Three-Parameter Kinetics of Self-organized Criticality on Twitter

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Abstract. A kinetic model is proposed to describe the self-organized criticality on Twitter. The model is based on a fractional three-parameter self-organization scheme with stochastic sources. It is shown that the adiabatic regime of self-organization to the critical state is determined by the coordinated action of a relatively small number of network users. The model is described the subcritical, self-organized critical and supercritical state of Twitter.

Keywords: Self-organized criticality · Social networks · Langevin equation

1 Introduction

Critical phenomena in complex networks have been considered in many papers (e.g., see the review [1] and references therein). In the network science, under the critical phenomena commonly understand the significant changes in the integral parameters of the network structure under the influence of external factors [1]. In the thermodynamics theory of irreversible processes, it is stated that significant structure reconstructions occur when the external parameter reaches a certain critical value and has the character of a kinetic phase transition [2]. The critical point is reached as a result of fine tuning of the system external parameters. In a certain sense, such critical phenomena are not robust. At the end of the 1980s, Bak, Tang and Wiesenfeld [3, 4] found that there are complex systems with a large number of degrees of freedom that go into a critical mode as a result of the internal evolutionary trends of these systems. A critical state of such systems does not require fine tuning of external control parameters and may occur spontaneously. Thus, the theory of self-organized criticality (SOC) was proposed. From the moment of the SOC model emergence, this model started to be applied to describe critical phenomena in systems regardless of their nature (e.g., see the review [5] with references). Not an exception is the application of the theory to the description of critical phenomena in social networks (e.g., see the works [6–9]).

The motivation of our investigation is the following. There is a number of studies (e.g., see the works [7, 9–17]), in which it is established that the observed flows of microposts generated by microblogging social networks (e.g., Twitter), are characterized

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by avalanche-like behavior. Time series of microposts (η_t) depicting such streams are the time series with a power law distribution of probabilities:

$$p(\eta) \propto \eta^{-\alpha} \tag{1}$$

where $\alpha \in (2,3)$.

Despite this, there are no studies on the construction and analysis of macroscopic kinetic models that explain the phenomenon of the emergence and spread of avalanche of microposts on Twitter.

2 One of the Possible Mechanisms of Twitter Self-organizing Transition in a Critical State

Let $\mathcal{N}$ be the total number of Twitter users, and let $S \ll \mathcal{N}$ be the number of users who follow a certain strategy. Let's call them strategically oriented users (SOUs). The remaining $\mathcal{N} - S$ users do not follow a single coherent strategy and, in this sense, are randomly oriented users (ROUs). Suppose that at each moment in time, one SOU goes on Twitter, i.e. social network is the open system. These users act in concert, trying to form some microposts in the network relevant to a certain topic. Gradually, a sub-network of SOUs is formed in the social network. ROUs that are SOUs subscribers in this case are also embedded in the emerging hierarchical network structure. As a result, local and predictable micropost flows are formed on Twitter, corresponding to the topic defined by SOUs. Such a behavior of the social network is simple, since the individual local flows of microposts are not interconnected. The formed hierarchical system of the social network by SOUs and ROUs are still not able to form an avalanche of microposts. Over time, the number of SOUs reaches a critical value S_c. In this state, the network can no longer be pumped by these users. In order to maintain a steady state, all network users, including ROUs, must follow a certain coordinated strategy in the distribution of microposts. Therefore, in a stationary system of users, global avalanches of microposts arise and distributed in the network. This is the SOC state of the social network, formed by the action of a small, compared with the total number of all users, the number of strategically oriented users. Instead of local flows of microposts, a global avalanche of microposts occurs, which is characteristic of the critical state of the network. The behavior of global avalanches spreading in the self-organized critical network is unpredictable based on the behavior of individual users. In this case, the social network has the property of emergence.

Let S_c be the number of SOUs in the stationary (critical) state of the social network. In relation to the critical state, three qualitatively different states of Twitter can be distinguished:

- $S < S_c$ is the subcritical (SubC) network state;
- $S = S_c$ is the SOC network state;
- $S > S_c$ is the supercritical (SupC) network state.

