A Fast Algorithm for Polynomial E-Pulse Synthesis

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Abstract— **In this paper, the E-pulse (extinction pulse) method known as aspect-independent ultra-wideband radar target discrimination technique is discussed. An alternative synthesis algorithm for the subsection polynomial E-pulse is introduced. The algorithm consists in building a skeleton E-pulse, its further extending and series of integration which all could be performed over the coefficients of basic functions. Not only the proposed algorithm performs up to a thousand times faster than direct matrix solution but it obtains the polynomial coefficients of the E-pulse sections avoiding the solution of a linear problem associated with ill-conditioned sparse matrix. It is proven that E-pulse signals synthesized by means of the fast algorithm and the direct one are exactly the same. To exposure the features of the E-pulse technique, two targets discriminating scheme has been simulated.**

Index Terms—**ultra-wideband radar, natural resonances, extinction pulse, E-pulse, radar target discrimination**

I. Introduction

In recent years, an approach to radar target discrimination based on digital processing time-domain responses of conducting targets to ultra-wideband incident waveforms has stimulated a growing interest. Since the singularity expansion method (SEM) theoretically reveals [1] the response of some conducting target in the late-time part of the response can be modeled as a finite sum of dumped sinusoids in the time domain [2]:

$$r(t) = \sum_{n=1}^{N} A_n e^{\sigma_n t} \cos(\omega_n t + \varphi_n), \quad t > T_L, \tag{1}$$

where $s_n = \sigma_n + j\omega_n$, is the aspect-independent natural complex frequency of the n-th target mode, and A_n, and φ_n, are the aspect-dependent amplitude and phase of the nth target mode, respectively. T_L denotes the beginning of the late-time part, and the number of modes in the response N is determined by the number of significant natural frequencies in the waveform returned from the excited target.

According to E. J. Rothwell, the author of the original technique [3], the E-pulse is defined as a finite duration waveform $e(t)$ which annihilates a preselected number of the natural resonances of a particular target. Mathematically, this means that the convolution product $c(t)$ of the target response $r(t)$ and the E-pulse matched to this target will vanish in the late-time part, starting immediately at the moment T_E where E-pulse ends, in other words, defined as the E-pulse duration:

$$c(t) = e(t) * r(t) = \int_0^{T_E} e(\tau) r(t-\tau) d\tau = 0, \, t > T_E. \tag{2}$$

Target discrimination approach based on E-pulse method proposes the energy estimation of the late-time part of the convolution product (2), which tends to be zero if the E-pulse is matched to the response of interest but it is essentially non-zero if it is not true. In the presence of noise, the problem of multi-target discrimination turns into the search for the convolution with minimum late-time energy among the output of E-pulse filter banks, that was known as the E-pulse target recognition scheme introduced in [4] and developed in [5, 8]. In order to make this comparison more robust from series of convolution results proceeded for several expected targets with specified natural resonances, the E-pulse discrimination number (*EDN*) was developed in [4, 5].

II. E-Pulse Synthesis

Following to [4], it was assumed that the E-pulse can be analytically represented as a weighted sum of M convenient basis functions $f_m(t; \boldsymbol{\theta}_m)$:

$$e(t) = \sum_{m=1}^{M} \alpha_m f_m(t; \boldsymbol{\theta}_m), \tag{3}$$

where α_m are the basis-function amplitudes, usually chosen to be independent from $\boldsymbol{\theta}_m$ and t, $\boldsymbol{\theta}_m$ – parameter vector of m-th basis function.

The most useful basis set, due to its natural simplicity, is one that is composed of subsectional polynomial time-bounded functions [6]:

$$e(t) = \sum_{m=1}^{M} \sum_{q=0}^{Q} k_{mq} t^q h_m(t), \tag{4}$$

where k_{mq} are unknown coefficients of polynomial basis function, Q is the E-pulse order (the maximum polynomial order), and $h_m(t) = u(t - m\Delta + \Delta) - u(t - m\Delta)$ is limiting functions composed of unit step functions with the equal sections of width Δ, given by $\Delta = T_E/M$

In order to synthesize the subsectional polynomial E-pulse (4), one has to determine all the coefficients k_{mq} which satisfy the necessary condition (2), formulated in the frequency domain in terms of Laplace transform:

