Research on the Model of Population Groups Human Capital Dynamics

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Abstract. This article describes the dynamic optimization model with human capital as a group educational characteristic (along with these groups population) and as the main factor of their production. The main feature of this model is inequality in qualification which leads towards the run for the middle as unlinear dynamics of educational effectiveness for different groups. The research of the simulation model in one specific regime allowed to describe two different scenarios. They include the development of the groups and run for the middle dynamics. These results allow stating conceptual usability of the model for real society dynamics description.

Keywords: Education · Human capital · Group human capital
Mathematical modeling · Optimization

1 Introduction

The modern post-industrial economy is the economy of knowledge and information. At the same time, it is the economy of the human factor. The key role of man in the modern economy can be attributed to him being a carrier of knowledge (in particular, the implicit knowledge) and ensures the service sector functioning. Thus, it is necessary to develop economic models, in which not physical capital, but human capital would play this key role.

The concept of "Human capital" means a combination of production factors, which are inalienable from man. This concept has been evolved for a long time, so various researchers use this term in a completely different sense. As a result, the term is used along with the concepts of "labor resources", "human resources", "human potential", "social capital", "intellectual capital" etc., often as synonyms. Among the various human capital conceptualizations, it is essential to distinguish three types of this term interpretations depending on the modeled subject (agent):

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- the individual human capital (see, for example, publications [1,3]);
- the group, community, strata human capital (see, for example, publications [4, 6]);
- the society, national economy human capital (see, for example, publications [1–4]).

It should be noted that the first interpretation is purely microeconomic, and the third one combines human capital with human potential. In this study, the second interpretation will be applied. Consequently, human capital is interpreted as the competence of the group, its ability of useful products creation. This interpretation tends to be applied to the firm's human capital (both in microeconomic and macroeconomic models). In this study, however, we propose to apply this interpretation to the simplest macroeconomic model in which households are the only economic agents. More specifically, these are large groups called clans. Each subject has exactly 2 characteristics: quantity and qualification.

The dynamic model is to describe their changes. Each clan at any given time distributes its members between production, education, and service in accordance with the chosen strategy. On the one hand, the description of society as the collection of large clans (families, communities) can be applied to human civilization in the early stages of development (starting from the primitive community and finishing with the Renaissance workshop). The completely different interpretation of this approach (like strata whose members collective behavior is regulated by general social norms) can be applied to modern society.

Let us briefly formulate this model basic premises:

1. physical capital (created goods) is a "fast" factor, fully used for consumption and training at the time of production;
2. human resources (the component of human capital associated with health) is a "slow" factor of production, which transfer in the next period occurs through the demographic function;
3. qualification (not health-related human capital component) is a "slow" factor of production which transfer in the next period occurs through the educational function;
4. economic entities are considered as autarky groups (clans) that produce and consume a homogeneous, non-stored product;
5. clans are not interested in trade because of material goods homogeneous and non-stored nature;
6. each clan's rational interest is to minimize the probability of extinction;
7. natural resources are not limited quantitatively;
8. clans do not come into conflict with each other for natural or labor force resources.

The last premises are not fundamental and can be eliminated with further model development. The model with the clans autarkic resources corresponds to the extensive social development stages, with the development of new territories, resources etc. The product autarky corresponds to communities with different cultural and economic habits. This difference makes useless one groups products for the other groups. Also, groups might be large enough for their internal labor division making external labor division benefit unimportant. Thus, with the further model development, it will be reasonable to introduce the struggle for resources concept to extend model applicability. But it is unnecessary in the case of the trade since the model is knowingly oriented toward the description of large groups (including aggregated ones). However, the clans in the model are not autarky in the general cultural level of the scientific development. This dependency will be described in the qualification section of this article.

