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Computational Aspects and Applications in Large-Scale Networks

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Methods of Criteria Importance Theory and Their Software Implementation

Andrey Pavlovich Nelyubin, Vladislav Vladimirovich Podinovski and Mikhail Andreevich Potapov

Abstract The article presents a general approach to the solution of the multicriteria choice problem by methods of the Criteria importance theory. The overview of methods of vector estimates comparison by preference using various types of information about the preferences of the decision-maker is given. These methods are implemented in the computer system DASS.

Keywords Multicriteria analysis · Criteria importance theory · Decision support system · Graphical-analytical methods

1 Introduction

Most of the real decision-making problems are inherently multicriterial. The decision-maker (DM) needs to assess his/her subjective preferences as accurately as possible to choose the best final alternative. For this purpose, he/she can utilize mathematical and computer tools that provide opportunities to use complex mathematical methods for multicriteria analysis and optimization.

Among other approaches to analyzing and solving multicriteria problems, the Criteria Importance Theory (CIT) developed in Russia has a number of special advantages [1–3]. In this theory, a formal definition of the relative importance of criteria is introduced, which makes it possible to correctly take into account the incomplete and inaccurate information about the preferences of DM: the relative importance of

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criteria and the change in preferences along the scale of criteria. This information can be expressed both qualitatively and quantitatively, in the form of intervals of possible values of preference parameters.

In this article, we will consider only those methods of CIT that are implemented in the computer decision support system DASS version 2.4 [3, 4]. This system is designed to help in solving the problem of multicriteria choice among a finite set of alternatives. There are some new features in the version 2.4 comparing with previous versions. One of them is implementation of conciliatory decisions [5] in case of incomplete information on preferences. Also some new algorithms were developed which will be described in this article.

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2 Methods of the Criteria Importance Theory

To describe the CIT methods, let us introduce the following mathematical model of the situation of making an individual decision under conditions of certainty:

$$M =< X, K, Z, R >,$$

where X is the set of decision variants (alternatives); $K = (K_1, \dots, K_m)$ is the vector of $m \geqslant 2$ individual criteria; Z is the range of values of the vector criterion K; R is a non-strict preference relation of the DM.

Criterion K_i is a function defined on X with a range of values on Z_i. It is assumed that all the criteria are homogeneous or reduced to such. This means that the criteria have a common scale and, in particular, they have a common range of values, which is the set of estimates $Z_0 = \{1, \dots, q\}$, $q \geqslant 2$. It is assumed that each of the criteria is independent by preference from the others and its larger values are preferable to smaller ones. The values of all the criteria $K_i(x)$ of variant x from X form a vector estimate of this variant $y = K(x) = (K_1(x), \dots, K_m(x))$. Vector estimates from the set $Z = Z_m^0$ can correspond to the available variants from X or be hypothetical.

Comparison of the options by preference is reduced to comparing their vector estimates. To do this, the non-strict preference relation R is introduced on the set of vector estimates of Z: the notation yRz means that the vector estimate y is no less preferable than z. The relation R is (partial) quasi-order (that is, it is reflexive and transitive) and generates on Z the indifference I and (strict) preference P relations:

$$yIz \Leftrightarrow yRz \wedge zRy; \qquad yPz \Leftrightarrow yRz \wedge \neg zRy.$$

Since the DM preferences increase along the scale of the criteria Z_0, the Pareto relation $R^{\varnothing}$ is defined on the set of vector estimates Z:

$$yRz \Leftrightarrow y_i \geqslant z_i, \quad i = 1, \ldots, m.$$

As a rule, it is not possible to solve the multicriteria choice problem only with the help of the Pareto relation. Therefore, it needs to be expanded, using additional information about the preferences of the DM. Let R^{Π} be the preference relation constructed on Z using the information obtained Π. If initially there is no such information ($\Pi = \varnothing$), then R^{Π} plays the role of $R^{\varnothing}$. CIT uses information about the relative importance of criteria for decision-makers, which can be expressed qualitatively or quantitatively [1].

Qualitative information on the importance of the criteria Ω consists of messages of the form "Criteria K_i and K_j are equally important" (denoted by $i \sim j$) and "Criterion K_i is more important than the criterion K_j" (denoted by $i \succ j$). An exact definitions of these concepts are given in [1]. Further in the article, we will assume that the information is consistent and complete, i.e., it allows you to order (rank) the importance of all the criteria. For convenience, we number the criteria in order of nonincreasing of their relative importance.

