Fuzzy Sets for Purchase Planning in Uncertain Conditions

O. Panchenko[1], Y. Kupriyanov[2], V. Taratoukhine[2], A. Zeier[1]
[1]Hasso Plattner Institute for Software Systems Engineering, Potsdam, Germany
[2]Higher School of Economics, Moscow, Russia
(panchenko@hpi.uni-potsdam.de)

Abstract - **This work is devoted to the development of models and algorithms for purchase planning problem. For accurate and flexible purchasing it is important to be able to effectively combine input data from various types of sources. This paper proposes a method based on fuzzy sets to solve the problem of effective exploitation of expert knowledge and statistics of demand to determine the optimal amount of a purchase order. A membership function is used to model demand for a product. This membership function can be built using statistics, expert knowledge, or both. After membership functions for each product are created, purchasing optimization takes place based on these functions. The proposed approach supports decision making process based on formal methods of fuzzy, uncompleted or permanently changing information. The developed models and algorithms can be used for optimum assortment determination of enterprises involved in production and distribution of various types of goods.**

Keywords – **Fuzzy sets, purchasing, demand model**

I. INTRODUCTION

Accurate demand prediction and purchase planning play an important role for reducing the "bullwhip effect" in supply chain management [1]. Numerous methods have been developed for its optimization, most of which are based on statistics [6]. However, often managers must handle under unclear conditions, where, for example, no statistics for a new product are available or demand is expected to be abruptly changed. For purchase planning for these products expert knowledge could be used. Nevertheless, for optimal profitability the purchasing should be optimized across all involved products. Therefore, there is a clear need to combine and process together expert knowledge and statistics.

A similar problem has been addressed by Cardoso and Gomide [7]. Nevertheless, their solution is focused on one product (newspapers) and aims at a more accurate demand prediction. In contrast to their approach, the problem addressed in this paper is broader and includes the selection of the optimal assortment taking into account product prices and profit.

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II. DESCRIPTION OF THE PROPOSED METHOD

The proposed method consists of two parts: the first part mathematically formalizes the demand using statistics, expert knowledge, or both; the second part optimizes the purchasing using formalized input from the first part.

A. Formalization of Demand Function Using Fuzzy Sets

This paper proposes a formal description of demand for finished goods produced by a company in the form of a fuzzy set of products to be sold. A membership function represents the degree of belonging $R(x)$ to the set of finished products, which will be sold in the next period of time, depending on their volume denoted by x (s. Fig 1). Obviously, $R(0)=1$, since 0 products belong to the set of products which will be sold in any case. Only a limited amount of products can be sold (1).

$$\lim_{x\to\infty} R(x) = 0 \tag{1}$$

Moreover, following the pattern of normal demand curve [5], the function decreases over the entire interval [0; ∞), because a smaller volume is more likely to be sold in comparison to a larger volume.

The remarkable feature of this function is the ability to be built using both expert knowledge and statistical data. Expert knowledge is often expressed in the form of a linguistic variable as a list of guessing statements [4], for example:

- "sale of 15 items is almost guaranteed"
- "sale of 25 items is very likely"
- "sale of 35 items is likely"
- "sale of 45 items is quite possible"
- "sale of 60 items is unlikely"
- "sale of 80 items is nearly improbable"

These statements naturally suit the form of the membership function and can be easily used for building it. First of all, degrees of belonging should be assigned to adjectives in the statements, then the linguistic variable can be presented as a set of points $(x, R(x))$ in a two dimensional space, where x is the number of units and $R(x)$ is the degree of belonging to the fuzzy set of products will be sold.

