Alexander Karpov

PATIENT MOBILITY, HEALTH CARE QUALITY, WELFARE AND REGULATION

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This paper justifies unequal health care quality in a model with two regions and patients differentiated by location and quality perception. Efficient distribution with unequal healthcare quality arises when there are low travel and/or quality provision costs. If costs are sufficiently low, then both regions win from inequality. Lump-sum transfers and price regulatory policies restore an efficient solution.

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1. Introduction

Globalisation, diminishing transport costs, and cross-border barrier reduction caused patient mobility expansion. The main incentive is the difference in the perceived quality of medical treatments. Ehrbeck et al. (2008) studied interviews with 49,980 patients who traveled abroad for medical treatment. They found that the vast majority of them sought quality and only 13% of the patients were motivated by lower-cost care for medically necessary and discretionary procedures. The main flow is from developing countries to developed countries.

The same phenomenon is observed in interregional movements. In 2009, 168,000 patients from southern Italy chose to be treated in the north and only 31,000 patients did the opposite, choosing to be hospitalised in a southern region despite their northern residence (Toth, 2014). When there are large interregional differences in health care quality, patients often spend time and money for travel to find better medical treatment.

Brekke et al. (2014) were motivated to study the effects of patient mobility on health care quality and welfare by new legislation in the European Union. The main results of this paper are based on the assumption of different costs of quality provision in two regions. Quality provision costs include costs on skilled doctors, medical facilities, and technologies, among other variables. These resources are traded on the common market with common prices. Looking at European countries and interregional studies, it is natural to assume that these costs are equal.

Under the Hotelling framework, unequal qualities in efficient distribution arise from the non-uniform density of a population (Aiura, 2013), unequal costs of quality provision (Brekke et al., 2014), and unequal production costs (Herr, 2011; Beitia, 2003). These reasons may not be the main cause of quality asymmetry and patient mobility within countries or across developed EU member states. The aim of this paper is to show that quality asymmetry arises even in countries and regions with equal productivity and uniform density.

Hospitals are faced with a highly heterogeneous set of patients. Some of these patients have mild diseases, while others have more serious diseases. The natural assumption is that the former have little concern regarding quality, and the latter fuss over the quality. Inequality in health leads to inequality in quality perception. All patients are differentiated by quality perception (marginal utility of quality), from indifferent to highly concerned about health care quality. This paper shows that a large variance in quality level is efficient in a world with low travel and/or quality provision costs. Equal quality becomes efficient when there are high costs. Market competition fails in effective solution implementation. Simple price regulation leads to an effective solution.

This paper contributes to the two strands of the literature: health care market regulation and price-quality competition. Health care market failure and price regulation have been surveyed by Dranove (2012). The regulation of the health care market with horizontal and vertical differentiation was analysed by Bardey et al. (2012), Beitia (2003), Brekke et al. (2006), Brekke et al. (2011), and Herr (2011). Models with simultaneous price and quality choice were developed by Brekke et al. (2010), Chioveanu (2012), and Dubovik and Janssen (2012). This paper, unlike other papers, models the differentiation of consumers (patients) by quality perception.

The rest of the paper is organised as follows. Section 2 presents the model. Equilibrium of the price-quality competition is derived in Section 3. Section 4 characterises price regulation. Section 5 provides concluding remarks. All proofs are given in the Appendix

2. Model

Two health care providers (hospitals) are located at the extremes of a $[0,1]$ linear city. Hospital 1 is located at point 0, and Hospital 2 is located at point 1. Patients of unit mass are uniformly distributed on the interval $[0,1]$. Each consumer consumes exactly one unit of service from one of the hospitals (intrinsic value v is quite large). Unit transportation costs are t. All consumers are exogenously divided in the two regions. The border between Region 1 and Region 2 is situated at the point $x = 0.5$.

In addition to different locations, hospitals also have potentially different health care quality. The health care quality of Hospital 1 is denoted by q_1 and that of Hospital 2 by q_2. Different consumers value quality differently according to their own quality perception y. Quality perception y is uniformly distributed on the interval $[0,1]$ for each location point. The location and quality perception distributions are independent. The consumer with location x and quality perception y has the following utility function

$$u(x,y) = \begin{cases} v + yq_1 - p_1 - \tau - tx & \text{if utilizes services from Hospital 1,} \\ v + yq_2 - p_2 - \tau - t(1-x) & \text{if utilizes services from Hospital 2.} \end{cases} \tag{1}$$

Prices under market provision are p_1, p_2, and τ is tax under public provision. Both hospitals have equal costs of quality provision $c(q_i) = 0.5\theta q_i^2$, and marginal treatment costs are normalised to zero. This cost structure stresses the importance of quality provision costs, which seems reasonable for the hospital market (Brekke et al., 2007).

2.1 The first best (centralised) solution

Utilitarian social welfare is the difference between consumer surplus and the costs of quality provision. The social planner could create a partition of location/quality perception square in any manner. Optimal social welfare is

$$W^{SP} = \max_{\substack{q_1,q_2 \geq 0, \\ \Omega_1,\Omega_2 .}} \left\{ v + \iint_{\Omega_1} yq_1 - txdydx + \iint_{\Omega_2} yq_2 - t(1-x)dydx - \frac{\theta}{2}\left(q_1^2 + q_2^2\right) \right\}, \quad (2)$$

s.t. $\Omega_1 \subseteq [0,1] \times [0,1]$, $\Omega_2 \subseteq [0,1] \times [0,1]$, $\Omega_1 \cup \Omega_2 = [0,1] \times [0,1]$, $\Omega_1 \cap \Omega_2 = \varnothing$, (3)

Ω_1, Ω_2 are measurable sets.