The positive numbers $\alpha_1, \ldots, \alpha_m$ in the sum equal to 1 are called the *coefficients of importance* of the criteria consistent with the information Ω if they satisfy the following conditions:

$$i \sim j \in \Omega \Rightarrow \alpha_i = \alpha_j, \qquad i \succ j \in \Omega \Rightarrow \alpha_i > \alpha_j.$$

Consistency of information Ω ensures the existence of coefficients of importance. The importance coefficients agreed with the full information Ω are called ordinal [2]. The set A^{Ω} of feasible values of the vector $\alpha = (\alpha_1, \ldots, \alpha_m)$ is given by a system of linear constraints (1, 2):

$$\alpha_1 + \cdots + \alpha_m = 1, \tag{1}$$

$$\alpha_i = \alpha_{i+1}, \ if\ i \sim i+1, \quad \alpha_i > \alpha_{i+1}, \ if\ i \succ i+1, \quad i = 1, \ldots, m-1. \tag{2}$$

Quantitative information on the importance of the criteria Θ consists of messages like "The criterion K_i is more important than the criterion K_j in h_{ij} times". An exact definition of this concept is given in [1, 6]. The information Θ is said to be complete and consistent, if it can be used to specify the quantitative or cardinal coefficients of the importance of the criteria α_i-positive numbers in the sum equal to 1 and possessing the property $h_{ij} = \alpha_i/\alpha_j$, $i, j = 1, \ldots, m$.

Quantitative information Θ allows qualitative information Ω to be specified. To do this, consider the degree of superiority in the importance h_i of each criterion K_i over the following criterion K_{i+1}:

$$h_i = \frac{\alpha_i}{\alpha_{i+1}} \geqslant 1, \quad i = 1, \ldots, m-1. \tag{3}$$

Here, $h_i = 1$ if $i \sim i+1 \in \Omega$, and $h_i > 1$ if $i \succ i+1 \in \Omega$.

Quantitative information on the criteria importance can be set not exactly, but with interval constraints:

$$1 \leqslant l_i \leqslant h_i \leqslant r_i, \quad i = 1, \ldots, m-1. \tag{4}$$

Here, $l_i = r_i = 1$ if $i \sim i+1 \in \Omega$, and $l_i > 1$ if $i \succ i+1 \in \Omega$. This interval quantitative information on the importance of the criteria is denoted by Ξ.

In addition to information on the relative importance of criteria, CIT takes into account information on the scale of criteria Z_0, namely its type and rate of growth of preferences along this scale. If we only know that the values $v(k) = v_k$ of the gradations k of the scale Z_0 increase with increasing k, then such a scale is called the ordinal scale:

$$v(1) < v(2) < \cdots < v(q). \tag{5}$$

For $q \leqslant 3$, information on the rate of growth of preferences along the criteria scale may serve as additional information. For this value increment between adjacent grades $d(k) = v(k+1)?v(k)$, $k = 1, \ldots, q-1$, are ranked by preference. With such information Δ, the scale of criteria is referred to as the first ordered metric scale [7]. In this article, we consider only the law of decrease (information $\Delta\downarrow$) of the growth rate of preferences:

$$\delta(1) > \delta(2) > \cdots > \delta(q-1) > 0. \tag{6}$$

The quantitative information on the criteria scale specifies the information $\Delta\downarrow$. An exact information V about the rate of growth of preferences can be specified by the ratio $\delta(k)/\delta(k+1)$, $k = 1, \ldots, q-2$. In this case, the criteria scale is a scale of intervals. Interval information [V] can also be specified:

$$1 \leqslant d_k \leqslant \frac{\delta(k)}{\delta(k+1)} \leqslant u_k, \quad k = 1, \ldots, q-2. \tag{7}$$

All these types of information about the preferences of the decision-maker and their combinations require their own methods of analyzing the problem and algorithms for comparing alternatives by preference. Within the framework of CIT, precise and effective methods, and algorithms for solving such problems have been developed. It was necessary to obtain solutions of a number of linear, nonlinear, and discrete problems. For some of these problems analytical solutions have been obtained. Here, is a brief overview of the methods developed.

