track of the Russian applied mathematics and to translate something on the topic.[20] He followed the advice and translated into English the book of Makarov and Rubinov on economic dynamics (Makarov and Rubinov 1977).

Krass was a mathematical economist who did some work in economic dynamics and turnpike theory at the Institute of Mathematics of the Siberian branch of the Academy of Sciences (see Krass 1976). McKenzie came to Novosibirsk for a conference and Krass managed to give him a note asking for help in his immigration to the US. McKenzie transferred this note to El-Hodiri who organized a visiting professorship for Krass at the University of Kansas.[21] After Krass came to the US, he wanted first to publish the correction of some minor mistakes he found in the paper of Polterovich that he knew quite well. But El-Hodiri together with McKenzie convinced Krass that it would be better to translate and publish the paper. The corrections as well as some clarifying notes that helped to reconcile Soviet and Western terminology and to legitimize Polterovich's work (as an important contribution to the current theoretical discussion,

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[20] According to his recollection, El-Hodiri also showed to Hurwicz the Pontryagin's book on optimal control that was later related by Hurwicz to Arrow and Uzawa. As we know from Pontryagin's memoirs, he was himself in contact with American mathematicians. His book on optimal control was published in the West as early as 1962.

[21] The details of Krass' emigration are related in the personal communication by El-Hodiri, as well as by the *Chronicle of Current Events* (No 51 and 52), an important semi-clandestine *samizdat* periodical. Krass managed to flee to the USA in December 1978. The laboratory of the Mathematics Institute in which he worked was dissolved (another colleague fled to Canada at the same time). Krass then went on to work for the Navy Personnel Research and Development Center in San Diego, CA.

though it did not cite a lot of work on the same topic that had appeared in the preceding years) were formulated as translators' remarks. McKenzie also helped to facilitate the revision process that did not change much in the substance of the paper and was managed by El-Hodiri.

The publication of the translated version of the Polterovich 1978 paper in *Econometrica* was accompanied by certain anomalies as compared to the regular publishing process, which highlight the singularity if this event. First, the paper appeared in No 3 of Vol. 51, but in the announcement of the accepted papers (published in No 5 of the previous volume) this text was attributed to El-Hodiri and Krass without mentioning Polterovich (see p. 1341). This mistake was perhaps caused by the tradition to put the translator on a par with the author of an original piece. Second, though a regular peer-review procedure was applied, Polterovich received only the proof of his paper accompanied by a letter in which he was asked whether he approved the publication of his work, without participating in the revision process.

What were the motivations behind this quite exceptional publication? Several considerations are in order. First, there was a high interest in what was going on in the East, notably expressed by McKenzie and Hurwicz (who was of Russian origin) and generally shared by the members of *Econometric Society*.[22] This, along with the personal interest by El-Hodiri, could explain why the work coming from the other side of the Iron Curtain was published by one of the world's most prestigious economic journals. Second, the result obtained in this paper was perceived as relevant for theoretical concerns of the Western mathematical economists. Indeed, Polterovich's "model admits a simple description and, at the same time, is rich in content and leads to an interesting theory" (Amir and Evstigneev 1999: 149). Nonetheless, this publication could well have not taken place without the personal involvement and efforts of several individuals brought together by an unforeseen, but happy contingency. Only afterwards did Polterovich realize that several other authors were moving in the same direction (Bewley 1982). He thus competed for a long time with (partly) unknown peers, which made the whole issue even more unusual.

Mitjushin–Polterovich criterion: common knowledge unpublished

At the same time Polterovich began collaboration with mathematicians Leonid Mitjushin and Vladimir Spivak. We will touch upon one of the results because it is widely recognized as a distinct theoretical contribution, but – unlike the previous paper – was never published in English.

[22] One of the signs of this interest was a Seminar in Moscow in summer 1976 where Soviet and American theorists (among them Robert Dorfman, Leonid Hurwicz, Franco Modigliani, Lawrence Klein, and Dale Jorgenson) shared their expertise and simply looked (with a lot of curiosity) at each other. See the proceedings: Shulman (1976).