Theorem 1 describes efficient quality/location distributions for $q_2 \geq q_1$. There is another solution for $q_2 \leq q_1$. It is symmetrical about the line $x = 0.5$.

Theorem 1. *If* $0 < t\theta \leq \frac{1}{3}$, *then*

$$q_1 \leq \frac{1}{12\theta}, \ q_2 \geq \frac{5}{12\theta}, \ q_2 + q_2 = \frac{1}{2\theta}, \quad (4)$$

$$\Omega_1 = \left\{ (x,y) \Big| 0 \leq x \leq \frac{1}{2}, 0 \leq y \leq \frac{t(1-2x)}{q_2 - q_1} \right\}, \ \Omega_2 = [0,1] \times [0,1] \setminus \Omega_1, \quad (5)$$

$$W^{SP} \geq v + \frac{1}{16\theta} - \frac{t}{4}. \quad (6)$$

If $t\theta \geq \frac{1}{3}$, *then*

$$q_1 = q_2 = \frac{1}{4\theta}, \quad (10)$$

$$\Omega_1 = \left\{ (x,y) \Big| 0 \leq x \leq \frac{1}{2}, 0 \leq y \leq 1 \right\}, \ \Omega_2 = [0,1] \times [0,1] \setminus \Omega_1. \quad (11)$$

$$W^{SP} = v + \frac{1}{16\theta} - \frac{t}{4}. \quad (12)$$

The proof for Theorem 1 and subsequent theorem are given in the Appendix. The second part of Theorem 1 confirms the results of Brekke et al. (2014) for equal quality provision costs. In the presence of high transportation costs and/or high quality production costs, all consumers from Region 1 utilise services from Hospital 1, and the same applies for Region 2. Patients do not have incentives to travel across jurisdictions. Lower costs change the situation. A transitional case is depicted in Fig. 1. In this case, asymmetrical quality provision becomes efficient. One

hospital specialises in high quality service, while another hospital is much smaller and specialised in low quality service. Patients from a low quality hospital region split between the two hospitals. In each location of this region, patients with low quality perception utilise the services of their own hospital, but patients with high quality perception (with more serious diseases) travelled to another hospital.
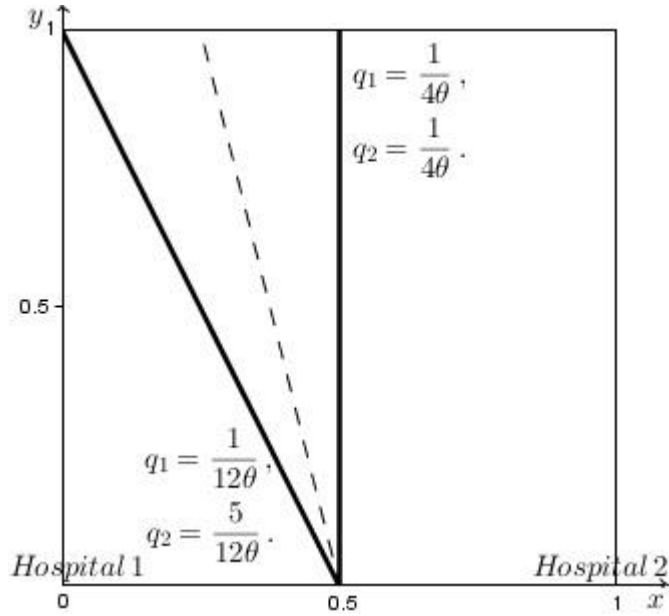

Fig. 1. Efficient patients partitions for $t\theta = \dfrac{1}{3}$. Solid lines separate the Hospital 1 area from the Hospital 2 area. The dotted line indicates patients who are indifferent to the two solutions.

For $t\theta = \dfrac{1}{3}$ there are two solutions with equal welfare. Under unequal quality solution, a quarter of the population has five times worse quality than the remaining portion of the population. Unequal quality solution is better for the majority of consumers. The dotted line in Fig. 1 separates consumers who prefer equal quality solution (left side) from the others. The lower costs increase the portion of the population which wins from unequal solution.

Unequal quality solution leads to quality specialization. In the case of health care provision one hospital (low quality) works basically with mild diseases. It serves the minority of their own region's patients. Another hospital is much larger and serves patients with serious diseases who require high quality. This hospital serves all patients from their own region and patients with serious diseases from another region.

2.2 No patient mobility across jurisdictions

Without mobility across jurisdictions each region has their own social welfare

$$W_1 = \frac{v}{2} + \frac{q_1}{4} - \frac{t}{8} - \frac{\theta}{2}q_1^2; \; W_2 = \frac{v}{2} + \frac{q_2}{4} - \frac{t}{8} - \frac{\theta}{2}q_2^2, \tag{13}$$

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which reaches the highest value at

$$q_1 = q_2 = \frac{1}{4\theta}.$$ (14)

In the case $t\theta \geq \frac{1}{3}$, this solution coincides with the first best solution. For lower cost levels, the solution with no patient mobility becomes inefficient. The region with higher quality of health care (lower quality) in the first best solution provides too low (high) quality. Assuming equal division of quality provision costs, we have the following conclusion:

Theorem 2. *If* $0 < t\theta < \frac{3}{125}\left(3\sqrt{21} - 8\right)$ *(approx. 0.14), then compared with decentralisation without patient mobility, both regions are better under a centralised solution with interregional patient mobility. If* $\frac{3}{125}\left(3\sqrt{21} - 8\right) < t\theta \leq \frac{1}{3}$, *then the higher quality (lower quality) region is better (worse) due to centralisation.*

The first part of Theorem 2 provides strong support for patient mobility and quality specialisation. The region with higher quality in the first best solution is superior because it is served by a higher quality health care producer and the quality provision costs are shared. The region with lower quality is superior because it is served by the higher quality health care producer, while there is only a modest increase in the quality provision costs. In the second case, the low quality region gains are limited and the low quality region would not approve a centralised solution if it had some effect on decision-making.