2 Variables

The model includes 7 types of variables:

1. N[i](t) the clan i population in year t (variable type: stock; dimension: man-hour above 0)
2. q[i](t) the clan i qualification in year t (stock; qualification units above 0)
3. L[i](t) the clan i labor force in year t (flow; man-hour above 0)
4. Y[i](t) the clan i production in year t (flow; money above 0)
5. C[i](t) the clan i consumption in year t (flow; money above 0)
6. E[i](t) the clan i education spending in year t (flow; money above 0)
7. q[a](t) society average qualification in year t (flow; qualification units above 0)

Neither of variables are negative nor discrete. To ensure non-discretion, the size of the clan is measured not in man, but in man-hours (non-integer values are considered as the partial capacity for some members of the clan). The stock variables are linked by dynamic equations, while flow variables by balance equations. The article also uses a large number of calibration constants. Some of them simultaneously carry the equation dimension. For the convenience of in-text orientation, we briefly list them, while a detailed description will be given directly together with the equations.

1. b and c0 are constants that reflect the positive and negative changes in the clan size (Eq. 1)
2. h and a are constants that carry the dimension and scale, and the efficiency of labor quantity substitution by its qualification in the production function (Eq. 2)
3. k1 and k2 are constants that reflect the time consumed by created goods consumption in the time balance (Eq. 3)
4. R - the discount factor in the minimized functional (Eq. 13) Used in the derivation of equations or re-designation for the simulation model, the other constants are introduced with appropriate constructions and calculations.

The [i] index reflects the variable belonging to a particular clan, and (t) index corresponds to a specific moment of time. Since the only variable that connects different clans q[a](t) does not have a clan index, the clan's membership index is omitted in all further formulas and calculations.

3 Functions

3.1 Demographic Function

The population dynamics of clans in the model is determined by consumption:

$$\frac{d}{dt}N(t) = N(t) * b * \frac{(c(t) - \frac{c0}{b})}{c(t)} \tag{1}$$

Here, c(t) = C(t)/N(t), i.e. consumption per capita. The constant c0 being strictly above 0 (money per capita) represents the required minimum level of consumption, without which the clan will extinct (so it can be interpreted as mortality and simultaneously as a level of consumption that makes it possible to compensate mortality). The calibration constant b being strictly above 0 (dimensionless) represents the consumption influence at the clan population (can be associated with natural birth rate). Consumption c(t) is considered as an investment in clan members health, including children. If b * c(t) is strictly below c0, the clan's population decreases, otherwise it increases. If b is big, the growth is rapid, and the extinction is slow. If b is small, the growth is slow, and the level of consumption which is necessary for any increment is much bigger. The nature of the clan sizes dependence from consumption is determined by the probability of death or withdrawal from the clan, as well as the birth of new clan members as a kinetic equation for a set of independent events. His detailed conclusion goes beyond this articles limits, and it does not have a fundamental scientific novelty. Essential limitations of the model are: the absence of any population pyramid (which makes it impossible to include external shocks like a drought in model); and the assumption that there is no possible mobility between the clans (caste system), or such movement occurs in a balanced manner (the same number of people leave the clan as join).

3.2 Production Function

The volume of each clans production in the model depends on two factors: the number of labor forces L and their skill level q. Since the product is considered homogeneous, we can use the simplest version (linearly homogeneous) of the classical Cobb-Douglas production function with constant returns to scale, whose properties are common knowledge [7]). The nonlinearity of the dependence reflects a negative effect: with the increase in the quantity, it is increasingly difficult to coordinate the work of people.

$$Y(t) = h * q(t)^a * L(t)^{1-a} \tag{2}$$

According to this, the (dimensionless) constant a (0 strictly below a strictly below 1) represents the efficiency of resources substitution (elasticity coefficient). It is intended for model calibration. Its value is considered externally specified. Finally, the constant h is used to restore the dimension (from exponent multiplication of man-hours and qualification units to money hence, reflects the estimation of these production factors value).

3.3 Qualification Function

The qualification dynamics is the most difficult part of the model, for the reason that the qualification is not only transmitted between periods but also connecting clans with each other through an average qualification:

$$q[a](t) = \frac{\sum_{i=1}^{x}(q[i](t) * N[i](t))}{\sum_{i=1}^{x}(N[i](t))} \tag{3}$$

The ratio between clan qualification q(t) and average qualification q[a](t) is a key tool for setting up the model. Therefore, it requires additional analysis. In general, this problem was called "race for the middle": how the strategy chosen by each subject (clan) affects the other clan's position. The similar problem was considered earlier by some researchers (e.g., Shananin [5]), but applied to technology and physical capital, and usually in models with the trade.