The SubC state is characterized by the small number of avalanches of microposts, which can be almost neglected. In the SOC state, microposts avalanche size is growing.

The appearance of such avalanches of microposts satisfies the power law distribution of probabilities (see Eq. (1)). In the SupC state, the number of SOUs, and, accordingly, the avalanche sizes of microposts continue to grow. This growth is unstable. In a response to a further increase in the number of microposts generated by SOUs entering the network, the number of "extra" microposts in the social network increases, reducing S to a critical level. The value S_c separates the chaotic and the ordered states of the network. Indeed, the almost zero flow of microposts, which occurs when $S < S_c$, can be considered as the result of lots of randomly directed flows of microposts, which are mutually balanced. When $S > S_c$, disorder gives way to order, which is expressed in the appearance of a dedicated flow direction (avalanche) of microposts. And, as a result, it becomes significant at the macro level. Both of these states correspond to the non-catastrophic behavior of the social network, since in these states the network is resistant to small impact. In a chaotic state, small perturbations still fade out quickly in time and space, and in an ordered state, perturbations can no longer have a noticeable effect on the avalanche size of microposts. In a critical state, in which only one added SOU can cause an avalanche of microposts of any size, catastrophes are possible.

As a result of self-organization in a critical state, a social network acquires properties that its elements did not have, demonstrating complex emergent behavior. At the same time, it is important that the self-organizing nature of emergent properties ensures their robustness. The SOC state is robust in relation to possible changes in the social network. For example, if the nature of interactions between users' changes, the social network temporarily deviates from the existing critical state, but after a while it is restored in a slightly different form. The hierarchical network structure will change, but its dynamics will remain critical. Every time, when trying to divert Twitter from the SOC state, the social network invariably returns to this state.

3 The Formalism

It is known (e.g. see works [18–20]) that the concept of self-organization is a generalization of the physical concept of critical phenomena, such as phase transitions. Therefore, the phenomenological theory that we propose is a generalization of the theory of thermodynamic transformations for open systems. Twitter self-organization is possible due to its openness, since there are incoming and outgoing network flows of its users constantly; its macroscopic, because includes a large number of users; its dissipation, because there are losses in the flows of microposts and associated information. Based on the synergetic principle of subordination, it can be argued that Twitter's self-organization in a critical state is completely determined by the suppression of the behavior of an infinite number of microscopic degrees of freedom by a small number of macroscopic degrees of freedom. As a result, the collective behavior of users of the social network is defined by several parameters or degrees of freedom: an order parameter η_t, its role is the number of microposts relevant to a certain topic that are sent by SOUs and, unwittingly following their strategies, by ROUs; a conjugate field h_t is information associated with microposts distributed in the network; a control parameter

S_t which is the number of SOUs of the networks. On the other hand, in Twitter's self-organization as the non-equilibrium system, the dissipation of flows of microposts in the network should play a crucial role, which ensures the transition of the network to the stationary state. In the process of self-organization in a critical state of the network, all three degrees of freedom have an equal character, and the description of the process requires a self-consistent view of their evolution. The restriction to three degrees of freedom is also determined by the Ruelle–Takens theorem, according to which a nontrivial picture of self-organization is observed if the number of selected degrees of freedom is, at least, three.

Kinetic equations and a detailed physical substantiation of the relations between its parameters are given in our paper [16]. The construction of the three-parameter self-organization scheme was based on the analogy between the mechanisms of functioning of a single-mode laser and the microblogging social network. The study of possible modifications of equations leading to models that are capable to describe critical phenomena on Twitter, in particular the SOC or the SupC states, is outside of the scope of this paper. These equations in dimensionless quantities have the following form:

$$\begin{cases} \dot{\eta}_t = -\eta_t^{\varepsilon} + h_t + \sqrt{I_\eta}\xi_t \\ \frac{\tau_h}{\tau_\eta}\dot{h}_t = -h_t + \eta_t^{\varepsilon} S_t + \sqrt{I_h}\xi_t \\ \frac{\tau_S}{\tau_\eta}\dot{S}_t = (S_0 - S_t) - \eta_t^{\varepsilon} h_t + \sqrt{I_S}\xi_t \end{cases} \tag{2}$$

where I_i is intensity of fluctuations (or noises) of each of the degrees of freedom $i = \eta, h, S$; τ_i is relaxation times of corresponding quantities; ξ_t is white noise due to random factors; S_0 is number of SOUs of the network at the initial time moment ($t = 0$) of the network evolution. The parameter S_0 determines the degree of external disturbance or pumping of the social network by strategically oriented users, which removes Twitter from the equilibrium state.