A Russian paper (Mitjushin and Polterovich 1978) developed the now well-known and "intriguing" (MasCollel, Winston, and Green 1995: 112) technical theorem in mathematical economics and microeconomics.[23] It states the condition of monotonicity for an individual demand function. This condition is important since it shows the properties necessary for a utility function to generate a monotone demand function that is, in turn, essential for generating the aggregate demand that would satisfy the law of demand property.

Why is this topic so important? The reason is that the monotonicity of individual demand functions is preserved on the aggregate and entails the fulfillment of the weak axiom for an aggregate demand, which, in turn, suggests the uniqueness of equilibrium. Otherwise, as is well known, price changes may lead to ambiguous results since income effects may outweigh substitution effects and the resulting change in demand may not follow the general law. But for Polterovich this work had another, more specific meaning: as we saw above, one needed monotonicity to integrate turnpike theory and equilibrium theory, and this criterion may be seen as a by-product of a more ambitious theoretical program that, however, turned out to be at least no less important than the program itself.

We present the result in its simplest form following Mas-Collel (1991). Let $h_i(p, w_i)$ be a demand function of the consumer *i*, *p* being a price vector and w_i - the wealth of the agent *i*. Let the income be normalized to one, i.e. for each *p* $ph(p) = 1$. The demand function is said to be (strictly) monotone if for different prices *p* and *q* $(p-q)(h(p)-h(q)) < 0$. The result obtained by Mitjushin and Polterovich states formally that if the demand *h* is generated by a twice differentiable, monotone and concave utility function *u* then the sufficient condition for the monotonicity of *h* is:

$$\sigma(x) = -\frac{x\partial^2 u(x)x}{x\partial u(x)} < 4 \quad \text{for all } x.$$

Interestingly, under some assumptions the coefficient $\sigma(x)$ is equivalent to the coefficient of the relative risk aversion. This played a role in some further research since the inequality turned out to be quite plausible given the data.[24] The result was also generalized, see Kannai (1989), and led to further interesting research questions (Kannai, Selden, 2012).

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[23] One should note that a comparable result was for the first time obtained by Jean-Claude Milleron (1974). His work was discussed at the Malinvaud's seminar on econometrics, but remained an unpublished manuscript (Milleron 1974), although publicly available (but surely not to Mitjushin and Polterovich). The result is therefore often referred to as a Milleron–Mitjushin–Polterovich criterion.

[24] "The original MMP result could be interpreted as saying that an agent generates a monotonic demand function if he is risk averse in commodity space but not too risk averse, in the sense that his coefficient of risk aversion at each point x, in the direction x, must not exceed 4." (Quah 2003: 715)

It is significant that in spite of its importance the Mitjushin and Polterovich 1978 paper was not translated into English for a long time: Dana (1995: 269) thanks Polterovich for providing a translation of his original proof. Although the text was cited by Mas-Colell (1985) and Mas-Colell and Freixas (1987) it was Werner Hildenbrand who first attracted the attention of the Western scholars to the Mitjushin–Polterovich criterion.[25] Thus the work of an unknown mathematical economist developed in the 1970s was discussed by a broader international community only in the second half of the 1980s.

Conclusion

The development of GET in the Soviet Union has to be considered in the broader context of mathematical economics brought to life by a political agenda of the early sixties favorable to the ideas of optimization. The concept of "optimal planning", in particular, served as an engine (and sometimes as an ideological veil) for the elaboration of different approaches in mathematical economics. It also opened the way to various interpretations of the optimal functioning of the socialist economy, including those in terms of decentralization and market.

It seems like the importance of mathematical-economic concepts and tools in the Soviet context belonged less to the realm of pragmatic concerns: nothing of the sort of an optimal plan was ever adopted by the planning authority, *Gosplan*, and the ideas of mathematical economists were applied only in a very limited way. At the same time, the development of "economic-mathematical methods", and the theory of optimal planning in particular, contributed to opening the Soviet intellectual space to the latest achievements of Western science (through translations of books and access to professional periodicals), in particular neoclassical economics;[26] to elaborating a radically new language for economics, incommensurable with the political economy of socialism; and to developing the culture of mathematical modeling in economics.

