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The Application of Stochastic Bifurcation Theory to the Early Detection of Economic Bubbles

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Abstract

The present research is devoted to the application of stochastic bifurcation theory to the early detection of economic bubbles. A nonlinear random dynamical system with the possible appearance of stochastic P-bifurcations with a fat-tailed probability density function is deduced. The possibility of application of chaotic bifurcation theory to the early detection of culminations of economic bubbles is investigated by the example of dot-com bubbles. For the increments of NASDAQ it is shown that the criterion of reaching the culmination for dot-com bubbles is a formation of a bimodal distribution with the subsequent conversion to a unimodal distribution as a result of codimension one P-bifurcation – a triple equilibrium point.

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Keywords: nonlinear random dynamical system; stochastic bifurcation; economic bubbles; dot-com bubbles; NASDAQ time series.

1. Introduction

The power law of the probability distribution of a signal is one of the distinguishing features for the most of complex systems, regardless of their origin, i.e. a probability density function (PDF) of the signal for $x \to \infty$ has the following form:

$$p(x) \sim x^{-(1+\gamma)}, \gamma > 0. \tag{1}$$

In this case, the complexity of the system is determined by the presence of catastrophic events: unexpected events that cannot be predicted, and extraordinary events that stand out from a series of related events. When describing catastrophic events the PDF (1) is the rule, practically without exception. The fundamental difference of the PDF (1), which belongs to the class of fat-tailed distributions [1], from compact distributions lies in the fact that the events occurring in the tail of the distribution are not rare enough to be neglected [2].

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The PDF (1) describes various kinds of catastrophic events with a good degree of accuracy. Among such events are, for example, dependence of the number of earthquakes on their energy [3]; relative mortality due to natural disasters [4]; a number of cases in epidemics [5]; an area of forest fires [6]; fluctuations in stock indices [7-11]; mass of avalanches [12], etc.

The existence of the PDF (1) is related to the presence of $1/f^{\beta}$ noise (power spectral density is $S(f) \sim 1/f^{\beta}$) in the signal, which is also a criterion of the system complexity. The presence of $1/f^{\beta}$ noise in a system means the possibility of giant fluctuations. This supposes that it stays in the vicinity of the critical point, or the bifurcation point, where such phenomena usually occur. There is a relatively new field in non-linear dynamics – a theory of self-organized criticality [13]. It was created to explain the similar phenomena in systems with the power-series distributions and $1/f^{\beta}$ noises. The existence of the $1/f^{\beta}$ noise in a system means the internal tendency to the catastrophic cases in a system. The theory of self-organized criticality studies the dynamical dissipative systems with the high range of discretion, which operate in the neighborhood of the critical point without the smallest external influence. If the system is in a critical configuration, than small fluctuations can lead to a random event of any “size” with the power-series distribution similar to (1).

If the signal $x(t)$ is the realization of random dynamical system (RDS) with some control parameter α , then stochastic bifurcations [14] may appear in the RDS. In [14] P-bifurcation or phenomenological bifurcation in the presence of noise refers to a qualitative change of $p(x)$ with a small change of control parameter α . For example, a similar bifurcation can lead to the appearance or disappearance of local maxima of the probability distribution for a certain value of the parameter $\alpha = \alpha_P$.

There are a lot of works [14-18] devoted to the study of stochastic bifurcations. However, the number of such studies is noticeably less than the number of publications in the other topical areas of nonlinear dynamics. Moreover, we do not know any papers devoted to the investigation of P-bifurcations in RDS, generating signals with the PDF (1), which is of undoubted interest in connection with the theoretical and practical significance of this distribution.

The practical importance of this study is determined by the possibility of modifying the algorithm for determining the position of the forthcoming bifurcation point and its type from the observed noise change (growth form, saturation level, a density of distribution) [19], taking into account the PDF (1).