3. Market provision

Under market provision, health care providers simultaneously choose prices and qualities. Hospitals use simple linear pricing and seek to maximise their own profit.

$$\pi_1 = D_1(p_1, p_2, q_1, q_2)p_1 - 0.5cq_1^2,$$ (15)

$$\pi_2 = \left(1 - D_1(p_1, p_2, q_1, q_2)\right)p_2 - 0.5cq_2^2,$$ (16)

where $D_1(p_1, p_2, q_1, q_2)$ is demand function of Hospital 1.

The indifference locus in location and quality perception space separates Hospitals' demand

$$x = \frac{(q_1 - q_2)y - p_1 + p_2 + t}{2t}.$$ (17)

We assume linear transportation costs but quadratic costs do not change the indifference locus and main results. This model incorporates the classic Hotelling model with maximum differentiation, the d'Aspremont et al. (1979) model in the case $y = 0$, and the Ma, Burgess (1993) model in the case $y = 1$.

The indifference locus (17) is constrained by bounds of the unit square. The intersections with the indifference locus belong to the interval [0,1] if and only if

$$-t \leq p_1 - p_2 \leq t \tag{18}$$

$$p_1 - p_2 - t \leq q_1 - q_2 \leq p_1 - p_2 + t \tag{19}$$

There are nine different areas of mutual arrangement of parameters shown in Table 1. Areas B, C, F have $q_1 > q_2$, and areas D, G, H have $q_1 < q_2$. In area A, with high quality and low prices, Hospital 1 is a monopolist. In area B, the hospital loses some low quality perception consumers. In area I, Hospital 1 has zero demand.

Table 1. Demand function for Hospital 1.

	Area*	Demand function
A.	$p_2 - t > p_1$, $q_1 - q_2 > p_1 - p_2 + t$	$D_1^A = 1$
B.	$p_2 - t \leq p_1 \leq p_2 + t$, $q_1 - q_2 > p_1 - p_2 + t$	$D_1^B = 1 - \dfrac{(p_1 - p_2 + t)^2}{4t(q_1 - q_2)}$
C.	$p_1 > p_2 + t$, $q_1 - q_2 > p_1 - p_2 + t$	$D_1^C = 1 - \dfrac{p_1 - p_2}{q_1 - q_2}$
D.	$p_2 - t > p_1$, $p_1 - p_2 - t \leq q_1 - q_2 \leq p_1 - p_2 + t$	$D_1^D = 1 - \dfrac{(q_1 - q_2 - p_1 + p_2 - t)^2}{4t(q_2 - q_1)}$
E.	$p_2 - t \leq p_1 \leq p_2 + t$, $p_1 - p_2 - t \leq q_1 - q_2 \leq p_1 - p_2 + t$	$D_1^E = \dfrac{q_1 - q_2 - 2p_1 + 2p_2 + 2t}{4t}$
F.	$p_1 > p_2 + t$, $p_1 - p_2 - t \leq q_1 - q_2 \leq p_1 - p_2 + t$	$D_1^F = \dfrac{(q_1 - q_2 - p_1 + p_2 + t)^2}{4t(q_1 - q_2)}$
G.	$p_2 - t > p_1$, $q_1 - q_2 < p_1 - p_2 - t$	$D_1^G = \dfrac{p_2 - p_1}{q_2 - q_1}$
H.	$p_2 - t \leq p_1 \leq p_2 + t$, $q_1 - q_2 < p_1 - p_2 - t$	$D_1^H = \dfrac{(p_1 - p_2 - t)^2}{4t(q_2 - q_1)}$
I.	$p_1 > p_2 + t$, $q_1 - q_2 < p_1 - p_2 - t$	$D_1^I = 0$

Because of continuity in some cases the borders belong to the both corresponding areas.

There is symmetry between parameter areas. Hospital i's area A corresponds with Hospital j's area I. Other correspondences are as follows: B and H, C and G, D and F, E and E. The demand function depends only on the difference between prices and the difference between qualities and $D_i = 1 - D_j$, therefore

$$\frac{\partial D_i}{\partial p_i} = -\frac{\partial D_j}{\partial p_i} = \frac{\partial D_j}{\partial p_j} = -\frac{\partial D_i}{\partial p_j},$$

(20)

$$\frac{\partial D_i}{\partial q_i} = -\frac{\partial D_j}{\partial q_i} = \frac{\partial D_j}{\partial q_j} = -\frac{\partial D_i}{\partial q_j}.$$

(21)

In the interior equilibrium $p_i^* > 0$, $q_i^* > 0$, the following condition holds

$$\frac{\partial \pi_i}{\partial p_i} = D_i^* + \frac{\partial D_i}{\partial p_i} p_i^* = 0,$$

(22)

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\partial D_i}{\partial q_i} p_i^* - c q_i^* = 0.$$

(23)

Because of Eq. (18-21) and $D_j^* + D_i^* = 1$ we have

$$\frac{p_i^*}{q_i^*} = \frac{p_j^*}{q_j^*},$$

(24)

$$D_j^* = \frac{p_j^*}{p_i^* + p_j^*} = \frac{q_j^*}{q_i^* + q_j^*}.$$

(25)

There is positive price/quality relationship. This is common in price/quality competition. In all efficient distribution, point $(x = 0.5, y = 0)$ belongs to the border between two hospital areas. In market provision, this point belongs to the indifference locus if and only if $p_1^* = p_2^*$. From Eq. (24) it follows that $q_1^* = q_2^*$. Asymmetrical solution, which arises if $t\theta \leq \frac{1}{3}$, cannot be implemented through market provision.