Nevertheless, it has to be stressed that this intellectual transfer was by far neither a mechanic borrowing nor a simple adaptation of a theoretical model, as the highly abstract character of GET (and of mathematical economics in general) may suggest. On the contrary, it required a work of interpretation from a mathematical language to an economic one and back again, done by individuals in view of their specific intellectual and cultural background, acting in specific institutional settings and practical contexts. Thus the works by Polterovich in equilibrium and disequilibrium theory, as well as in various other research projects, were

[25] Werner Hildenbrand, personal communication.

[26] SOFE analysis elaborated at the CEMI also used a language and axiomatic of neoclassical economics (scarcity of resources, subjective theory of value, equilibrium expressed in prices etc.) See Katsenelinboigen, 1980.

motivated by the perceived necessity of reforming the economy and improving economic policy. From the beginning (block programming) until today (theory of institutional traps, see Polterovich 2008) the main direction of his research efforts has been guided by this agenda of reformist *Weltanschauung* that originates in the engineering techno-political tradition he stemmed from.

As the case of Polterovich shows, his theoretical work on GE was an answer both to the political concerns of his time and to the conceptual developments of mathematical economics in the West and in the Soviet Union. The analysis of this special case is instructive because many issues important for the development of Soviet mathematical economics and GE analysis in general are illustrated here. There were generally two kinds of motivations behind the work done by Polterovich in GET. The first was, of course, to derive new mathematical results. But there was another one: to defend equilibrium analysis within the Soviet context of optimal planning, to prove that decentralizing decisions could be quite compatible with overall strategic perspective on Soviet economy.

As mentioned above, at the CEMI, the idea of decentralization as a principle for optimal planning already had its proponents by the time Polterovich wrote the papers we discussed. In fact, there was a controversy between those who insisted on the possibility of introducing better computing techniques and methods of optimization for designing a "global" optimal plan and those who realized that one needs feedback loops and the decentralization of information. Polterovich belonged to the latter party. GET was for him a mathematical tool to tackle the logic of decentralized decisions. Equilibrium prices provided, first, the *incentives* to fulfill the optimal plan and, second, performed their normal function balancing supply and demand. But it was obviously a specific language as well that could sometimes conceal – behind the technical complications of mathematical models – the disappointment with the overarching socialist planning.[27] Equilibrium was a shibboleth for those who realized that alternative decentralizing decisions were to be implemented in order to manage the economy[28].

The description of the subsequent development in Polterovich's work exceeds the intention of the present paper. However, it is worth noting that this development was, again, in a large part influenced by his concerns with the current economic situation. In particular, by the

[27] "I know from conversations with one of the authors several years ago that there was a period in the USSR when even "equilibrium" was a dirty word" (Gale 1978: 668).

[28] Just to what extent this reinterpretation was not only a theoretical, but also an ideological issue can be illustrated by the story Polterovich related in the interview: "It was the end of the 80s [In another interview (http://polit.ru/article/2006/06/26/polterovich/) where the same story emerges, Polterovich speaks of 1976 which is much more plausible – I. B., O. K.]. I wrote a paper originally entitled 'Non-uniqueness of optimal decisions and the problem of decentralization'. […] After I got the first proof I was astonished, everything was correct but for the title. There was 'the problem of centralization'. I thought it was a typo, I corrected it and sent back. But in the second proof the title was again changed the same way. I called the proofreader and asked her: 'What is going on?' The answer was: 'Victor, you may write whatever you want in the text, but the title should not be suspicious since the titles are checked. You don't need a cavil, do you?'"

end of the 1970s Polterovich abandoned GET in the form we described above and switched to the theory of disequilibrium (that, in fact, was a subfield of GET). He realized that the Soviet practices of rationing could fit well into the framework of disequilibrium story (assuming that the prices are fixed, which was clearly the case in the planned Soviet economy). Further research in this vein led Polterovich (1993) to discuss queues and black markets as exemplary phenomena of market imperfection characteristic for the Soviet economy of the 1980s.

