Theoretical Study of Self-organized Phase Transitions in Microblogging Social Networks

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Abstract. A simple sociophysical model is proposed to describe the transition between a chaotic and a coherent state of a microblogging social network. The model is based on the equations of evolution of the order parameter, the conjugated field, and the control parameter. The self-consistent evolution of the networks is presented by equations in which the correlation function between the incoming information and the subsequent change of the number of microposts plays the role of the order parameter; the conjugate field is equal to the existing information; and the control parameter is given by the number of strategically oriented users. Analysis of the adiabatic approximation shows that the second-order phase transition, which means following a definite strategy by the network users, occurs when their initial number exceeds a critical value equal to the geometric mean of the total and critical number of users.

1 Introduction

Since the second half of the 20th century, a general trend in the development of science has been the propagation of the ideas and methods of physics into natural and traditional humanities. The methods of physical modeling are often used in sciences such as demography, sociology and linguistics.

In the mid 1990s, there was an interdisciplinary research field known as Econophysics, which sought to apply theories and methods originally developed by physicists to solving problems of economics, usually those including uncertainty or stochastic processes and nonlinear dynamics. The term “econophysics” was coined by H. Eugene Stanley to classify a large number of papers written by physicists on the problems of stock markets and other types of markets.

The interest of physicists in social sciences is not new. Daniel Bernoulli, to give but one example, was the inventor of utility-based preferences. Sociophysics is the study of social phenomena from the physics perspective, often using the human atom, social atom, or human particle perspective (for sociophysics reviews see Refs. [1, 2]). The main objective of this new field of natural sciences consists in the analysis of objectively measurable and formalizable laws that define various social processes.

Social networks have the longest history of research in comparison with other network types. It is noteworthy that it was in a study of social networks that D. Price [3] empirically discovered the power law of distribution of nodes by the number of ties

L. M. Aiello et al. (Eds.): COMPLEX NETWORKS 2018, SCI 813, pp. 236–245, 2019.
https://doi.org/10.1007/978-3-030-05414-4_19

(which is one of the signs of network complexity) for the first time in 1965. In 1999, at the University of Notre Dame, US physicists A.L. Barabasi and R. Albert established [4, 5] that in a large number of networks, the distribution of nodes tends to obey a power law, instead of the expected probabilistic Poisson distribution of nodes. Among the most recent papers related to our research, [6, 7] should be mentioned. A comparison of the results obtained in the aforementioned literature to our research results will be shown later as required in the corresponding sections of the present paper. Some other relevant works in this area are those of Refs. [8–12].

Critical phenomena in complex networks (phase transitions being among them) have been considered in many papers, (e.g., see the review [13] and references therein). Research on the phase transitions in social networks [6–9, 14–18] is not an exception. Nowadays the most interesting models of the phase transitions in social networks are the ferromagnetic Ising model, the model of condensation transition, Potts model, the XY model, the Kuramoto model, the reaction-diffusion model, and the co-evolution model, among others. Lately an interest emerged for the application of self-organized criticality theory to the analysis of critical phenomena in social networks (e.g., [6]).

In spite of the existence of a large number of various models of phase transitions mentioned above, there has been no research of phase transitions in social networks within a synergetic framework that would generalize the picture of phase transitions.

Self-organized phase transitions in microblogging social networks are part of our research.

Therefore, this paper is organized as follows. A brief theoretical background is provided in Sect. 2. The results of the three-parameter analysis of the kinetics of phase transitions applied to social networks are presented in Sect. 3. In Sect. 4, our conclusions are summarized.

2 Brief Theoretical Background

First of all, let us briefly consider the main terms of the synergetic theory of phase transitions [19–23], which are required for a complete understanding of the analysis below.

Considering that the concept of self-organization is a generalization of the physical concept of a phase transition, the empirical theory must be derived as an expansion of a theory of thermodynamic transformations onto open systems. Within the limits of the synergetic concept, a phase transition is realized as a result of coordinated behavior in three degrees of freedom represented by the following parameters:

- Order parameter (η_t), which represents the density of a conserved quantity in the closed subsystem,
- Conjugate field (h_t), which is the gradient of the corresponding flow.
- Control parameter (S_t), which is determined by the external influence and defines the state of the system.