Theorem 3. *There is unique symmetric equilibrium if and only if* $t\theta \geq \frac{5}{64}$, *and, whenever it exists*

$$p_1^* = t, \ p_2^* = t,$$

(27)

$$q_1^* = \frac{1}{4\theta}, \ q_2^* = \frac{1}{4\theta}.$$

(28)

This equilibrium implements efficient solution for $t\theta \geq \frac{1}{3}$. Theorem 2 generalises the results of Ma and Burges (1993) and Brekke et al. (2010). The optimal profit $\pi^* = \frac{t}{2} - \frac{1}{32\theta}$ is increased with respect to t and θ. Despite the negative direct effect of θ increase, the indirect effect of competition weakening is greater and it increases profit. The possibility of changing quality intensifies competition, but the equilibrium prices and revenue are similar to the standard model without quality (d'Aspremont, et al. 1979). The model presented in this section converges with the standard Hotelling model with firms located in extreme points if θ goes to infinity.

4. Price regulation

Price and quality competition is inefficient. Efficiency can be restored by lump-sum transfers and price regulatory policies. Under this policy, all patients pay the same amount τ. It can be tax or a social insurance contribution. Government pays r_1, r_2 to Hospital 1 and Hospital 2 for each treatment and charges T_1, T_2 as lump sum payments. Hospitals simultaneously choose quality levels.

The demand for Hospital 1 depends on $q_1 - q_2$. There are three cases

$$D_1 = \begin{cases} 1 - \dfrac{t}{4(q_1 - q_2)} & \text{if } q_1 - q_2 > t, \\[2mm] \dfrac{q_1 - q_2 + 2t}{4t} & \text{if } -t \leq q_1 - q_2 \leq t, \\[2mm] \dfrac{t}{4(q_2 - q_1)} & \text{if } q_1 - q_2 < -t. \end{cases} \tag{29}$$

Facing this demand, hospitals maximize their profits

$$\pi_1 = D_1(q_1, q_2)r_1 - 0.5\theta q_1{}^2 - T_1, \tag{30}$$

$$\pi_2 = (1 - D_1(q_1, q_2))r_2 - 0.5\theta q_2{}^2 - T_2, \tag{31}$$

where T_1, T_2 are lump sum payments (royalty). The aim of government is to find r_1, r_2, T_1, T_2, τ to incentivise hospitals to provide efficient quality levels.

Theorem 4. *For any $t\theta \leq \frac{1}{3}$ if government chooses*

$$r_1 = \frac{4\theta\hat{q}_1(\hat{q}_2 - \hat{q}_1)^2}{t}, \; r_2 = \frac{4\theta\hat{q}_2(\hat{q}_2 - \hat{q}_1)^2}{t}, \tag{32}$$

$$T_1^* = \hat{q}_1\left(0.5 - 2.5\theta\hat{q}_1\right), \quad T_2^* = \hat{q}_2\left(\frac{4\theta(\hat{q}_2 - \hat{q}_1)^2}{t} + 0.5 - 2.5\theta\hat{q}_2\right). \tag{33}$$

$$\tau = \frac{\theta}{2}\hat{q}_1{}^2 + \frac{\theta}{2}\hat{q}_2{}^2, \tag{34}$$

then in equilibrium

$$q_1^* = \hat{q}_1, \quad q_2^* = \hat{q}_2, \tag{35}$$

where $\hat{q}_1$, $\hat{q}_2$ *efficient quality levels.*

For any $t\theta \leq \dfrac{1}{3}$ there exists price regulation and flat rate tax, leading to efficient distribution. In equilibrium, health care providers have zero profits. Asymmetric solution benefits are shared unequally. By fairness and regional sovereignty reasons, the financing scheme can include transfers. Because joint welfare is increased in comparison with the decentralised solution, there exists a transfer mechanism, which leads to welfare improvement in both regions. Some redistribution schemes are discussed in Brekke et al. (2014).

5. Concluding remarks

There are two efficient regimes of health care quality provision. In the presence of high travel and quality provision costs, qualities should be equal. Reducing these costs results in an unequal distribution of quality in an effective solution. High health care inequality under a centralised solution is better for the majority of the population as well as for both regions, if the costs are sufficiently low. As long as utility is linear in income it does not depend on difference on income levels in two regions. Market equilibrium implements an efficient solution only when there are high travel and quality provision costs. Lump-sum transfers and price regulatory policies restore efficiency in the case of low costs.

Unequal health care quality provision can be observed in Italy, where the already significant gap between the health care systems of the northern and southern regions has increased within a decade (1999–2009) (Toth, 2014). It is reasonable to suggest that transport and quality provision costs declined during this decade and therefore increased unequal health care quality provision is efficient.

Appendix

Proof of Theorem 1.