The qualification dynamics is determined by education investments per capita E(t)/N(t) multiplied by some efficiency coefficient. However, in the model, this coefficient is considered to be not constant, but depending on the ratio between clan qualification q(t) and the average qualification q[a](t).

$$\frac{d}{dt}q(t) = q1(\frac{q(t)}{q[a](t)}) * \frac{E(t)}{N(t)} \tag{4}$$

The choice of this dependence nature determines the race for the middle dynamics, consequently, it should be specially researched. Let us consider extreme cases. The q(t)/q[a](t) ratio is strictly above zero since both variables in it are strictly above zero. For q(t)/q[a](t) tending to 0, the clans development stays heavily behind from the general level of society culture. For q(t)/q[a](t) tending to infinity, the clan is heavily ahead of the general level of society culture. Depending on whether the development gap and advancement being considered factors that increase or decrease the effectiveness of education, the race for the middle takes different trajectories.

First of all, we should normalize q1:

$$q1(\frac{q(t)}{q[a](t)}) = y * q2(\frac{q(t)}{q[a](t)}) \tag{5}$$

Here y being strictly above 0 is the calibration constant of investments in qualification efficiency (with the dimension: reverse investment per capita). Then q2 is a dimensionless function 0 strictly below q2 strictly below 1 that determines the race after the middle trajectory, i.e. relative effectiveness of education.

3.4 Race for the Middle

In this study, we simulate two effects which affect it:

1. The observation and repetition effect. The stronger the clan falls behind in development from the average level, the easier it is for him to improve his skills, adopting the most widely known practices.

2. The competencies concentration effect. Clans that form the intellectual elite are the easiest to come up with fundamentally new technologies, scientific discoveries, etc., which makes them innovation drivers.

These oppositely acting effects can be described by a single function as two exponentials sum. One of them reaches a maximum which is equal to 1, while it approaches 0, and the other while it tends to infinity. On the contrary, clans with "mediocre" qualification indicators growth with the greatest difficulty. Obviously, such qualification model, which takes into account the heterogeneity of the society qualification distribution, works correctly only with the number of clans x being above 2.

$$q2(\frac{q(t)}{q[a](t)}) = g3^{g2*\frac{q(t)}{q[a](t)}} + g1*(1 - g3^{\frac{q(t)}{q[a](t)}}) \tag{6}$$

Here g1 being strictly above 0, g2 being strictly above 0 and g3 being strictly above 0 are calibration constants, with g3 = g3 (g1, g2) in such a way that min (q2) = q2(1): g3 = (g1/g2)exponent(1/(g2-1)); min(q2) = q2(1) = g3exponent(g2) + g1exponent(1-g3); lim(q2, q(t)/q[a](t) tends to infinity) = g1. This function is continuous and smooth. It is integrable and differentiable on any segment, including the q2(1) point. The race after the middle can be visually represented (Fig. 1) on the graph q2(q(t)/q[a](t)) because any clan can be uniquely identified one of the points on the function trajectory.

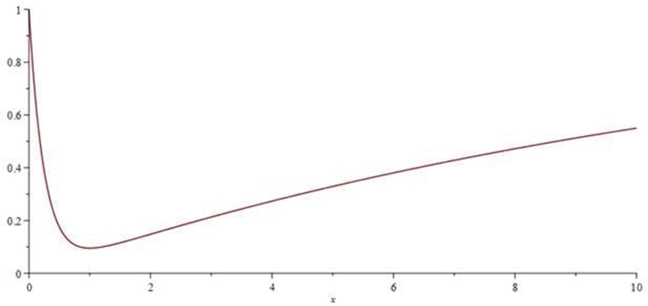

Fig. 1. Dependence between q2 (y-axis) and q(t)/q[a](t) (x-axis)

Substituting q2 in the initial qualification function, we get:

$$\frac{d}{dt}q(t) = y * g3^{g2*\frac{q(t)}{q[a](t)}} + g1*(1 - g3^{\frac{q(t)}{q[a](t)}}) * \frac{E(t)}{N(t)} \tag{7}$$