The simplest (from the mathematical point of view) way to describe a self-organizing system is the well-known Lorenz system, which features three differential equations linking the rates of the parameter variation to their values:

$$\begin{cases} \dot{\eta}_t = -\frac{\eta_t}{\tau_\eta} + \gamma h_t \\ \dot{h}_t = -\frac{h_t}{\tau_h} + g_h \eta_t S_t \\ \dot{S}_t = \frac{S_0 - S_t}{\tau_S} - g_S \eta_t h_t \end{cases} \quad (1)$$

where $z_t = z(t)$, $\dot{z}_t = \frac{dz}{dt}$.

The main characteristic of Eq. (1) is the fact that all of those differential equations include dissipative components. The values of these components are inversely proportional to the corresponding relaxation times τ_η, τ_h, τ_S, i.e., the time intervals over which an excitation in an off-balance physical system decreases by a factor of e. The Le Chatelier principle is of importance: because the reason of the self-organization is the increase of the control parameter, then the order parameter and the conjugate field must vary in such a way as to prevent the control parameter from growth. Formally, this fact means the existence of a negative feedback loop between the values η_t and h_t. This feedback loop makes the stationary value smaller in comparison with its value fixed by the external stress.

The positive feedback loop between the order parameter and the control parameter is extremely important, because it leads to the growth of the conjugate field. This feedback loop can lead to the self-organization of the system, which in turn might cause a phase transition.

In Eq. (1) γ is a kinetic coefficient; the positive constants g_h, g_S represent the strength of feedback, and S_0 is the parameter of external influence.

The advantage of the synergetic approach lies in the fact that it allows one to employ the Le Chatelier principle without specifying a narrowly defined model.

The adiabatic approximation is normally used when studying the thermodynamics of phase transitions. During the system's evolution, the conjugated field and the control parameter change so fast that they can follow the slow change of the order parameter. At the same time, the evolution of the system is described by the Landau–Khalatnikov equation. The synergetic potential plays the role of the free energy in this case.

In summary, it should be noted that the physical meaning of the order parameter is the correlation function that defines a measure of the long-range order.

3 Three-Parameter Kinetics of the Phase Transitions

3.1 Self-organized Scheme

A microblogging social network is an open system, which means the existence of a continuous influx of external information – for example, from other mass media – which preserves the possibility of phase transitions from chaos (uncorrelated state) to an ordered (coherent) state. The open nature of the network allows us to consider it in a synergetic approach that generalizes the picture of phase transitions. In what follows, we propose a simple theory that describes the transition between a chaotic and a coherent state of a microblogging social network. This theory is based on the equations of evolution of the order parameter, the conjugated field, and the governing parameter.

Let us assume that Twitter is a dynamical system that consists of a large number of users $(n_\Sigma > > 1)$, where $n_t \leq n_\Sigma$ are strategically oriented (maintaining a definite strategy), and the remaining $n_\Sigma - n_t$ are randomly oriented (acting in a random way).

According to [24], among the strategically oriented users there can be business users and spam users. Business users follow a marketing and business agenda on Twitter. The profile description strongly depicts their motive, and a similar behavior can be observed in their tweeting behavior. Spammers mostly post malicious microposts at a high rate. Most of the time, automated computer programs (bots) run behind a spam profile and randomly follow users, expecting some users to follow them in return.

Personal users and professional users can be considered to be randomly oriented users. Personal users are casual home users who create their Twitter profile for entertainment, learning, to obtain news, etc. These users neither strongly advocate any type of business or product, nor are their profiles affiliated with any organization. Generally, they have a personal profile and show a low to moderate activity in their social interaction. Professional users are home users with professional intent on Twitter. They share useful information about specific topics and engage in healthy discussions related to their area of interest and expertise.

The most popular microblogging network, Twitter, can be considered to be an open system, so it is reasonable to study the microblogging social network within the framework of the synergetic approach that generalizes the concept of phase transitions. The simplest possible theory is based on the evolution of the order parameter (c_t), conjugate field (i_t), and control parameter (n_t).

Let us define the variables of the self-organized scheme as applied to microblogging social network.