Assuming $\varepsilon = 1$, and also neglecting random factors ξ_t, Eqs. (2) is a well-known system of Lorenz equations, which dynamic variables describe the self-consistent behavior of the order parameter, the conjugate field and the control parameter. In such a system, the functions η_t/τ_η, h_t/τ_h, $(S_t - S_0)/\tau_S$ describe autonomous relaxation of the number of microposts, of conjugate information and the number of strategically oriented network users to stationary values $\eta_t = 0$, $h_t = 0$, $S_t = S_0$. The Lorenz system takes into account the Le Chatelier's principle: since the reason for self-organization is the growth of the control parameter S_t, the values of η_t and h_t should be changed in such a way as to prevent the growth S_t. Finally, the positive feedback between the order parameter η_t and the control parameter S_t, which leads to an increase in the conjugate field h_t, is fundamentally important. The feedback intensity indicator ε in the system of Eqs. (2), which also distinguishes it from the Lorenz system, is an indicator of the disturbance of Twitter's ordering on its self-consistent behavior. From a physical point of view, replacing the order parameter normalized to one ($\eta_t \in (0, 1]$) with a larger value η_t^{ε} ($\varepsilon < 1$) means that the ordering process affects Twitter's self-consistent behavior more than in the ideal case, when $\varepsilon = 1$.

Equations (2) does not have an exact analytical solution. When certain conditions are met, Eqs. (2) in an adiabatic approximation can be quite acceptable approximation. The adiabatic self-organization mode corresponds to a phase transition process for which the stationary value of the control parameter does not reduce to the pump parameter (e.g. see works [18–20]). In the adiabatic approximation, the characteristic relaxation time of the number of microposts τ_η far exceeds the corresponding relaxation times of the information associated with microposts and the number of strategically oriented users: τ_h and τ_S. This means that the information $h_t \cong h(\eta_t)$ and the number of strategically oriented users $S_t \cong S(\eta_t)$ follow the changes in the number of microposts η_t on Twitter. When the conditions $\tau_\eta \gg \tau_h, \tau_S$ are fulfilled, the principle of subordination makes it possible to neglect the fluctuations of the quantities $h(\eta_t)$ and $S(\eta_t)$ in the system of Eqs. (2), i.e. assume $\frac{\tau_h}{\tau_\eta}\dot{h}_t = \frac{\tau_S}{\tau_\eta}\dot{S}_t = 0$.

The use of the adiabatic approach to Twitter as an open nonequilibrium system means that, when the value of the social network pumping by strategically oriented users tends to zero ($S_0 \to 0$), there is a slow decrease in the flow of microposts η_t and a rapid decrease in the associated information h_t, as well as in the number of strategically oriented users S_t, who sending microposts. Using the adiabatic approximation allows to reduce the dimension of the phase space, i.e. transit from the analysis of a three-dimensional dynamic system with additive noise (2) to the analysis of a one-parameter stochastic system with multiplicative noise:

$$\tau_\eta \dot{\eta}_t = f_\eta + \sqrt{I_\eta}\xi_t. \tag{3}$$

In the Langevin Eq. (3), the drift and diffusion parts are determined by the following values:

$$f_\eta \equiv -\eta_t^\varepsilon + S_0\eta_t^\varepsilon\mu_\eta, I_\eta \equiv I_\eta + \left(I_h + I_S\eta_t^{2\varepsilon}\right)\mu_\eta^2. \tag{4}$$

where $\mu_\eta \equiv 1 + \eta_t^{2\varepsilon}$.