This dependence creates the problem of the clans expectations: changing one of the clans policies leads to unexpected changes in the average qualification. The clan may lose the advantage because of the rapprochement with the average qualification, which is possible both due to changes in the qualifications of other clans and due to their quantity changes. The task of analyzing expectations can be examined in detail with Game theory, but we confine ourselves to the simplest assumption: each clan extrapolates the other clans existing policies for the future (which is as natural for people as primitive). This approach is based on the assumption on the existence of the turnpike effect, which is ordinary for economic systems [8]. This assumption is performed for some model regimes provided by relatively realistic initial choices of expectations in the model. As will be shown below in the simulation model study, the turnpike effect exists. However, unrealistic (for example, random) initial expectations choice can introduce a noticeable imbalance, causing high mortality in the first few periods. But even after that, the model reaches the main path.

4 The Balance of Time and Products

The total clan population N(t) is distributed between the labor force L(t) and the services: the consumption services (C) and the educational services (E).

$$L(t) + k1 * C(t) + k2 * E(t) \leq N(t) \tag{8}$$

The calibration constants k1strictly being above 0 (with the dimension man-hour/money) and k2 being strictly above 0 (man-hour/qualification unit) represent the number of services required to use the created unit of consumer goods (C) and educational goods (E). Created by production factors product is distributed between consumption and training.

$$0 = Y(t) - C(t) - E(t) \tag{9}$$

The balance of the time is in the form of inequality since formal logic does not prevent the clan from leaving part of the people who are unemployed, not engaged in any useful labor. However, it is not difficult to show that under existing model design it is always more advantageous for the clan to redistribute unemployed between creating educational goods and providing educational services.

The qualification growth cannot be disadvantageous to the clan since qualification is a non-decreasing stock (in contrast to the demographic function that allows the decrease, if there are not enough produced goods).

The bime balance (in equality form) and the products balance are technically reducible to one equation through the production function. However, a separate entry is more convenient for an understanding of models basic dependencies.

5 Functional and Optimization

For the case of optimization, we should add the control to the model.

$$L(t) = u(t) * N(t) \tag{10}$$

Also we need to renormalize E, y, and $k2$, transferring the coefficient to the products balance equation (here and below the coefficient h is taken as 1 and omitted, since any other of its values can be compensated by remaining coefficients renormalization):

$$N(t) = L(t) + k1 * C(t) + E(t) \tag{11}$$

$$Y(t) = C(t) + p * E(t) \tag{12}$$

The given control u value (0 below u(t) below 1) uniquely determines remaining variables dynamics, taking into account the above equations that give dq(t)/dt and dN(t)/dt, but solutions with nonnegative variables N, E and C exist not for all controls. Therefore, there is an optimal solution problem.

$$J(t) = c0 * \int_t^{\infty} \frac{e^{-Rs}}{c(s)} ds, t > 0 \tag{13}$$

The solution optimality principle is determined through the functional minimized by the clan, which is the discounted inverse relative consumption per capita c(t)/c0. The calibration factor R (between 0 and 1) reflects the consumption significance for the clans future generations. In real calculations, the infinite upper limit is replaced by some maximum time T. This functional optimization corresponds to a decrease in the clans death probability. More precisely, for some calibration constants values, minimizing this functional, the clan maximizes the duration of its life (until the clan members quantity reaches 0), while in others it makes a safety margin that can be needed when the external conditions worsen. At any time t, each clan selects the control u(t), minimizing the value of J(t) provided that for any s above t the u(s) other clans remain values are constant and equal to u(t), i.e. other clans continue the current moment economic policy. Therefore it can be shown that:

$$E(t) = \frac{(1 - u(t)) * N(t) - k1 * Y(t)}{1 - k1 * p} \tag{14}$$

$$C(t) = \frac{Y(t) - p * (1 - u(t) * N(t))}{1 - k1 * p} \tag{15}$$

Since for u, (0 below u(t) below 1) is valid (u(t) below u(t)exponent(a)), so the requirement p * E(t) below Y(t) leads to the control lower limit. And the upper control limit follows from the requirement E(t) above 0 (or C(t) above 0).

$$u(t) > u_{\min}(t) = (1 + \frac{(\frac{q(t)}{N(t)})^{1-a}}{p})^{-\frac{1}{a}} \tag{16}$$

$$u(t) \leq u_{\max}(t) = (1 + k1 * (\frac{q(t)}{N(t)})^{1-a})^{-1} \tag{17}$$

6 Simulation Model

6.1 Simulation Model Construction

The model uses many standard functions, but the qualification function requires non-standard solutions. So we constructed a dynamic simulation model for main model primary research and actual processes describing usage verification. It is discrete (time becomes a discrete variable, so integration and differentiation are replaced by the corresponding summation operations).