Let us assume that the total number of microposts p_t with a specific topic at time t changes over an interval of time τ by $\delta p_{t+\tau}$. The strategically oriented users/partners make a decision to post microposts with content relevant to a certain topic depending on the variation of the respective information δi_t at the preceding moment of time t in other mass media, as well as Twitter. As a result, the random variables $\delta p_{t+\tau}$ and δi_t turn out to be statistically dependent, and the correlation function

$$\langle \delta p_{t+\tau}, \delta i_t \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T \delta p_{t+\tau} \delta i_t dt$$

equals zero.

An essential change in Twitter, meaning that an agreement on the choice of a definite strategy has been reached, occurs when $\langle \delta p_{t+\tau}, \delta i_t \rangle$ takes finite values asymptotically at infinite times of the information registration and the complete number of microposts, i.e., at $\tau \to \infty$. In this case, the correlation function takes the following form:

$$c_t = \lim_{\tau\to\infty} \langle \delta p_{t+\tau}, \delta i_t \rangle = \lim_{\tau\to\infty} \lim_{T\to\infty} \frac{1}{T} \int_0^T \delta p_{t+\tau} \delta i_t dt. \quad (2)$$

Thus, the difference between the strategically oriented and randomly oriented users consists in the fact that $c_t \neq 0$ for the former group, and $c_t = 0$ for the latter group.

In this case (2) can be considered to be an order parameter that determines the choice of strategy in the actions of users. Consequently, the conjugate field (i_t) is information, and the control parameter (n_t) is the number of strategically oriented users. This means that, with an increase in n_t above the critical value, the network transits from the uncorrelated state $c_t = 0$ to the coherent state $c_t \neq 0$, which is defined by the choice of strategy of a small number of users $n_t << n_\Sigma$.

In the uncorrelated state, the strategically oriented users act randomly and independently of each other. In this case, the information immediately affects the number of microposts on a certain topic; the time series of such microposts behaves as a random time series. In this case, therefore, the microblogging social network is unpredictable.

A coherent social network is characterized by such fundamental trends as an exponential growth of the number of microposts caused by coordinated action of a relatively small number n_t of partners/users and their followers, followers of their followers, and so on (Fig. 1).

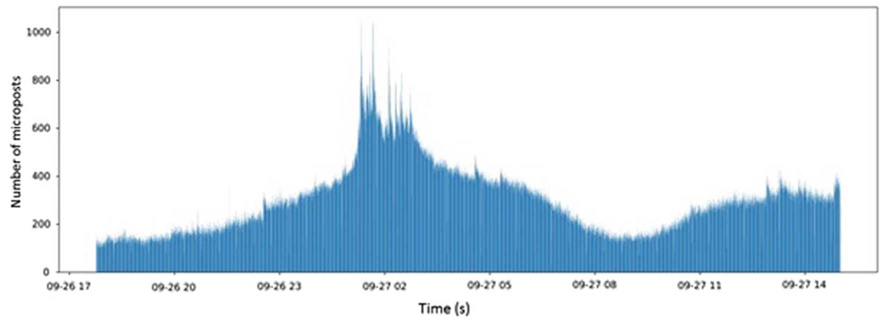

Fig. 1. Twitter time series. The data were obtained by hydrating of the list of 3,183,202 tweet IDs from a set of 12 lists of IDs distributed by Harvard University.

Within the framework of the synergetic approach, the evolution of a self-organized system is determined by the self-consistent equations connecting the rates of change of parameters $\dot{c}_t$, $\dot{i}_t$, $\dot{n}_t$ to their values c_t, i_t, n_t:

$$\begin{cases} \tau_c \dot{c}_t = -c_t + a_c i_t \\ \tau_i \dot{i}_t = -i_t + a_i c_t n_t \\ \tau_n \dot{n}_t = (n_0 - n_t) - a_n c_t i_t \end{cases} \quad (3)$$

In Eq. (3), n_0 is the initial number of strategically oriented users, a_c is a kinetic coefficient, positive constants a_i, a_n are the feedback coefficients of the network. The functions c_t/τ_c, i_t/τ_i, $(n_0 - n_t)/\tau_n$ describe autonomous relaxation of the correlation function (2), the information and the number of strategically oriented users of the network to their stationary values $c_t = 0$, $i_t = 0$, $n_t = n_0$ with the relaxation times τ_c, τ_i, τ_n. According to the Le Chatelier–Braun principle [25], if a system deviates from the state of the stable equilibrium, then the forces arise that try to return the system back to the equilibrium state.