First, consider the information Ω on ordering the criteria by importance. For various types of criteria scale (5) and (6), analytical decision rules were developed in CIT [2, 8, 9]. To formulate them, for each $k = 1, \ldots, q-1$, we introduce the vector $\alpha^k(y) = (\alpha_{1k}, \alpha_{2k}, \ldots, \alpha_{mk})$ whose elements are:

$$\alpha_{ik}(y) = \begin{cases} \alpha_i, & y_i > k, \\ 0, & y_i \leqslant k, \end{cases} \quad i = 1, \ldots, m. \tag{8}$$

Let $\psi^{(n)}(x)$ be a vector-function that orders the components of the n-dimensional vector x in the order of their nonincreasingness:

$$\psi_1^{(n)}(x) \geqslant \psi_2^{(n)}(x) \geqslant \cdots \geqslant \psi_n^{(n)}(x).$$

The analytical decision rule specifying the ratio R^{Ω} is:

$$yR^{\Omega}z \Leftrightarrow \psi_i^{(m)}\left(\alpha^k(y)\right) \geqslant \psi_i^{(m)}\left(\alpha^k(z)\right),\ i = 1, \ldots, m;\ k = 1, \ldots, q-1. \quad (9)$$

If in (9) all the non-strict inequalities $\geqslant$ are satisfied as equalities, then $yI^{\Omega}z$, and if at least one $\geqslant$ is satisfied as strict $>$, then $yP^{\Omega}z$.

The analytical decision rule defining the ratio $R^{\Omega\&\Delta\downarrow}$ is similar in form (9), but instead of the vector $\alpha^k(y)$, it uses the vector $\alpha^{[1,k]\downarrow}(y)$ composed of the vectors $\alpha^1(y), \alpha^2(y), \ldots, \alpha^k(y)$ [9].

In case of exact quantitative information Θ about the relative importance of criteria having an ordinal scale (5), the decision rule that defines the preference relation R^{Θ} can be represented as follows [6]:

$$yR^{\Theta}z \Leftrightarrow B_k(y) \geqslant B_k(z), \quad k = 1, \ldots, q-1, \quad (10)$$

here $B_k(y) = \sum_{i=1}^{m} \alpha_{ik}(y)$.

If, however, only a set A of possible values of the vector α is known (for example, under constraints (1–4)), then $yR^{\mathrm{A}}z$ is true if and only if the inequalities (10) hold for any $\alpha \in \mathrm{A}$, or equivalently:

$$yR^{\mathrm{A}}z \Leftrightarrow \inf_{\alpha\in\mathrm{A}} (B_k(y) - B_k(z)), \quad k = 1, \ldots, q-1. \quad (11)$$

In the case of exact quantitative information Θ about the relative importance of criteria having a first ordered metric scale (6), the decision rule defining the preference relation $R^{\Theta\&\Delta\downarrow}$ can be represented as follows [8]:

$$yR^{\Theta\&\Delta\downarrow}z \Leftrightarrow D_k(y) \geqslant D_k(z), \quad k = 1, \ldots, q-1,$$

here $D_k(y) = \sum_{t=1}^{k} B_t(y) = \sum_{t=1}^{k} \sum_{i=1}^{m} \alpha_{it}(y)$.

If only a set A of possible values of the vector α is known, then

$$yR^{\mathrm{A}\&\Delta\downarrow}z \Leftrightarrow \inf_{\alpha\in\mathrm{A}} (D_k(y) - D_k(z)), \quad k = 1, \ldots, q-1. \quad (12)$$

To construct decision rules that use quantitative (in particular, interval) information about the growth of preferences along the scale of criteria (7), we use the additive value function:

$$F\left(y \mid \alpha, v\right) = \sum_{i=1}^{m} \alpha_i v(y_i).$$

Let us introduce the matrix $C(y, z)$ with the elements:

$$c_{ik}(y,z) = \begin{cases} 1, & if z_i \leq k < y_i, \\ -1, & if y_i \leq k < z_i, \\ 0, & otherwise, \end{cases} \quad i = 1, \ldots, m; k = 1, \ldots, q-1. \tag{13}$$

With its help, it is possible to express the difference in the value functions for the vector estimates y and z being compared:

$$G(y, z \mid \alpha, v) = F(y \mid \alpha, v) - F(z \mid \alpha, v) = \sum_{i=1}^{m} \alpha_i \left(v(y_i) - v(z_i)\right) =$$
$$\sum_{i=1}^{m} \left(\alpha_i \sum_{k=1}^{q-1} c_{ik}(y,z)\delta_k \right) = \alpha^T C(y,z)\delta = G(y, z \mid \alpha, \delta).$$

If the values of the importance coefficients α_i and the increments of the values of gradations of the scale $\delta(k)$ are known exactly, then the following decision rule can be used:

$$y P^{\Theta \& \mathrm{V}} z \Leftrightarrow G(y, z \mid \alpha, \delta) \geqslant 0.$$

And if only (non-empty) sets A and Δ of possible values of vectors α and δ are known, then

$$y P^{\mathrm{A} \& \Delta} z \Leftrightarrow \inf_{(\alpha,\delta) \in \mathrm{A} \times \Delta} G(y, z \mid \alpha, \delta) \geqslant 0. \tag{14}$$

The decisive rules (11), (12), and (14) requires solving of optimization problems. Even with a relatively small number of variants (several dozen), this makes it difficult to use them. For interval information about the relative importance of the criteria, it was shown in [10, 11] how to solve these optimization problems with the help of recurrence formulas.