Since J(t) is a complex nonlinear function, the minimization is performed by searching for u(t), umin(t), umax(t), over a uniform net with a given nodes number.

The model dynamics was researched for various constants values and starting conditions. This research's details are omitted in this publication. At this research stage, it is essential the very existence of a model regime that allows clans population and qualification realistic dynamics. So for further analysis, we chose a particular model regime with the following coefficients values:

a = 0,4; h = 1; b = 0.01; c0 = 0,01; g1 = 1; g2 = 100; g3 = 0,95455; k1 = 0,1; k2 = 1 means p = 1; y = 100; R = 0,01;

In this regime, we considered two typical scenarios. They describe the race for the average with an uneven society qualifications distribution. Both scenarios assume the presence of two clans: Clan 1 includes an elite with high qualification and small population, and Clan 2 includes a common people with a low qualification and a big population. The difference between the scenarios is in this unevenness scale.

6.2 Simulation Model First Scenario

The race for the middle dynamics In the first scenario, the clans characteristics are:

q[1](0) = 50; q[2](0) = 1000; n[1](0) = 99; n[2](0) = 1;

This combination of factors allows Clan 1 to gradually build up its qualification (Fig. 3) while losing its population (Fig. 2). Meanwhile, the second clan rapidly increase both its population and qualification.

If we describe the race for the middle on the graph, where the X-axis is the q[i](t)/q[a](t) ratio, and q2(q[i]/q[a]) is located along the Y-axis, we will get the following dynamics (Fig. 4):

Obviously, Clan 1 is initially very close to the Y-axis minimum, i.e. its qualification investment is ineffective (which leads to its slow population reduction), while Clan 2 is in a moderately effective investment zone. However, no matter how much Clan 2 invests in qualification, the effectiveness of its investments gradually decreases, as its increase in population also increases the average qualification q[a]. On the contrary, Clan 1 is gradually staying behind the average and therefore shifting in the high investment efficiency range direction. This allows him after a while to begin its population growth.

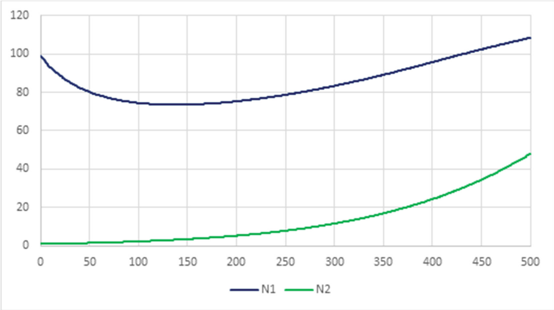

Fig. 2. Clan 1 and Clan 2 population dynamics in the first (normal) scenario

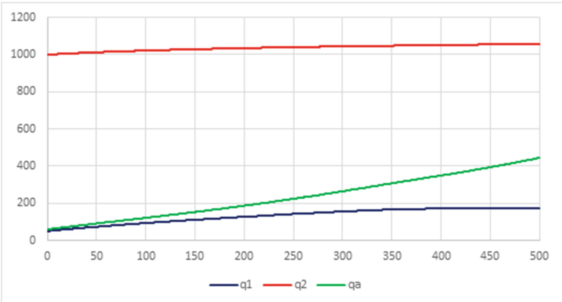

Fig. 3. Clan 1 and Clan 2 qualification and average qualification dynamics in the first (normal) scenario