3.2 Kinetics of the Phase Transition

A social network may exhibit a self-consistent behavior when the relaxation time of the correlation function (2) is considerably larger than the corresponding relaxation times for the information and the number of strategically oriented users: $\tau_c > > \tau_i, \tau_n$. This means that the information $i_t \approx i(c_t)$ and the number of strategically oriented users $n_t \approx n(c_t)$ follow the evolution of the correlation function c_t. If $\tau_c > > \tau_i, \tau_n$, then the principle of collateral subordination allows one to assume that $\tau_i \dot{i}_t = \tau_n \dot{n}_t = 0$ in Eq. (3). In other words, if $\tau_c > > \tau_i, \tau_n$ then the fluctuations of $i_t \approx i(c_t)$ и $n_t \approx n(c_t)$ can be neglected, assuming that $\dot{i}_t$, $\dot{n}_t = 0$ in (3). Here we consider an adiabatic process in which slow changes in the external conditions (the information and the number of strategically oriented users) are accompanied by the faster change in the state of the social network. As far as the operation of a microblogging social network is concerned, the adiabatic approximation means that when the influx of external information, e.g., from other mass media, tends to zero, the correlation function (2) decays slowly, and simultaneously the amount of information on a specific topic and the number of strategically oriented users decrease quickly.

In this case we obtain equations that express the conjugate field and the control parameter through the order parameter:

$$i_t = \frac{A_i n_0 c_t}{1 + c_t^2/c_m^2},\ n_t = \frac{n_0}{1 + c_t^2/c_m^2}, \tag{4}$$

where

$$c_m^2 \equiv 1/(A_i A_n),\ A_z \equiv \tau_z a_z,\ z = c, i, n. \tag{5}$$

When $c_t < < c_m$, the first of the Eqs. (4) has a linear form. With the growth of the order parameter up to $c_t = c_m$ the dependence $i(c_t)$ approaches saturation, and when $c_t > c_m$ it decreases. Thus, a constant c_m, which is defined by Eq. (5), has a meaning of the maximum value of the control parameter.

The second of the Eqs. (4) describes the decrease of the control parameter from the maximum value $n_t = n_0$ at $c_t = 0$ down to the minimum of $n_t = n_0/2$ at $c_t = c_m$. The decreasing character of $n(c_t)$ is a demonstration of the Le Chatelier principle, which predicts a negative feedback loop between c_t and i_t.

Thus, the increase of the correlation function (2) within the interval limited by c_m leads to an increase of the information i_t and to a decrease of the number of strategically oriented users n_t below that of its initial value, n_0.

Substitution of the first of the Eq. (4) into the first of the Eq. (3) yields the Landau–Khalatnikov equation

$$\tau_c \dot{c}_t = -\frac{\partial E}{\partial c_t} \tag{6}$$

with the synergetic potential

$$E = \frac{c_t^2}{2}\left\{1 - \frac{n_0}{n_m}\left(\frac{c_t}{c_m}\right)^{-2} \ln\left[1 + \left(\frac{c_t}{c_m}\right)^2\right]\right\}. \tag{7}$$

Equation (6) describes the evolution of a social network with the critical value of strategically oriented users $n_m = 1/(A_c A_i)$.

When the initial number of strategically oriented users is low, then the synergetic potential has a minimum at $c_t = 0$, which corresponds to the uncorrelated state of the social network where the connection between the incoming information and the network's behavior is absent. When n_0 increases to a value n_m above the critical value, there is a minimum (see Fig. 2) at

$$c_0 = c_m \sqrt{\frac{n_0}{n_m} - 1}. \tag{8}$$

The state of the social network, which is defined by the minimum value of the correlation function, corresponds to the condition of coherence, under which the social network evolves in accordance with the strategy chosen by a relatively small number of users $n_t << n_\Sigma$. Within the synergetic theory, such a state corresponds to the ordered phase, where the information has a stationary value $i_{st} = c_0/A_c$. At the same time, the stationary number of strategically oriented users decreases to the critical value $n_m < n_0$.