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$$E(p)=\sum_{m=1}^{M}\sum_{q=0}^{Q}k_{mq}\left\{\int_{(m-1)\Delta}^{m\Delta}t^{q}\exp(-pt)dt\right\}=0. \quad (5)$$

An additional requirement for the subsectional polynomial E-pulse that makes its spectrum more compact is its continuous and smoothness at the edges of all sections that is meant as continuity of its derivatives up to Q–1 order. It is important to notice that continuous and smooth E-pulse has a strict relation between the number of poles N which such E-pulse is built for, the number of sections M and the polynomial order Q:

$$M=N+Q+1. \quad (6)$$

The straightforward method of solving this problem [4, 6] assumes that the solution can be obtained by constructing and solving the system of linear algebraic equations in the form of $\mathbf{Ax}=\mathbf{b}$, where $\mathbf{A}$ and $\mathbf{b}$ are respectively the system matrix and right-hand side vector, and $\mathbf{x}$ is a vector of solutions made up of all the elements of the vectors $\boldsymbol{\theta}_{\boldsymbol{m}}$ i.e. polynomial coefficients k_{mq}. It was shown in [7] that the matrix $\mathbf{A}$ is a block matrix with two-level hierarchy and has a complicated structure. It consists of several groups of equations: first of all, equations, related to the necessary conditions for the existence of the E-pulse, the conditions of continuity and smoothness of the E-pulse at section edges and conditions of linearity that define the total energy of E-pulse. Also the matrix can be supplemented by equations involving synthesis of natural or forced E-pulse and E-pulse with zero area (so called DC E-pulse) [3].

III. Fast E-Pulse Synthesis

The complex structure of the system matrix $\mathbf{A}$ in the matrix equation demands significant efforts in the algorithmic formulation, as well as the evaluation of its elements would be computationally demanding [6]. In addition, with increasing the order of polynomial basis functions forming sections and the number of sections, the system of linear equations tends to become ill-conditioned so long as the decimal order of the condition numbers depends of the number of poles pairs and polynomial order almost-linear. In addition, the matrix is sparse because of the growing number of all-zero submatrices in the main structure of the matrix $\mathbf{A}$ and a lot of null elements in the most of other submatrices.

To avoid solving the linear problem related to the polynomial coefficients of the E-pulse, we introduce the method for constructing polynomial functions for each section of the E-pulse with an alternative approach, which allows obtaining the same signal without solving a set of equations at all. The proposed method could be performed in three steps: synthesis of skeleton E-pulse, its extending and the series of integrations.

In the first step one should synthesized E-pulse, which we call *skeleton*, consisting of N delta functions delayed by time multiple by section width Δ:

$$e_{SK}(t)=\sum_{n=0}^{N}b_{n}\delta(t-n\Delta). \quad (7)$$

Although the coefficients $\{b_n\}$ have to satisfy the necessary condition (2) there is no need to solve corresponding equations. With the known set of poles $\{s_n\}$, the E-pulse (7) ought to be factorized as a convolution product, denoted by capital “C”:

$$e_{SK}(t)=\mathop{\mathrm{C}}_{q=1}^{N}\left\{\delta(t)-\exp(s_{n}\Delta)\delta(t-\Delta)\right\}, \quad (8)$$

and the obtaining of the set of $\{b_n\}$ turns into simple algebraic calculation.

In the second step, the E-pulse has to be extended in the way that will guarantee its finite duration while the further transition from delta-functions to polynomial sections is done. The extension is performed by the $Q+1$ convolutions with the following sequence $\psi(t)$ consisted of delta functions:

$$\psi(t)=\delta(t)-\delta(t-\Delta). \quad (9)$$

Convolution commutative property allows defining the extended skeleton E-pulse using the expression that results in the sum of delta-functions with weights $\{v_n\}$:

$$e_{EXT}(t)=e_{K}(t)*\mathop{\mathrm{C}}_{q=1}^{Q+1}\psi(t)=\sum_{n=0}^{N+Q+1}v_{n}\delta(t-n\Delta). \quad (10)$$

The desired waveform $e(t)$ could be obtained by a series of Q+1 integrations of $e_{EXT}(t)$ that transforms it from delta-function basis up to the polynomial of order Q:

$$e(t)=\int_{0}^{t}\int_{0}^{\tau_1}\dots\int_{0}^{\tau_Q}e_{EXT}(\tau_{Q+1})d\tau_{Q+1}d\tau_{Q}\dots d\tau_{2}d\tau_{1}. \quad (11)$$