2. Stochastic bifurcations in the RDS

First of all, let us define the explicit form of Ito process driven by the standard Wiener process $W(t)$ as an RDS [20]:

$$dx(t) = \mu(x(t),t)\,dt + \sigma(x(t),t)\,dW(t). \tag{2}$$

In the RDS (2) $\mu(x(t),t)$ is a drift coefficient, $D(x(t),t) = \sigma^2(x(t),t)$ is a diffusion coefficient. The Fokker–Planck equation for a PDF of the random variable $x(t)$ is the following equation:

$$\frac{\partial}{\partial t} p(x(t),t) + \frac{\partial}{\partial x} G(x(t),t) = 0, \tag{3}$$

with the following probability current:

$$G(x(t),t)=\mu(x(t),t)\,p(x(t),t)-\frac{1}{2}\frac{\partial}{\partial x}\left[D(x(t),t)\,p(x(t),t)\right]. \tag{4}$$

When constructing the explicit form of the RDS (2) with implementations in the form of the PDF (1), we used an RDS, presented in a series of works [21-23], as the fundamental one:

$$dx=\sigma^2\left(\eta-\frac{\nu}{2}\right)x^{2\eta-1}dt+\sigma x^{\eta}dW\ . \tag{5}$$

Here, x is a signal, η is an exponent of the multiplicative noise, and ν defines the behavior of a stationary probability distribution. The fact, that the RDS (5) generates a signal with the power-series distribution (1) and with the occurrence of the $1/f^{\beta}$ noise [18], is the important feature of the RDS (5). It is determined by the existence of degree $2\eta-1$ in the drift term and degree η in the diffusion term.

Given that there are no stochastic bifurcations in the RDS (5), let us consider the following nonlinear RDS:

$$dx(t)=af^{n}(x(t))\,df(x(t))+f^{\frac{m}{2}}(x(t))\,dW(t)\ . \tag{6}$$

Here, a is a constant, n is an odd number, m is an even number ($n=m-1$), $f(x)$ is a function of a normal form of a saddle-node, transcritical and pitchfork bifurcations [21].

By the zero boundary conditions for the probability current (4)

$$G(-\infty,t)=G(\infty,t)=0,\ p(-\infty,t)=p(\infty,t)=0\ , \tag{7}$$

the stationary equation (3) for the RDS (6) has the following form:

$$\frac{d}{dx}\left[f^{m}(x)\,p_{st}(x)\right]-2af^{n}(x)\frac{df(x)}{dx}p_{st}(x)=0\ . \tag{8}$$

One-dimensional steady-state probability density $p_{st}(x)=\lim_{t\to\infty}p(x,t)$ does not depend on time t and on the initial distribution $p_0(x)$. Therefore, the probability current $G=\text{const}$ in a stationary state.

The general solution (8) as the steady-state PDF has the following form:

$$p_{st}(x)=\frac{C}{f^{m}(x)}\exp\left[2a\int\frac{f^{n}(x)}{f^{m}(x)}df(x)\right]\ . \tag{9}$$

Here, C is a constant, which is determined from the normalization condition:

$$\int_{-\infty}^{+\infty}p_{st}(x)\,dx=1\ . \tag{10}$$

Let us consider the influence of noise on the following local bifurcations of equilibrium states: a codimension one bifurcation – to the triple equilibrium point, and a codimension one bifurcation – to the five-fold equilibrium point. By definition [22], the codimension of a bifurcation is the smallest number of parameters for which a bifurcation occurs. Further, without losing generality, we will consider codimension one bifurcations. This choice is due to the fact that the bifurcation data is the most typical for most of dynamical systems, and also because of the fact that the most of bifurcation multidimensional problems can be reduced to one-dimensional or two-dimensional systems [23].

Let us consider a codimension one bifurcation – a triple equilibrium point (type I).

In the deterministic case, the bifurcation is described by the following autonomous dynamical system (DS):

$$\dot{x} = \left(x^2 + \alpha\right)^n x. \tag{11}$$

Here α is a control parameter. For $\alpha < 0$ there exist a stable equilibrium point at zero and two unstable equilibrium points $\pm\sqrt{|\alpha|}$. At $\alpha = 0$ a bifurcation occurs, as a result of which, for $\alpha > 0$ the equilibrium becomes unstable at zero (there are no non-zero equilibrium points).