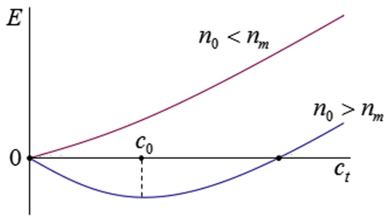

Fig. 2. Dependence of the synergetic potential on the number of strategically oriented users for the second-order phase transition at different critical values of the number of strategically oriented users

When the social network drastically transits to the domain characterized by the value $n_0/n_m > 1$, then over the time scale of

$$\tau = \tau_c \left(\frac{n_0}{n_m} - 1 \right)^{-1} \tag{9}$$

it approaches the stationary value of the number of strategically oriented users as follows:

$$c_t = c_0 \left(1 - e^{-t/\tau} \right). \tag{10}$$

3.3 Stochastic Behavior of the Phase Transitions

The behavior of users of a social network can be considered to be a random walk with standard deviation $\sigma = \sqrt{s}$, where s is the number of steps [26]. In this case, only $\sqrt{n_\Sigma} << n_\Sigma$ random moves out of the total number of n_Σ random moves will be coherent (in the case of a one-dimensional random walk). Based on this analogy, it can be argued that the initial number of users n_0 acting in a consistent (coherent) way is of the order of $\sqrt{n_\Sigma}$. Thus, a social network can remain in a coherent state if the following condition is met:

$$n_0 < \sqrt{n_\Sigma n_m}. \tag{11}$$

In a general case, the behavior of users of a social network can be considered as an anomalous diffusion [27]. In the case of the anomalous diffusion, the system's evolution can be reduced to a Levy flight superdiffusion or subdiffusion. For such a system, the right-hand site of Eq. (11) has a more complicated form $(n_\Sigma n_m)^\alpha$ defined by the dynamic parameter α ($\alpha < 2$ for superdiffusion and $\alpha > 2$ for subdiffusion). As a result, the condition (11) can be generally written down as

$$n_0 < (n_\Sigma n_m)^\alpha. \tag{12}$$

Therefore, the social network follows a definite strategy if the initial number of strategically oriented users exceeds a critical value equal to the geometric mean of the total and critical number of users.

Thus, a stochastic consideration shows that taking into account the information about the previous state of the social network is essential for determining its current state. If the initial number of strategically oriented users $n_0 < (n_\Sigma n_m)^\alpha$ choose a definite strategy of changing the topic, then the social network goes into a coherent state corresponding to the minimum of the synergetic potential (7), where the correlation function (2) has the stationary value (8). In this case, the remainder of users (the number of which $n_\Sigma - n_0 \leq n_\Sigma$ is much greater than the initial number $n_0 << n_\Sigma$ of strategically oriented users) starts to act following the chosen strategy. In other words, coherent actions of a small part of users $n_0/n_\Sigma << 1$ can spontaneously impose their decision onto the major part of users n_Σ.

4 Conclusions

As a result of the present study, it can be noted that microblogging social networks, and Twitter in particular, are complex networks. Social networks are characterized by a non-trivial behavior, i.e., they are able to exhibit phase transitions.

Within the framework of a Lorenz scheme, the kinetics of first-order phase transitions in a microblogging social network was analyzed.

The results are as follows:

- An increase of the correlation function (an increase in a measure of order in the network) in the range limited by the maximum number of strategically oriented users leads to an increase in the information and a decrease in the number of strategically oriented users to below their initial number.
- When the initial number of strategically oriented users is small, the social network remains in a disordered state in which there is no connection between the incoming information and the network's behavior. As the initial number of strategically oriented users increases above a critical value, there appears a minimum of their number that corresponds to an ordered state of the network. In the ordered state, the network evolves in accordance with the strategy chosen by a relatively small number of users.
- The social network follows a definite strategy when the initial number of strategically oriented users exceeds a critical value, which is given by the geometric mean of the total and critical values of the number of users. In the meantime, accounting for the information about the previous state of a social network may change its current state considerably.

Acknowledgments. This work was supported by the Russian Foundation for Basic Research (grant 16-07-01027).

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