Suppose that the social network Twitter is self-organized into a critical state as a result of the agreed actions of SOUs and ROUs. Such a saturated network state by strategically oriented users and information is characterized by the following features. Firstly, by the significant intensity of stochastic interactions between strategically oriented users ($I_S \gg I_\eta, I_h$) Secondly, by the significant impact of the streamlining process on Twitter's self-consistent behavior. In this state, even a negligible external disturbance ($S_0 \cong 0$) is enough to spread avalanches of microposts in the social network.

Therefore, Eq. (3), describing the being of the social network in the SOC state, will be in the following form:

$$\tau_\eta \dot{\eta}_t = -\eta_t^\varepsilon + \sqrt{I_S}\eta_t^\varepsilon\mu_\eta\xi_t. \tag{5}$$

Suppose that the homogeneous process (5) occurs on the interval $(\eta_{\min}, 1)$ and $\varepsilon < 1$, where $\eta_{\min}$ is the minimum number of microposts for which power-law for probability density function (PDF) is performed. Then a non-normalized solution of the corresponding stationary Fokker-Planck equation with reflecting boundaries is:

$$p_s(\eta) \propto I_S^{-1} \eta^{-2\varepsilon} \mu_\eta^{-2} \exp\left[-I_S^{-1} \int_{\eta_{\min}}^{\eta} \eta'^{-\varepsilon} \mu_{\eta'}^{-2} d\eta'\right]. \tag{6}$$

The integral in PDF (5) has the following form, rather bulky for analysis:

$$\left\{ \frac{\eta^{1-\varepsilon}}{2(\varepsilon-1)\mu_{\eta'}} \left[(3\varepsilon-1)\mu_{\eta'\,2}F_1 + \eta'^{2\varepsilon} - \varepsilon\left(3\eta'^{2\varepsilon}+2\right)\right] \right\}\Bigg|_{\eta_{\min}}^{\eta}. \tag{7}$$

where ${}_2F_1\left(1, \frac{\varepsilon-1}{2\varepsilon}; \frac{3\varepsilon-1}{2\varepsilon}; -\eta'^{-2\varepsilon}\right)$ is hypergeometric function.

The graph of the unnormalized PDF (6) in a log-log scale for $\eta \in (\eta_{\min}, 1), \eta_{\min} = 0.001$ and the values of $\varepsilon \in (0, 1)$ is presented in the Fig. 1.

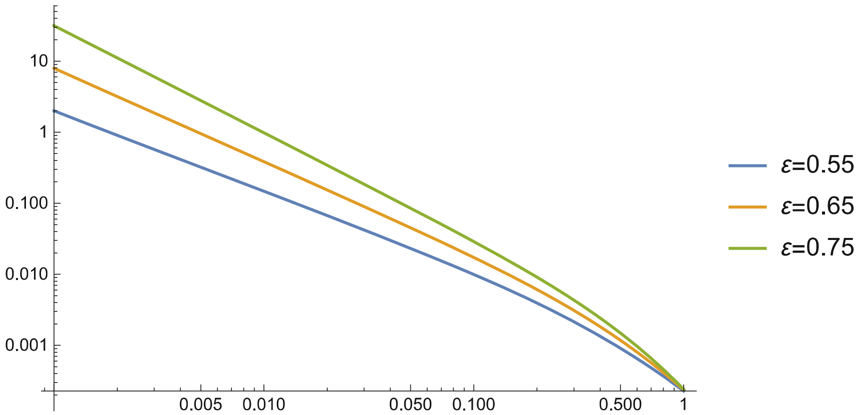

Fig. 1. Log-log plot of the distribution (14) for three different $\varepsilon = 0.55, 0.65, 0.75$.

Distribution (6) is a power-law PDF with indicators α, corresponding to indicators of feedback intensity ε. Increasing of the feedback intensity leads to an increase in the stationary probability $p_s(\eta)$ for all $\eta \in (0.001, 1)$.