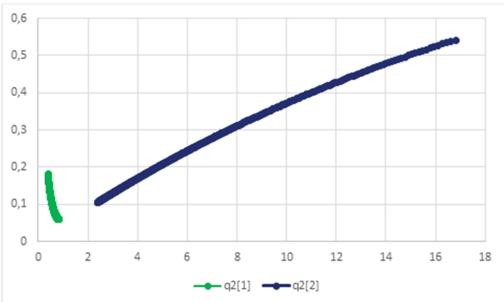

Fig. 4. Clan 1 and Clan 2 relative education effectiveness in the first (normal) scenario

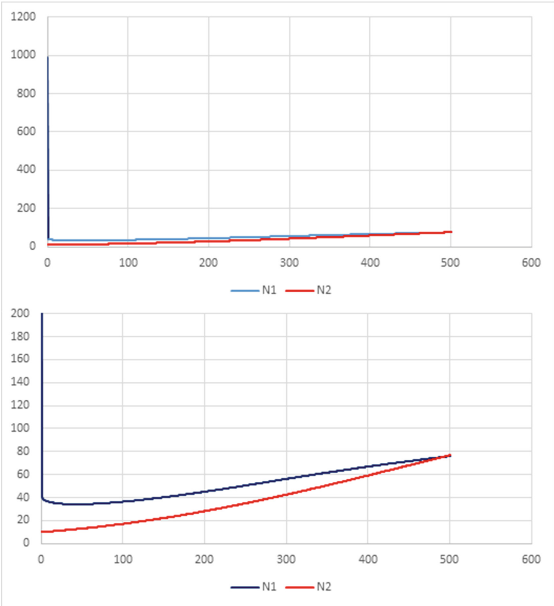

Fig. 5. Clan 1 and Clan 2 population dynamics in the second (catastrophic) scenario in different scales

6.3 Simulation Model Second Scenario

In the second scenario, the clans characteristics are:

q[1](0) = 5; q[2](0) = 100; n[1](0) = 990; n[2](0) = 10;

Clan 1 here is in a state where the population requires such a quantity of consumer goods, which can't be provided by the entire clan size due to its low qualification. At this point, the optimal choice is a large part of the population rapid death (Fig. 5) (which can be interpreted, for example, as a civil war for the few remaining consumer goods), while all possible resources are used to improve clans qualification (Fig. 6). The population reduction rate is determined by the b and c0 ratio.

The race for the middle nature is also different (Fig. 7): instead of moving uniformly along the curve, we observe a sudden increase in the average qualification, which immediately places Clan 2 into a low education efficiency zone, and Clan 1 - into a relatively high-efficiency zone (from where it then gradually returns to the low-efficiency zone).

6.4 Scenarios Interrelation

The considered scenarios are interrelated and can transform from one into another. Obviously, the model provides an equalization of clans unrealistic initial population values, and under realistic values allows you to simulate a

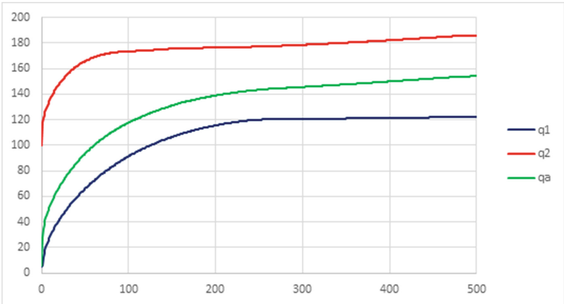

Fig. 6. Clan 1 and Clan 2 qualification and average qualification dynamics in the second (catastrophic) scenario

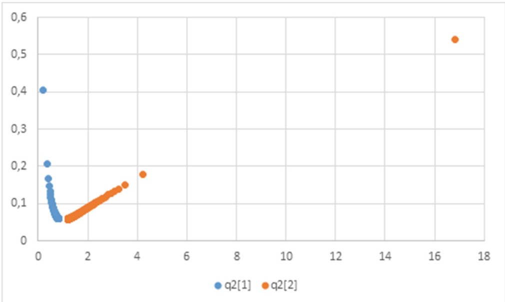

Fig. 7. Clan 1 and Clan 2 relative education efficiency in the second (catastrophic) scenario

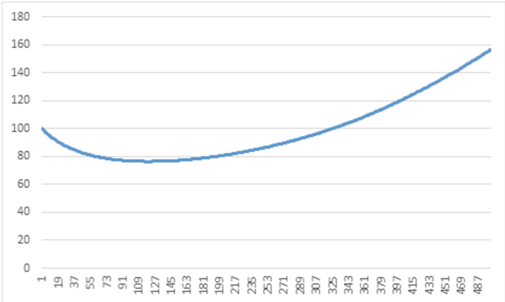

Fig. 8. The first (normal) scenario total population

demographic transition as a result of the gradual population and qualification accumulation in more qualified clans (Fig. 8).