This transition from equilibrium to disequilibrium problems occurred also due to a change in the theoretical contexts. Having first ignored the pioneering work of Braverman (1972) that was motivated by problems in the Soviet economy Polterovich then came back to it and acknowledged its importance since, in the meantime, a lot of work on the topic had appeared in the West. It was mainly the French school of disequilibrium economics that tried to use equilibrium analysis for describing the economies of a Keynesian type with rigid prices (Benassy 1975; Drèze 1975; but also Barro and Grossman 1971). Thus Polterovich was motivated not only by real observations and pragmatic needs, but also by conceptual shifts in Western science.

Although perfectly aware of the developments in the West, as many other advanced Soviet scholars, Polterovich spent the most productive years of his career in institutional isolation from the Western science, with no involvement whatsoever in the theoretical competition and the priority issues. He was eventually well received by the neoclassical tradition due to the originality of his research but also, as we have seen, because of a happy coincidence of working on theoretical problems that were in the center of the research agenda in the mainstream of the 1960s and 1970s. A personal interest of certain US economists and contacts established via émigrés also played an important role in drilling a hole in the Iron Curtain.

These observations lead us to relativize a thesis according to which the "Détente" policy created the conditions for a transnational dialogue among economists sharing a common mathematical language of neoclassical economics (Bockman 2007). Although mathematical economists were often concerned with similar topics and problems, and some intellectual transfers (in both directions) have effectively taken place, a regular scientific communication between the East and the West was barely existent beyond rare official Soviet-Western events (like conferences on mathematical economics and applied mathematics held since the 1960s), sporadic individual visits of distinguished scholars and some personal contacts. Differences in intellectual and practical contexts accountable for a specific research agenda as well as institutional factors (the lack of incentives to publish abroad, censorship and other limitations) may, to a greater extent, explain why Soviet scholarly production, even while possessing a high theoretical relevance and meeting the Western standards, have often remained unnoticed in the larger international academic community.

Acknowledgements: We are most grateful to Victor Polterovich for an interview and subsequent comments. The help from Hildegard Dierker, Mohamed El-Hodiri, Werner Hildenbrand, and Yakar Kannai, Alexander Karpov was also invaluable. All the usual caveats apply.

References

Amir, Rabah, and Igor Evstigneev. 1999. "Stochastic version of Polterovich's model: exponential turnpike theorems for equilibrium paths." *Macroeconomic dynamics* 3(2): 149–166.

Barro, R.J., and H.I. Grossman. 1971. "A General Disequilibrium Model of Income and Employment." *American Economic Review* 61(1): 82–93.

Benassy, J.P. 1975. "Neo-Keynesian Disequilibrium Theory in a Monetary Economy." *Review of Economic Studies* 42: 503–523.

Bewley, Truman. 1982. "An integration of equilibrium theory and turnpike theory." *Journal of Mathematical Economics* 10: 233–267.

Bockman, Johanna. 2007. "The Origins of Neoliberalism Between Soviet Socialism and Western Capitalism: 'a Galaxy Without Borders'." *Theory and Society* 36(4): 343–371.

Bockman, Johanna, and Michael A Bernstein. 2008. "Scientific Community in a Divided World: Economists, Planning, and Research Priority During the Cold War." *Comparative Studies in Society and History* 50(3): 581–613.

Braverman, Emmanuil. 1972. "Model" proizvodstva s neravnovesnymi tsenami." [Production Model with Disequilibrium Prices] *Ekonomika i matematicheskie metody* [Economics and Mathematical Methods] 8: 40–64 (in Russian).

Codenotti, Bruno, and Kasturi Varadarajan. 2007. "Computation of market equilibria by convex programming." In: Noam Nisan, Tim Roughgarden, Éva Tardos, Vijay V. Vazirani (eds.) *Algorithmic Game Theory*. Cambridge etc.: Cambridge University Press.

Dana, R. 1995. "An extension of Milleron, Mitjushin and Polterovich's result." *Journal of Mathematical Economics* 24: 259–269.