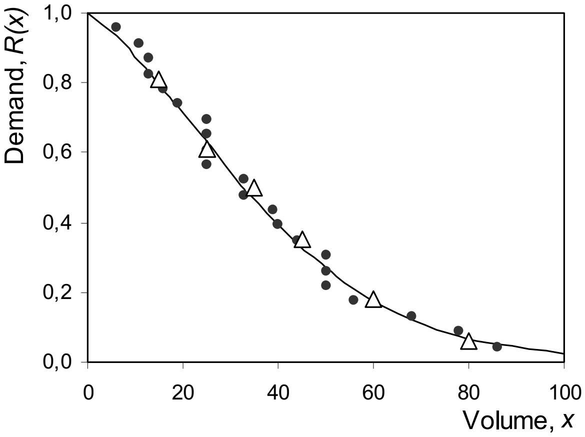

Fig. 1. Form of the membership function for one product

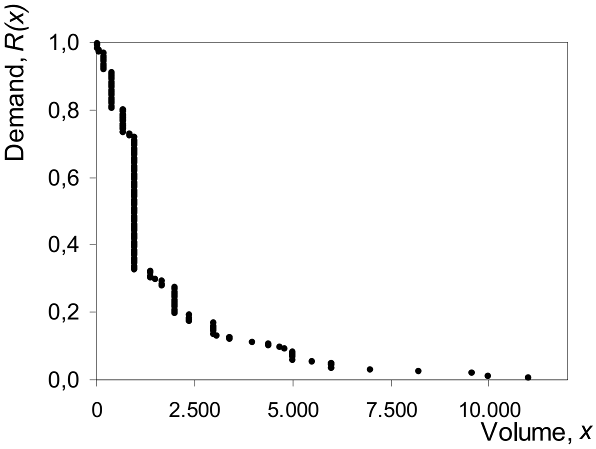

Fig. 2. Real demand example

For further optimization it is better to have an explicit formal description of the function. Numerous functions have been considered for use as a membership function [3]. For illustrating purpose in this paper a minor modification of the half-logistic cumulative distribution function [2] is suggested.

$$R(x) = 1 - \frac{1 - e^{-\alpha x^{\beta}}}{1 + e^{-\alpha x^{\beta}}} \quad (2)$$

This function has two parameters (α– scale, and β – growth rate), which can be easily calculated from the linguistic variable using the least-squares method. Nevertheless, other functions can be used as well.

The formalization of statistical data using this membership function is equally easy. The degree of belonging to the set of products to be sold is calculated from the frequency of the fact that correspondent number of product have been sold in the past. The set should be sorted by the volume. Then, the interval [0, 1] is divided into M equal parts, where M is the number of records in the statistic. Finally, the records are sequentially marked on the graph starting with the greatest *R(x)* value and with the smallest x value.

$$R(x_i) = 1 - \frac{j}{M}, (j = orderOf(x_i))(j = \overline{1, M}) \quad (3)$$

For the successful calculation of the parameters only few statistic values or linguistic variable values are required. Moreover, linguistic variables and statistics for one product can be combined for demand description. The values of the linguistic variable are depictured on Figure 1 with triangles. The resulting membership function is represented by a solid line.

Although statistics often have outliers in both directions these are naturally absorbed by the proposed form of the membership function.

Figure 2 exhibits demand for a material to be purchased in a real company. The data was collected during one year and appears to be a dataset of 170 completed purchase orders for materials used in the company's production process.

In some cases company's demand for materials has characteristics of an integer function with a predetermined purchase order volume. One of the reasons could be the agreement with the supplier to supply materials in certain portions. However, often a purchase manager has no time or analytical capacity to select the volume of the purchase order carefully and creates a purchase order intuitively, using one of the predefined purchase volumes. An example of such a behavior is presented in Figure 3. This strategy has an obvious disadvantage because the existing resources (money, inventory, etc.) are not used optimally: the excessive purchase volume may negatively affect the firm's working capital or, at worst case scenario, may turn into scrap, i.e., net losses [1, 5]. Undercharged orders result in wasted opportunity. The orders that caused losses are marked with rectangles in Figure 3.

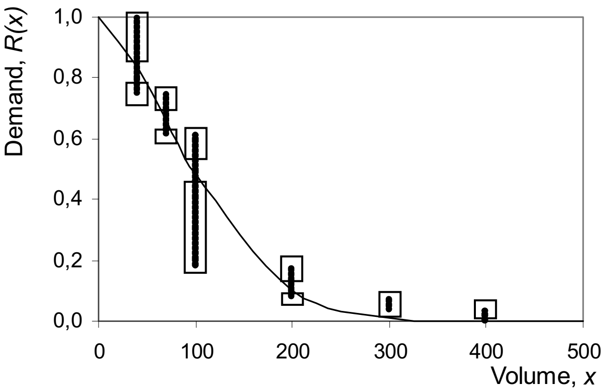

Fig. 3. Demand of integer type

B. Optimization Step

Once the membership functions for each product have been built, the optimization can begin. Given c_i is a price a company has to pay to purchase material *i* (cost of material *i*) whose contribution to the finished product sold at price p_i can be shown with a coefficient f_i (for the purpose of model simplicity), optimization criteria for product *i* or profit can be presented as follows:

$$F_i\ (x_i) = p_i f_i x_i R(x_i) - c_i x_i, (i = \overline{1, N}) \tag{4}$$

where the first summand describes gain for all units from the fuzzy set of products to be sold, the second summand is the cost of corresponding materials, and *N* is the number of products. It should be specially noted that in our model we consider a situation where exactly one unit of a material is required for production of one unit of a finished product. Finally, the optimal solution, in the matrix form to present an optimal unit quantity of all materials to be purchases given (5) is (6).

$$X = \{x_i\}, P = \{p_i\}, F = \{f_i\}, C = \{c_i\}, \left(i = \overline{1, N}\right) \tag{5}$$

$$X^* = \underset{X}{\operatorname{argmax}} \begin{cases} \left[P \times F^T\right] \times \left[X \times R(X)^T\right] - \\ C \times X^T \end{cases} \tag{6}$$

The optimization can be done using the quickest descent method. As a constraint a resource available for purchasing is introduced:

$$C < \sum_i c_i x_i \tag{7}$$

More complex optimization criteria can be used for the reproduction of the enterprise's goals, for example a goal function, which takes into account possibility that the products can be sold later with discount, etc. This can be done by introducing a coefficient *d* which represents the discount applied to the product price after a certain period of time. $d \in [0, 1]$, whereas d_i=1 means that product *i* cannot be sold after the planning period, and d_i =0 means that products *i* can be sold after a given period at the same price. Figure 4 illustrates how the value of the coefficient influences the profit function.