Let $\Omega_1^*, \Omega_2^*, q_1^*, q_2^*$ be a solution of a social planner problem (Eq. (2-3)). Without loss of generality, $q_1^* \le q_2^*$.

Suppose there exist measurable sets $A_1 \subseteq [0,1] \times [0,1]$, $A_2 \subseteq [0,1] \times [0,1]$, and number $\varepsilon > 0$ such that $A_1 \subseteq \Omega_1^*$, $A_2 \subseteq \Omega_2^*$, and $(x,y) \in A_1$ if and only if $(x - \varepsilon, y) \in A_2$. Defining new sets $\tilde{\Omega}_1 = \Omega_1^* \cup A_2 \setminus A_1$, $\tilde{\Omega}_2 = \Omega_2^* \cup A_1 \setminus A_2$ we obtain

$$-\iint_{\tilde{\Omega}_1} txdydx - \iint_{\tilde{\Omega}_2} t(1-x)dydx =$$
$$= -\iint_{\Omega_1^*} txdydx - \iint_{\Omega_2^*} t(1-x)dydx + \iint_{A_1} (2tx - t)dydx + \iint_{A_2} (t - 2tx)dydx \ge \qquad (36)$$
$$\ge -\iint_{\Omega_1^*} txdydx - \iint_{\Omega_2^*} t(1-x)dydx$$

$\tilde{\Omega}_1, \tilde{\Omega}_2$ is also an optimal partition.

Suppose there exist measurable sets $A_1 \subseteq [0,1] \times [0,1]$, $A_2 \subseteq [0,1] \times [0,1]$, and number $\varepsilon > 0$ such that $A_1 \subseteq \Omega_1^*$, $A_2 \subseteq \Omega_2^*$, and $(x,y) \in A_1$ if and only if $(x, y - \varepsilon) \in A_2$. Defining new sets $\hat{\Omega}_1 = \Omega_1^* \cup A_2 \setminus A_1$, $\hat{\Omega}_2 = \Omega_2^* \cup A_1 \setminus A_2$ we obtain

$$\iint_{\hat{\Omega}_1} yq_1 dydx + \iint_{\hat{\Omega}_2} yq_2 dydx =$$
$$= \iint_{\Omega_1^*} yq_1 dydx + \iint_{\Omega_2^*} yq_2 dydx + \iint_{A_2} y(q_1 - q_2)dydx + \iint_{A_1} y(q_2 - q_1)dydx \ge \qquad (37)$$
$$\ge \iint_{\Omega_1^*} yq_1 dydx + \iint_{\Omega_2^*} yq_2 dydx$$

$\hat{\Omega}_1, \hat{\Omega}_2$ is also an optimal partition.

There exist optimal partition Ω_1, Ω_2 such that for any $y \in [0,1]$ if $(x_1, y) \in \Omega_1$ and $(x_2, y) \in \Omega_2$ then $x_1 < x_2$ and for any $x \in [0,1]$ if $(x, y_1) \in \Omega_1$ and $(x, y_2) \in \Omega_2$, then $y_1 < y_2$. Social welfare for this partition is equal to

$$\max_{\substack{q_1, q_2 \ge 0, \\ 0 \le f(x) \le 1}} \left\{ v + \int_0^1 \int_0^{f(x)} yq_1 - txdydx + \int_0^1 \int_{f(x)}^1 yq_2 - t(1-x)dydx - \frac{\theta}{2}\left(q_1^2 + q_2^2\right) \right\} =$$

$$\max_{\substack{q_1, q_2 \ge 0, \\ 0 \le f(x) \le 1}} \left\{ v + \int_0^1 \frac{f^2(x)}{2}q_1 + \frac{1 - f^2(x)}{2}q_2 - t(1-x) + t(1-2x)f(x)dx - \frac{\theta}{2}\left(q_1^2 + q_2^2\right) \right\}, \quad (38)$$

where $f(x)$ is a non-increasing function (mapping). The first order condition for this problem is

$$f(x) = \frac{t(1-2x)}{q_2 - q_1}, \tag{39}$$

$$q_1 = \int_0^1 \frac{f^2(x)}{2\theta} dx, \tag{40}$$

$$q_2 = \int_0^1 \frac{1 - f^2(x)}{2\theta} dx. \tag{41}$$

Because $f(x)$ is constrained by the interval of $[0,1]$, for all $x \geq 0.5$ $f(x) = 0$ and in some cases there is an interval with $f(x) = 1$. Let us denote $z = \int_0^1 f^2(x)dx$ and consider three cases.

1. $-q_2 + q_1 + t > 0$. There is an interval with $f(x) = 1$. From Eqs. (39-41), we have

$$z = 0.5 + \frac{-1 + 2z}{4t\theta} + \int_{0.5 + \frac{-1+2y}{4t\theta}}^{0.5} \frac{4t^2\theta^2(1-2x)^2}{(1-2z)^2} = 0.5 + \frac{-1+2z}{4t\theta} + \frac{1-2z}{12t\theta}. \tag{42}$$