Drèze, Jacques. 1975. "Existence of an Exchange Equilibrium under Price Rigidities." *International Economic Review* 16: 301-320.

Eisenberg, Edmund, and David Gale. 1959. "Consensus of Subjective Probabilities: The Pari-Mutuel Method." *Annals of Mathematical Statistics* 30: 165–168.

Eisenberg, Edmund. 1961. "Aggregation of Utility Functions." *Management Science* 7(4): 337–350.

Ellman, Michael. 1973. *Planning problems in the USSR: the contribution of mathematical economics to their solution 1960-1971*. CUP Archive.

Frenkel, Edward. 2012. "The Fifth problem: math & anti-Semitism in the Soviet Union." *The New Criterion* Vol. 31, October, p. 4 (Last accessed November 18, 2012 at: http://www.newcriterion.com/articles.cfm/The-Fifth-problem--math---anti-Semitism-in-the-Soviet-Union-7446).

Gale, David. 1960. *The Theory of Linear Economic Models*. N. Y.: McGraw-Hill. [Russian translation: Teoriya lineinykh ekonomicheskikh modelei. Moscow: Izd. Inostrannoi Literatury, 1963]

Gale, David. 1978. Review: V. L. Makarov and A. M. Rubinov, Mathematical theory of economic dynamics and equilibria *Bulletin of the American Mathematical Soc*iety 84(4): 665–671.

Gamp, Jan-Philip. 2012. Games and their Relation to Markets. Inaugural-Diss. zur Erlangung des Grades eines Doktors der Wirtschaftswissenschaften (Dr. rer. pol.) an der Fakultät für Wirtschaftswissenschaften der Universität Bielefeld.

Gerovitch, Slava. 2002. *From Newspeak to Cyberspeak: A History of Soviet Cybernetics*. MIT Press.

Hands, D. Wade. 1985. "The Structuralist View of Economic Theories: A Review Essay: The Case of General Equilibrium in Particular." *Economics and Philosophy* 1(2): 303–335.

Kannai, Yakar. 1989. "A Characterization of Monotone Individual Demand Functions." *Journal of Mathematical Economics* 18: 87–94.

Kannai, Yakar and Selden, Larry. 2012. "Violation of the Law of Demand." Columbia Business School Research Paper No. 12-9. Available at SSRN: http://ssrn.com/abstract=1952088 or http://dx.doi.org/10.2139/ssrn.1952088.

Karlin, Samuel. 1959 *Mathematical methods and theory in games, programming, and economics.* Addison-Wesley Pub. Co. [Russian translation: *Matematicheskie metody v teorii igr, programmirovanii i ekonomike.* M.: Mir, 1964.]

Katzenelinboigen, Aron, Ovsienko, Juri, and Efim Faerman. 1966. *Metodologicheskie voprosy optimal"nogo planirovamija sotsialosticheskoj ekonomiki* [Methodological problems of optimal planning of the socialist economy]. Moscow: TSEMI AN SSSR.

Katsenelinboigen, Aron. 1980. *Soviet Economic Thought and Political Power in the USSR*. Pergamon Press.

Krass, Iosif A. 1976. *Matematicheskiye Modeli Ekonomicheskoi Dinamiki* [Mathematical Models of Economic Dynamics.] Moscow: Sovetskoe Radio.

Makarov, Valeri. 1962. "Ob uslovii ravnovesiya v modeli Neimana" [On the Equilibrium Conditions of the von Neumann Model]. *Sibirskij matematicheskij zurnal* 3(3): 476–478.

Makarov, Valeri, and Aleksander Rubinov. 1973. *Matematicheskaya Teoriya Ekonomicheskoy Dinamiki i Ravnovesiya*. Moscow: Nauka.

Makarov, Valeri, and Aleksander Rubinov. 1977. *Mathematical theory of* economic dynamics *and equilibria*. New York, Heidelberg, Berlin: Springer (English translation of Makarov and Rubinov 1973).

Mas-Colell, A. 1985The Theory of General Economic Equilibrium: A Differentiable Approach, Cambridge

University Press,

Mas-Colell, A., and X. Freixas. 1987. Engel Curves Leading to the Weak Axiom in the Aggregate, *Econometrica* 55(3): 515-531.