The SubC state of Twitter is a chaotic state characterized by the presence of a negligible number of avalanches of microposts and, therefore, $p_s(\eta)$ is not a distribution with heavy tails. Also SubC state is characterized by resistance to small disturbances. In this state, minor chaotic directed flows of microposts are created by all users of the social network, regardless of the size of its pumping by strategically oriented users. The social network functions in the SubC state until S_t reaches a certain critical value S_c. In this state, the streamlining process has almost no effect on the self-consistent behavior of the network, i.e. the feedback intensity indicator $\varepsilon = 1$, and the fluctuation intensity of each of the degrees of freedom are comparable (let's take $I_\eta = I_h = I_S = I$).

Therefore, the SubC state of Twitter is described by the following Langevin equation:

$$\tau_\eta \dot{\eta}_t = -\eta_t + S_0 \eta_t \mu_\eta + \sqrt{I\left(1+\mu_\eta^3\right)}\,\xi_t. \tag{8}$$

where $\mu_\eta \equiv 1+\eta_t^2$.

Assuming that the process (8) is on the interval $(0,1)$, the solution of the corresponding stationary Fokker-Planck equation is a PDF of the following form:

$$p_s(\eta) \propto I^{-1}\left(1+\mu_\eta^3\right)^{-1} \exp\left[I^{-1}\int_0^\eta \frac{S_0 \eta' \mu_{\eta'}^2 - \eta'}{1+\mu_{\eta'}^3} d\eta'\right]. \tag{9}$$

The integral in the distribution (9) has the following form:

$$\left\{\frac{1}{12I}\left[2\sqrt{3}(S_0-1)\tan^{-1}\left(\frac{2\eta'^2+1}{\sqrt{3}}\right) - (S_0+1)\ln\frac{(2+\eta'^2)^2}{\eta'^4+\eta'^2+1}\right]\right\}\Bigg|_0^\eta. \tag{10}$$

Distribution graph (9) is presented in the Fig. 2 in a log-log scale.

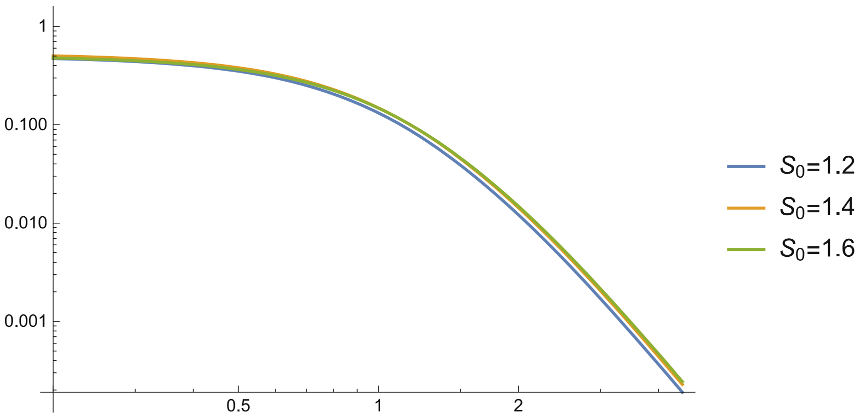

Fig. 2. Log-log plot of the distribution (17) for three different $S_0 = 1.2, 1.4, 1.6$

It is obvious that presented in the Fig. 2 PDF is not a power-law PDF and, therefore, such a distribution is not a distribution with heavy tails. In addition, an increase in the network pumping S_0 leads to an increase, although not significant, in the frequency of appearance of relatively large numbers of microposts.

Thus, it is reasonable to assume that a further increase in the number of SOUs to a certain critical value S_c, accompanied by a significant increase in the intensity of stochastic interactions between them ($I_S \gg I_\eta, I_h$) will lead the network to self-organization in a critical state.

If $S > S_c$, then chaos changes in order, and instead of insignificant chaotic directed flows of microposts, a dedicated directional flow (avalanche) of microposts appears in the network. The distribution of the number of microposts which is characterized to the SupC state of Twitter is presented in the Fig. 3 in a log-log scale. PDFs are presented (see distribution (6)) for $\varepsilon = 0.75$ and various values of the network pumping parameter.