The solution is $z = 0.5$. For this solution

$$q_2 = q_1 = \frac{1}{4\theta}, \tag{43}$$

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 0.5, \\ 0 & \text{if } 0.5 < x \leq 1 \end{cases} \tag{44}$$

$$W = v + \frac{1}{16\theta} - \frac{t}{4}. \tag{45}$$

2. $-q_2 + q_1 + t = 0$. From Eqs. (39-41), we have

$$z(1-2z)^2 = \frac{2t^2\theta^2}{3}. \tag{46}$$

For $-q_2 + q_1 + t = 0$ we have $z = \frac{1}{6}$, $t\theta = \frac{1}{3}$ and

$$q_1 = \frac{1}{12\theta}, \quad q_2 = \frac{5}{12\theta}, \tag{47}$$

$$f(x) = \begin{cases} (1-2x) & \text{if } 0 \leq x \leq 0.5, \\ 0 & \text{if } 0.5 < x \leq 1. \end{cases} \tag{48}$$

$$W = v + \frac{1}{16\theta} - \frac{t}{4}. \tag{49}$$

3. $-q_2 + q_1 + t < 0$. For this case from Eqs. (39-41), we obtain the same equation as Eq. (46). For $t\theta < \frac{1}{3}$ Eq. (46) has a unique root that is smaller than $\frac{1}{6}$. For this root

$$W = v + \frac{1}{\theta}\left(-\frac{3z^{*2}}{4} + \frac{z^*}{4} + \frac{1}{8} - \frac{t\theta}{2}\right). \tag{50}$$

Social welfare in this case is higher than in the first and the second cases if and only if

$$z^* \in \left(\frac{1 - 2\sqrt{1 - 3t\theta}}{6}, \frac{1 + 2\sqrt{1 - 3t\theta}}{6}\right). \tag{51}$$

Substituting $\frac{1 - 2\sqrt{1 - 3t\theta}}{6}$ and $\frac{1}{6}$ to Eq. (46) we have for $t\theta < \frac{1}{3}$

$$\frac{2}{27}\left(-2 + 9t\theta - 2(1 - 3t\theta)^{1.5}\right) < \frac{2t^2\theta^2}{3} < \frac{2}{27}. \tag{52}$$

The root of Eq. (46) belongs to the interval (51). Because of $q_1 + q_2 = \frac{1}{2\theta}$, then in this

case $q_1 < \frac{1}{12\theta}$, $q_2 > \frac{5}{12\theta}$, $f(x) < (1 - 2x)$ for $0 \le x < 0.5$, $W > v + \frac{1}{16\theta} - \frac{t}{4}$. $\blacksquare$

Proof of Theorem 2.

Without loss of generality in the centralised solution with interregional patient mobility $q_1^* \le q_2^*$. Assuming equal division of quality provision costs under a centralised solution we have

$$W_1 = \frac{v}{2} - \frac{5z^2}{8\theta} + \frac{z}{4\theta} + \frac{1}{16\theta} - \frac{3t}{8}, \tag{53}$$

$$W_2 = \frac{v}{2} - \frac{z^2}{8\theta} + \frac{1}{16\theta} - \frac{t}{8}, \tag{54}$$

where $z = \int_0^1 f^2(x)dx$. Under a decentralised solution each region has $W = \frac{v}{2} + \frac{1}{32\theta} - \frac{t}{8}$. Having

Eq. (46) $W_1 > W$ is equivalent to

$$-20z^2 + 8z - 8(1 - 2z)\sqrt{\frac{3}{2}z} + 1 > 0. \tag{55}$$

If $0 < t\theta < \frac{3}{125}\left(3\sqrt{21} - 8\right)$ then $W_1 > W$, $W_2 > W$. If $\frac{3}{125}\left(3\sqrt{21} - 8\right) < t\theta \le \frac{1}{3}$ then $W_1 < W$,

$W_2 > W$. $\blacksquare$

Proof of Theorem 3.

The only symmetric equilibrium can be in area E (hereinafter in the proofs numbering from Table 1). In this area, the first order condition for Hospital 1 problem is

$$q_1 = \frac{p_1}{4ct}, \quad p_1 = \frac{q_1 - q_2 + 2p_2 + 2t}{4}. \tag{56}$$

Having similar condition for Hospital 2, we have $p_i^* = t$, and $q_i^* = \frac{1}{4c}$, $\pi_i^{E*} = \frac{t}{2} - \frac{1}{32c}$, $i = \{1,2\}$. This point belongs to area E. Let us check possible profitable deviations in other areas. Because of equilibrium symmetry, we check only possible deviations of Hospital 1.

A. Because of $p_2^* = t$ area A is empty.

B. Because of $\pi_1^E - \pi_1^B = \frac{p_1(-q_1 + q_2 + p_1 - p_2 + t)^2}{4t(q_1 - q_2)} > 0$, π_1^E is greater than π_1^B for all

$q_1 > q_2$.

C. Taking derivative with respect to p_1 we have $\frac{\partial \pi_1^C}{\partial p_1} = 1 - \frac{(2p_1 - p_2)}{q_1 - q_2} = 0$. Because of

$\frac{\partial^2 \pi_1^C}{\partial p_1^2} < 0$, and $\lim_{q_1 \to \infty} \pi_1^C = -\infty$ the highest value of π_1^C in this area is situated on the line

$q_1 - q_2 + t - 2p_1 = 0$. Having $p_1 = 0.5(q_1 - q_2 + t)$, $p_2 = t$, $p_1 > p_2 + t$ and

$q_1 - q_2 > p_1 - p_2 + t$ we obtain $\pi_1^E - \pi_1^C = \frac{(q_1 - q_2 + t)(q_1 - q_2 - 2t)}{8(q_1 - q_2)} > 0$.