Mas-Colell, A. 1991. "On the Uniqueness of Equilibrium Once Again." In *Equilibrium Theory and Aplications* (W. A. Barnett, B. Cornet, C. d"Aspremont, J. Gabszewicz, and A. Mas-Colell, eds.), Cambridge University Press.

Mas-Colell, A., M. Whinston and J. Green. 1995. *Microeconomic Theory*. N. Y.: Oxford University Press.

Milleron, J.C. 1974. *Unicité et stabilité de l'équilibre en économie de distribution*. Séminaire d'Econométrie Roy-Malinvaud. Unpublished seminar paper.

Mirowski, Philip. 2002. *Machine Dreams*. N.Y. etc.: Cambridge University Press.

Mitjuschin, Leonid G. and Viktor. M. Polterovich. 1978. "Kriterij monotonnosti funktsij sprosa" [Criterion for Monotonicity of Demand Functions]. *Ekonomika i matematicheskie metody* [Economics and Mathematical Methods] 14: 122-128 (in Russian).

Nash, J. F. 1950. "The bargaining problem." *Econometrica* 18:155–163.

Polterovich, Victor. 1970. "Ob odnoj modeli pereraspredelenija resursov." [A Model of Resource Redistribution] *Economics and Mathematical Methods* / *Ekonomika i matematicheskie metody* 6(4): 583-593. (In Russian).

Polterovich, Victor. 1971. "A model of Resource Redistribution." *Matecon* 7 (3): 245-262.

Polterovich, Victor. 1973. "Ekonomichskoye ravnovesie i optimum." [Economic Equilibrium and the Optimum] *Economics and Mathematical Methods / Ekonomika i matematicheskie metody* 9 (5): 838-845.

Polterovich, Victor. 1973a. "Economic Equilibrium and the Optimum." *Matekon* 5: 3-20.

Polterovich, V.M. 1978. "Equilibrium trajectories of economic growth." In B.A.Efimov (ed.), *Methods of Functional Analysis in Mathematical Economics*, pp. 56–97. Moscow: Nauka.

Polterovich, Victor. 1983. "Equilibrium trajectories of economic growth." *Econometrica* 51(3): 693-730. (English translation of Polterovich 1978).

Polterovich, Victor. 1993. "Rationing, Queues, and Black Markets." *Econometrica* 61(1): 1-28.

Polterovich, Victor. 2006. “Iskusstvo reform. Lektsija.” [The Art of Reform. A public lecture] *Polit.ru*. Retrieved August 6, 2012 (http://polit.ru/article/2006/06/26/polterovich/).

Polterovich, Victor. 2008. “Institutional Trap.” *The New Palgrave Dictionary of Economics*, Second Edition.

Quah, J. K.-H. 2003. “The law of demand and risk aversion.” *Econometrica* 71: 713–721.

Shulman, S. (ed.) 1976. *Mathematical Models in Economics:* Papers and Proceedings of US-USSR Seminar. NBER.

Volkonsky, V. A. 1967. *Model’ optimal’nogo planirovnija i vzaimosviazi ekonomicheskih pokazatelej* [The optimal planning model and interconnections of the economic indicators]. M.: Nauka.

Walker, Donald A. (ed.) 2000. *Equilibrium*, Cheltenham: Elgar.

Weintraub, E. Roy. 1985. *General Equilibrium Analysis: Studies in Appraisal*. Cambridge University Press.

Weintraub, E. Roy. 1991. *Stabilizing Dynamics: Constructing Economic Knowledge*. N.Y. etc: Cambridge University Press.

Weintraub, E. Roy. 2002. *How Economics Became a Mathematical Science*. Durham, NC: Duke University Press.

Weintraub, E. Roy and Düppe, Till (forthcoming). *Finding Equilibrium: Living Through the Transformation of Postwar Economic Theory.* Princeton: Princeton University Press.

Zauberman, Alfred. 1975. *The mathematical revolution in Soviet economics*. Oxford University Press.

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