Immediate integrations could be replaced by the iterative algorithm with Q+1 steps. The steps are numbered by w going up from 0 to Q. The result of each intermediate step w is a subsection polynomial time function, described by the formula

$$e_{w}(t)=\sum_{m=1}^{M}\sum_{q=0}^{w}k_{w}^{q}[m]\cdot t^{q}\cdot h_{m}(t), \quad (12)$$

where $k_{w}^{q}[m]$ is the coefficient of the q-th degree term of t in the m-th section obtained at the w-th step. Step zero ($w=0$) transforming the extended skeleton E-pulse into the rectangular should be made in the following way:

$$k_{0}^{0}[m]=\sum_{n=0}^{m-1}v_{n}. \quad (13)$$

The coefficients of the polynomial sections of signals $e_w(t)$ obtained at further steps, i.e. for $w \geq 1$, can be calculated by the following formula:

$$k_w^q[m] = \begin{cases} q^{-1} \cdot k_{w-1}^{q-1}[m], & \textit{if } 1 \leq q \leq w \\ 0, & \textit{if } (q=0) \textit{ and } (m=1) \\ k_w^q[m-1] + \sum_{s=1}^{w} \left(k_w^s[m-1] - k_w^s[m]\right) \cdot \left[(m-1)\Delta\right]^s, & \textit{if } (q=0) \textit{ and } (m \geq 2) \\ 0, & \textit{if } (q<0) \textit{ or } (q>w) \textit{ or } (m \leq 0) \textit{ or } (m>M) \end{cases} \quad (14)$$

The final step ($w = Q$) will bring one a set of coefficients referred to the desired polynomial E-pulse of order Q owning the property of continuity and Q–1 order smoothness at any point within its duration.

We should emphasize the fact that the proposed fast method allows to obtain the E-pulse equal to the signal obtained using the direct algorithm in [6, 7]. Due to the linearity property inherited by E-pulse as a filter, both resulting E-pulses would be the same waveform, since the coefficients of the sections polynomials are considered identical provided the normalization of their energies is made.

IV. Appication for Radar Target Discrimination

The simulation made in this work tests the proposed fast algorythm by the well-known discrimination problem. The one consists of two simplified aircraft scale models – McDonnel Douglas F-18 and Boeing 707 (B-707) which should be distinguished by employing the E-pulse technique, based on annigilation of known target dominant natural frequencies extracted from the late-time portion of the each target response [3,4].

As an example in this paper the set of E-pulses were synthesized for the radar response of F-18 aircraft model with a known set of poles. E-pulses are composed from polynomials of various order Q using the techniques of the fast algorithm and normalized energy-wise. Those continuous and smooth E-pulse waveforms with finite duration are shown in fig. 1.

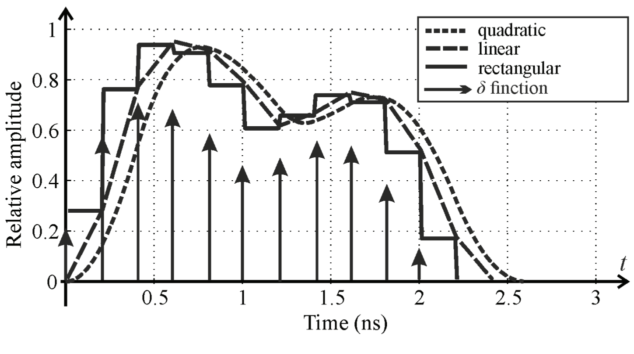

Figure 1. E-pulses constructed for F-18 response

It should be noted that the computation speed by using the fast algorithm is significantly higher. Thus, the synthesis of a E-pulse of 7th order polynomials for the 10-pole F-18 scaled target by using the straight algorithm has taken as much as 170 ms on 2.4 GHz PC while the fast algorithm has required 280 µs only.

Conclusion

Proposed in the present work algorithm for extinction pulse (E-pulse) synthesis can be applied for enhancing the system solving the problem of radar target identification based on invariant features contained in the impulse response of the observed targets. This algorithm provides one with the same waveform as well-known direct solution based on linear problem formulation but has two main benefits. It avoids solving an ill-conditioned system of linear equation with sparse matrix and the computational time required for the E-pulse synthesis is three orders of magnitude shorter than for the direct algorithm.

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