D. Because of $p_2^* = t$ area D is empty.

E. By construction, (p_1^*, q_1^*) is the point with the highest value in this area.

F. Having $p_1 = p_2 + t = 2t$ we obtain $\pi_1^E = \pi_1^F = 0.5(q_1 - q_2) - 0.5cq_1^2$. Having $q_1 - q_2 = p_1 - p_2 + t$ we obtain $\pi_1^C = \pi_1^F = t - 0.5cq_1^2$. There is no discontinuity in the borders between areas E and F, C and F. Taking derivative with respect to p_1 we have $\frac{\partial \pi_1^F}{\partial p_1} = \frac{(q_1 - q_2 - p_1 + 2t)(q_1 - q_2 - 3p_1 + 2t)}{4t(q_1 - q_2)} \le 0$. The values of the profit function in area F are lower than the values in areas C and E.

G. Because of $p_2^* = t$ area G is empty.

H. Having $q_1 - q_2 = p_1 - p_2 - t$ we obtain $\pi_1^E = \pi_1^H = \frac{(-p_1 + 2t)p_1}{4t} - 0.5cq_1^2$. There is no discontinuity in the border between areas E and H. Taking derivative with respect to p_1 we have $\frac{\partial \pi_1^H}{\partial p_1} = \frac{2(p_1 - 2t)p_1}{4t(q_2 - q_1)} + \frac{(p_1 - 2t)^2}{4t(q_2 - q_1)} = 0$, $p_1 = \frac{2}{3}t$.

If $ct > \dfrac{3}{16}$ line $p_1 = \dfrac{2}{3}t$ does not intersect area H. In this case in area H $\dfrac{\partial \pi_1^H}{\partial p_1} < 0$ and

profit in area E is greater. If $ct \le \dfrac{3}{16}$ then $\dfrac{\partial^2 \pi_1^H}{\partial p_1^2} < 0$ and the possible maximum is on the line

$p_1 = \dfrac{2}{3}t$ with the value of profit function

$$\tilde{\pi}_1^H = \frac{1}{c}\left[\frac{\dfrac{8}{27}c^2 t^2}{\left(\dfrac{1}{4} - cq_1\right)} - 0.5c^2 q_1^2 \right], \quad cq_1 \in \left[0, \frac{1}{4} - \frac{4}{3}tc\right]. \tag{57}$$

There exists a point with the higher value of function (57) than $\pi_i^{E*} = \dfrac{1}{c}\left(\dfrac{ct}{2} - \dfrac{1}{32}\right)$ if and only if

$ct < \dfrac{5}{64}$. If $ct \ge \dfrac{5}{64}$, there is no profitable deviation.

I.   The highest value of the profit function in this area is equal to $\pi_1^{I*} = 0$. The deviation is

not profitable if and only if $\pi^{E*} = \dfrac{t}{2} - \dfrac{1}{32c} \ge 0$. It holds for $ct \ge \dfrac{1}{16}$. ∎

Proof of Theorem 4.

Let consider the case $q_1 - q_2 < -t$. If $\hat{q}_1 + \hat{q}_2 = \dfrac{1}{2\theta}$, and $\hat{q}_1 > 0, \hat{q}_2 > 0$, then for $q_1 = \hat{q}_1$,

$q_2 = \hat{q}_2$

$$\frac{d\pi_1}{dq_1} = 0, \ \frac{d\pi_2}{dq_2} = 0, \ \frac{d^2\pi_1}{dq_1^2} < 0, \ \frac{d^2\pi_2}{dq_2^2} < 0. \tag{58}$$

This lead to

$$\pi_1^* = \hat{q}_1(0.5 - 2.5\theta\hat{q}_1) - T_1, \tag{59}$$

$$\pi_2^* = \frac{0.5 - \theta\hat{q}_1}{\theta}\left(\frac{(1 - 4\theta\hat{q}_1)^2}{t\theta} + 0.5 - 2.5(0.5 - \theta\hat{q}_1) \right) - T_2. \tag{60}$$

Having

$$T_1^* = \hat{q}_1(0.5 - 2.5\theta\hat{q}_1), \ T_2^* = \hat{q}_2\left(\frac{4\theta(\hat{q}_2 - \hat{q}_1)^2}{t} + 0.5 - 2.5\theta\hat{q}_2 \right). \tag{61}$$

we obtain $\pi_1^* = 0$, $\pi_2^* = 0$.

Let try to find profitable deviations for $q_1 - q_2 \geq -t$. For $t \geq q_1 - q_2 \geq -t$ the highest profit of Hospital 1 is equal to

$$\pi_1 = \hat{q}_1 \left[\frac{(-0.5 + \theta\hat{q}_1 + 2t\theta)(0.5 - 2\theta\hat{q}_1)^2}{t^2\theta^2} + 0.5\theta\hat{q}_1 \left(\frac{(0.5 - 2\theta\hat{q}_1)^2}{t^2\theta^2} \right)^2 \right], \qquad (62)$$

if $\dfrac{\theta\hat{q}_1(0.5 - 2\theta\hat{q}_1)^2}{t^2\theta^2} \leq 0.5 - \theta\hat{q}_1 + t\theta$;

$$\pi_1 = \hat{q}_1 \left(\frac{3(0.5 - 2\hat{q}_1)^2}{t\theta} - \frac{(0.5 - \theta\hat{q}_1 + t\theta)^2}{2\theta\hat{q}_1} \right), \qquad (63)$$

if $\dfrac{\theta\hat{q}_1(0.5 - 2\theta\hat{q}_1)^2}{t^2\theta^2} \geq 0.5 - \theta\hat{q}_1 + t\theta$.