The analytical decision rule specifying the relation R^{Ξ} is:

$$y R^{\Xi} z \Leftrightarrow \gamma_k^* \geqslant 0, \quad k = 1, \ldots, q-1. \tag{15}$$

Here, γ_k^* are the quantities consecutively calculated for each k according to the recurrence formulas:

$$\gamma_{1k} = \begin{cases} l_1 c_{1k}, & c_{1k} \geqslant 0, \\ r_1 c_{1k}, & c_{1k} < 0; \end{cases}$$

$$\gamma_{ik} = \begin{cases} l_i \left(c_{ik} + \gamma_{i-1,k}\right), & \left(c_{ik} + \gamma_{i-1,k}\right) \geqslant 0, \\ r_i \left(c_{ik} + \gamma_{i-1,k}\right), & \left(c_{ik} + \gamma_{i-1,k}\right) < 0, \end{cases} \quad i = 1, \ldots, m-1; \tag{16}$$

$$\gamma_k^* = c_{mk} + \gamma_{m-1,k}.$$

The decision rules $yR^{\Xi\&\Delta\downarrow}z$ and $yR^{\Xi\&\mathrm{V}}z$ are similar in form to (15–16), but instead of the numbers $c_{ik}(y, z)$ (introduced in (13)), they use the numbers $d_{ik}(y, z) = \sum_{t=1}^{k} c_{it}(y, z)$ and $e_i = \sum_{k=1}^{q-1} c_{ik}(y, z)\delta_k$, respectively.

When combining interval information on the importance of the criteria Ξ and interval information on the scale V, the decision rule (14) requires solving the bilinear programming problem. The algorithm for solving this problem was proposed in [12], it uses the extreme point formulas of the sets A and Δ given by constraints (1–4) and (5–7), respectively.

3 Software Implementation of the Methods

To solve multicriteria choice problems using the CIT methods considered, the authors develop the computer system DASS, which is freely available at http://www.mcodm.ru/soft/dass. In this system, the solution of the choice problem is organized in the form of an iterative process, during which the DM gradually specifies information about his/her preferences in the interactive mode [3, 4]. At each step of this process, the system calculates the results of comparisons of alternatives, based on the available information on the DM preferences and using the methods described above.

First, the DM introduces basic information about the problem: a set of criteria, decision variants, and their evaluations by each of the criteria. At this stage, the Pareto relation R is constructed on the set of vector estimates of the variants. Nondominant (Pareto-optimal) variants are singled out, among which the choice is to be made taking into account the preferences of the DM.

The DM does not need to immediately indicate the exact information about his/her preferences. First, with the help of special methods developed, qualitative information on the importance of the criteria [1] is inquired. Using the decision rule (9), the system tries to compare variants from the set of Pareto-optimal ones among themselves. As a result, some of these variants turn out to be dominant by the relation R. Thus, the number of nondominated options decreases.

To further narrow the set of nondominant variants, one can begin to inquire, using special methods, quantitative information about the relative importance of criteria in the form of intervals (4), consistent with the qualitative information [1]. Alternatively, one can begin to refine the type of the criteria scale Z_0 in the form (7) or (8). Using the appropriate decision rules, the system will again try to compare by preference the nondominant variants obtained in the previous step. And so on, until the one nondominant variant is obtained. In practice, for this purpose it is not necessary to inquire exact values for the importance coefficients α_i of the criteria and the exact values $v(k)$ for the gradations of the criteria scale Z_0.

The preference relation R at each step of the iterative process of solving the choice problem can be represented as an incomplete oriented graph. The vertices of the graph are variants or their vector estimates. The arcs represent the preference relation R.

Incomparable by the relation of R variants are not connected on the graph. In addition, to analyze the solution, it is convenient to arrange the vertices of the graph in the plane so that all the arcs have a common direction, for example, downwards. Thus, the nondominant variants will be on top.

Such a representation of intermediate results in the form of an oriented graph refers to graphical-analytical methods for solving the choice problem. Graphic image promotes a comprehensive perception of a large number of complex information. The visual representation of the set of variants allows to reveal the relationship existing between them. When a DM sees this picture of the relationship as a whole, he/she can best plan the further course of the solution of the problem. As a result, visualization tools improve the quality of decision-making.

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