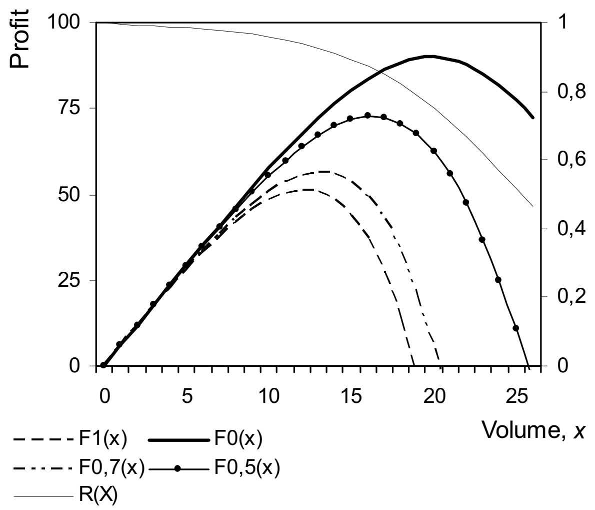

Fig. 4. Goal functions with different values of *d*

However, to model this situation we will need two membership functions for representing the degree of belonging to the fuzzy set of products to be sold at the first period (R_1) and at the second period (R_2). In that case the optimization criteria will look as follows:

$$\begin{aligned} &F_i\ (x_i) = p_i f_i x_i R_1(x_i) - \\ &c_i x_i + \\ &p_i(1 - d_i) f_i (x_i - x_i R_1(x_i)) R_2(x_i - x_i R_1(x_i)), \\ &(i = \overline{1, N}) \end{aligned} \tag{8}$$

where the first summand describes gain for all units from the fuzzy set of products to be sold at first period before discount, the second summand is the cost of corresponding materials, the third summand represents the revenue of products sold at the second period. To provide an optimal solution in the matrix form we need to introduce a variable *g*, s.t.

$$g = 1 - d \tag{9}$$

Given $G = \{g_i\}, \left(i = \overline{1, N}\right)$ and the available volume of products to be sold at the second period equals $\widetilde{X} = X - R_1(X)$ the extended optimal solution with two-period discount is:

$$X^* = \underset{X}{\arg\max} \begin{Bmatrix} \left[P \times F^T\right] \times \left[X \times R_1(X)^T\right] - \\ C \times X^T + \\ \left[P \times G^T\right] \times \left[F \times \widetilde{X}\right] \times R_2(\widetilde{X})^T \end{Bmatrix} \tag{10}$$

V. CONCLUSION AND FURTHER WORK

This paper describes basics of the developed method and provides a foundation for further approval and investigation. First of all should be proved which formal function should be used for the description of the membership function. Some categories of products have demand distribution which differs for the form presented in this paper. In these cases demand function should be adjusted. Alternatively, a table form of the function can be used instead of the explicit formal description.

Secondly, the method could be extended with an integer valued optimization. This makes sense for products which can be shipped in certain portions only.

The proposed fuzzy formulation is generally more effective than the deterministic methods or intuitive approaches for handling the situations where precise or certain information is not available for demand and purchase planning.

REFERENCES

[1] D. Simchi-Levi, P. Kaminsky, and E. Simchi-Levi (2003). *Managing the Supply Chain: The Definitive Guide for the Business Professional*. McGraw-Hill.

[2] N., Balakrishnan (1992). *Handbook of the Logistic Distribution*. Marcel Dekker, New York.

[3] S. Mitaim and B. Kosko, “The Shape of Fuzzy Sets in Adaptive Function Approximation” *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 4, pp. 637–656, August 2001.

[4] L. A. Zadeh, “The concept of a linguistic variable and its application to approximate reasoning - I”, *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.

[5] H. R. Varian (1992), *Microeconomic Analysis*. Norton & Company, Inc.

[6] G. E. P. Box , G. M. Jenkins (1994). *Time Series Analysis: Forecasting and Control*, Prentice Hall PTR, Upper Saddle River, NJ.

[7] G. Cardoso and F. Gomide, “Newspaper demand prediction and replacement model based on fuzzy clustering and rules”, *International Journal Information Science*, vol. 177, no. 21, pp. 4799–4809, November 2007.