For $t \geq q_1 - q_2 \geq -t$ the highest profit of Hospital 2 is equal to

$$\pi_2 = \frac{0.5 - \theta\hat{q}_1}{\theta} \left[\frac{(-\theta\hat{q}_1 + 2t\theta)(0.5 - 2\theta\hat{q}_1)^2}{t^2\theta^2} + 0.5(0.5 - \theta\hat{q}_1) \left(\frac{(0.5 - 2\theta\hat{q}_1)^2}{t^2\theta^2} \right)^2 \right], \qquad (64)$$

$$t\theta \geq \theta\hat{q}_1 - \frac{(0.5 - \theta\hat{q}_1)(0.5 - 2\theta\hat{q}_1)^2}{t^2\theta^2} \geq -t\theta. \qquad (65)$$

Profitable deviation for Hospital 2 does not exist.

For $t \leq q_1 - q_2$ the highest profit of Hospital 1 is achieved at the point

$$q_1 = \hat{q}_2 + \frac{\sqrt{\hat{q}_1(4\hat{q}_2 - 3\hat{q}_1)} - \hat{q}_1}{2} \text{ if } \frac{\sqrt{\theta\hat{q}_1(2 - 7\theta\hat{q}_1)} - \theta\hat{q}_1}{2} \geq t\theta, \qquad (66)$$

$$q_1 = \hat{q}_2 + t \text{ if } \frac{\sqrt{\theta\hat{q}_1(2 - 7\theta\hat{q}_1)} - \theta\hat{q}_1}{2} \leq t\theta. \qquad (67)$$

The highest profits of Hospital 1 are equal to

$$\pi_1 = \hat{q}_1 \left[\frac{(1 - 4\theta\hat{q}_1)^2}{t\theta} - \frac{2(0.5 - 2\theta\hat{q}_1)^2}{\sqrt{\theta\hat{q}_1(2 - 7\theta\hat{q}_1)} - \theta\hat{q}_1} - \frac{(1 + \sqrt{\theta\hat{q}_1(2 - 7\theta\hat{q}_1)} - 3\theta\hat{q}_1)^2}{8\theta\hat{q}_1} \right] \qquad (68)$$

if $\dfrac{\sqrt{\theta\hat{q}_1(2 - 7\theta\hat{q}_1)} - \theta\hat{q}_1}{2} \geq t\theta$;

$$\pi_1 = \hat{q}_1 \left(\frac{3(0.5 - 2\theta\hat{q}_1)^2}{t\theta} - \frac{(0.5 - \theta\hat{q}_1 + t\theta)^2}{2\theta\hat{q}_1} \right) \qquad (69)$$

if $\dfrac{\sqrt{\theta\hat{q}_1(2 - 7\theta\hat{q}_1)} - \theta\hat{q}_1}{2} < t\theta$.

If $t \le q_1 - q_2$ then $\dfrac{d\pi_2}{dq_2} = \dfrac{\theta \hat{q}_2 (\hat{q}_2 - \hat{q}_1)^2}{(\hat{q}_1 - q_2)^2} - \theta q_2 > 0$. There is no profitable deviation for Hospital 2.

From Eq. (46) all efficient distributions for $t\theta \le \dfrac{1}{3}$ are described by equation

$$\sqrt{3\theta\hat{q}_1} \left(1 - 4\theta\hat{q}_1\right) = t\theta . \tag{70}$$

Figure 2 depicts profitable deviations and efficient quality distributions. The intersection of these sets for $0 < \theta\hat{q}_1 < \dfrac{1}{12}$ is empty.
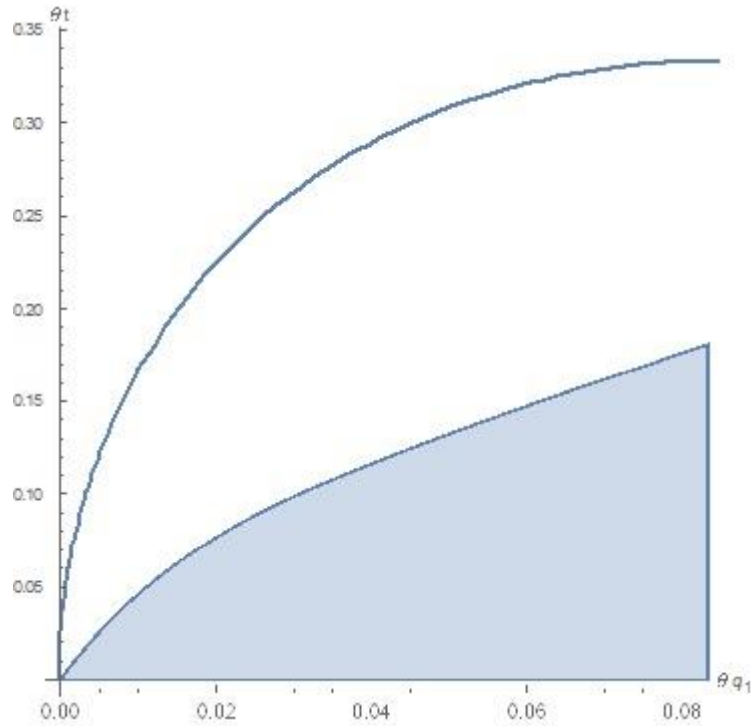

Figure 2. Profitable deviations of Hospital 1 are marked in shaded area. The line depicts all efficient distributions.

Balancing governments spending we have

$$\tau = \frac{\theta}{2}\hat{q}_1{}^2 + \frac{\theta}{2}\hat{q}_2{}^2 . \blacksquare \tag{71}$$

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