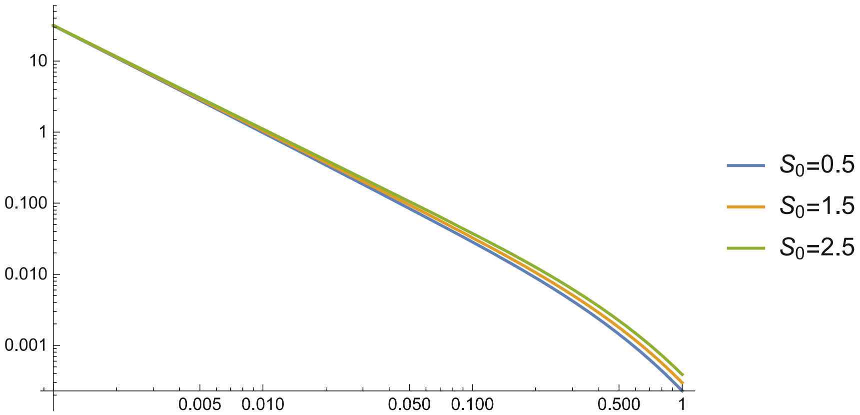

Fig. 3. Log-log plot of the distribution (14) for three different $S_0 = 0.5, 1.5, 2.5$.

The distributions shown in this figure correspond to the distributions with heavy tails. Moreover, the weighting of the distribution tails is due to the increased pumping of the network by the strategically oriented users. If Twitter is in the SupC state, then the number of SOUs, and, accordingly, the microposts avalanche sizes continue to grow.

4 Discussion

The proposed three-parameter scheme of Twitter self-organization (see Eq. (2)) in the adiabatic approximation (see Eq. (3)) describe not only the functioning of Twitter in the SubC, the SOC, and the SupC states, but also the conditions for the transition from one critical state to another. The latter clearly demonstrates by the Table 1.

Table 1. Critical states of Twitter.

State	I_i	ε	S_0
SubC	$I_\eta = I_h = I_S$	1	>0
SOC	$I_S \gg I_\eta, I_h$	$(0, 1)$	0
SupC	$I_S \gg I_\eta, I_h$	$(0, 1)$	>0

SubC states of Twitter is a typical social network state. Indeed, the network consists of a large number of users $\mathcal{N}$, each of which follows its own strategy, not related to the strategies of other network users. In a certain sense, the network has the classic Brownian movement of microposts, the distribution of which is characterized by the absence of heavy tails. The intensity of fluctuations of each of the degrees of freedom are commensurate, and the indicator of the intensity of feedback is equal to 1. The network pumping by SOUs does not change the network functioning mode qualitatively.

5 Conclusion

In conclusion, we formulate important questions, the answers to which cannot be gotten in the analysis of the model (see Eq. (3)) of Twitter self-organization in the adiabatic approximation, and we also indicate the possible ways to find the solution.

When discussing Twitter's self-organization mechanisms in a critical state (see the Sect. 2), it was indicated the need to build a hierarchical structure by strategically oriented users as a necessary condition for the formation of microposts avalanches relevant to some Tweet Ids. There are two scenarios for building such hierarchical structure.

The first simplest scenario assumes that strategically oriented users are built into a hierarchical structure in which some strategically oriented user sends microposts to its subscribers following the same strategy. These subscribers send relevant microposts to their subscribers, etc. Accidental network users, subscribers of strategically oriented users, are unwittingly come into this structure. When a certain critical number of strategically oriented users is reached, an avalanche of microposts relevant to some theme.

The second scenario involves building a hierarchical network structure as a result of the detection of communities and their influential people by strategically oriented users, and, next, the construction of the hierarchy of strategically oriented users and influential people with their subscribers.

The answer to the question which of the scenarios is more effective can be obtained only as a result of a detailed analysis of Twitter data, which is not confined to the analysis of the corresponding time series of microposts. In any case, the coordinated action of strategically oriented users as a necessary condition for self-organization in a critical state requires the construction of a hierarchical network structure. Only agreed acting users of the network can form an avalanche of microposts, either in the first scenario or in the second one. In contrast, uncoordinated user actions lead to the chaotic SubC state of Twitter.

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