The race for the middle common feature in both scenarios is the qualification gap (q[2]-q[1])/q[a] reduction. In the catastrophic scenario, it happens much faster (Fig. 9).

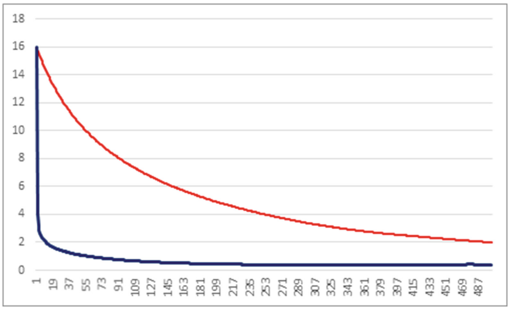

Fig. 9. Qualification gap between Clan1 and Clan 2 in normal (top line) and catastrophic (bottom line) scenarios

Considering other model regimes, cyclic jumps in the clans population are possible as they approach the critical numbers.

7 Conclusion

The article describes a dynamic model of the development of social groups and communities, taking into account their qualification and population. The different group's dynamics are linked through the average qualification indicator. It is shown that there is a model's mode (the combination of the calibration coefficients values) which provides realistic groups dynamic.

On the basis of the series of simulation experiments, we choose two typical scenarios of the groups mutual influence (race for the middle): "normal" and "catastrophic". It is shown that in the normal scenario, the educational effectiveness of the highly qualified group is gradually slowing down, and for the low-qualified group is increasing, while the qualification gap between them is slowly decreasing. In the catastrophic scenario, when a low-qualified group suddenly reduces its population, the effectiveness of its education initially increases, after which it gradually decreases, while the highly qualified clan dynamics remains the same.

The following questions are of the greatest interest in the further model research and development:

1. The coefficients multidimensional metric analysis, their identification and the establishment of their restrictions in accordance with the available social data.
2. The calibration conditions, ensuring realistic population and race for the middle dynamics.
3. The calibration conditions that determine the cyclic amplitude of the model.
4. The conditions of initial clans characteristics, which determine their development in "normal" and "catastrophic" dynamics.
5. The nature of the dynamics of interaction between 3 or more clans.
6. The changes of models dynamics on the assumption of including several clans with the same characteristics.

References

1. Kamenev, I.G.: Problems and tasks of human capital modeling. In: Modeling the Nature and Society Co-evolution: Problems and Experience. To the 100th N.N. Moiseev Academician Birth Anniversary (Moiseyev-100) Works of the All-Russian Scientific Conference, pp. 227–237. GBI FRC CC of RSA, Moscow (2017)
2. Kapelyushnikov, R.I.: The human capital of Russia: evolution and structural features. Bull. Public Opin. Data Anal. Discuss. **4**, 46–54 (2005)
3. Koritsky, A.V.: Human Capital as a Russian Regions Economic Growth Factor. Siberian University of Consumer Cooperatives, Novosibirsk (2010)
4. Korchagin, Y.A.: Human Capital is an Intensive Individual, Economy, Society, and Country Social and Economic Development Factor. NRU HSE, Moscow (2011)
5. Shananin, A.A.: Technological changes on the macroeconomic level – mathematical modeling. Discret. Dyn. Nat. Soc. **6**, 291–304 (2001)
6. Tuguskina, G.N.: Evaluation of enterprises investments effectiveness in the human capital. Pers. Manag. **3**, 73–77 (2009)
7. Cobb, C.W., Douglas, P.H.: A theory of production. Am. Econ. Rev. **18**, 139–165 (1928)
8. McKenzie, L.: Turnpike theory. Econometrica **44**, 841–865 (1976)