Let us consider the P-bifurcation in the DS (11) with multiplicative noise. The bifurcation is described by the RDS (6) as follows:

$$dx = \left(x^2 + \alpha\right)^n x dt + \left(x^2 + \alpha\right)^{\frac{m}{2}} dW. \tag{12}$$

A steady-state PDF of the RDS (12) has the following form:

$$p_{st}\left(x\right) \sim \frac{\left|x^2 + \alpha\right|}{\left(x^2 + \alpha\right)^m}. \tag{13}$$

Fig. 1 illustrates the change in the steady-state PDF of RDS (13) with the change of the control parameter α For $\alpha < 0$ PDF p_{st} has a smooth minimum at the point $x = 0$, $p_{st} \to \infty$ for $x \to \pm\sqrt{|\alpha|}$. At $\alpha = 0$, there is a P-bifurcation, as a result of which, p_{st} has only a smooth maximum at the zero point.

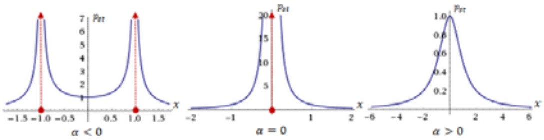

Fig. 1. P-bifurcation in random dynamical system (12).

Thus, if the parameter α passes through zero, then the bimodal PDF with singularities at nonzero points becomes a unimodal PDF.

Now, let us consider a codimension one bifurcation – a triple equilibrium point (type II).

In the deterministic case, the bifurcation is described by the following autonomous dynamical system (DS):

$$\dot{x} = \left(\alpha x - x^2\right)^n \left(\alpha - 2x\right). \tag{14}$$

For $\alpha < 0$, there exist a stable equilibrium point $-|\alpha|/2$ and two unstable equilibrium points: 0 and $-\sqrt{|\alpha|}$. At $\alpha = 0$, there is a bifurcation in the form of inversion: there is a stable equilibrium point $\alpha/2$ and two unstable equilibrium points: 0 and $+\sqrt{\alpha}$. The corresponding RDS looks as follows:

$$dx = \frac{1}{2}\left(\alpha x - x^2\right)^n \left(\alpha - 2x\right) dt + \left(\alpha x - x^2\right)^{\frac{m}{2}} dW \quad . \tag{15}$$

A steady-state PDF of the RDS (15) has the following form:

$$p_{st}\left(x\right) \sim \frac{\left|\alpha x - x^2\right|}{\left(\alpha x - x^2\right)^m}. \tag{16}$$

Fig. 2 illustrates the change in the steady-state PDF of the RDS (15) with the change of the control parameter α For $\alpha < 0$, the PDF has a smooth minimum at the point $x = -\alpha/2$, $p_{st} \to \infty$ for $x \to 0$ and $x \to -\alpha$. At $\alpha = 0$ $p_{st} \to \infty$ for $x \to 0$.For $\alpha > 0$ PDF has a smooth minimum at the point $x = \alpha/2$, $p_{st} \to \infty$ for $x \to 0$ and $x \to \alpha$.

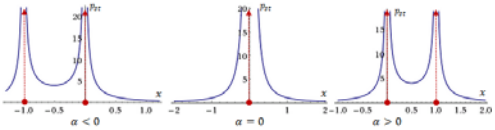

Fig. 2. Changing probability density function in a random dynamical system (15)

Thus, if the parameter α passes through zero, then there is a PDF displacement without any qualitative changes. P-bifurcation in the RDS (15) does not exist.

And now, let us consider a codimension one bifurcation – a five-fold equilibrium point.

In the deterministic case, the bifurcation is described by the following autonomous dynamical system (DS):

$$\dot{x} = \left(\alpha x - x^3\right)^n \left(\alpha - 3x^2\right). \tag{17}$$

For $\alpha \le 0$, there is a zero unstable equilibrium point. For $\alpha = 0$ bifurcation occurs, as a result of which, for $\alpha > 0$ the equilibrium at zero remains unstable, there appear two unstable equilibrium points $\pm\sqrt{\alpha}$ and two stable points $\pm\sqrt{\alpha/3}$. The corresponding RDS looks as follows:

$$dx = \frac{1}{2}\left(\alpha x - x^3\right)^n \left(\alpha - 3x^2\right) dt + \left(\alpha x - x^3\right)^{\frac{m}{2}} dW \, . \tag{18}$$

A steady-state PDF of the RDS (15) has the following form:

$$p_{st}\left(x\right) \sim \frac{\left|\alpha x - x^3\right|}{\left(\alpha x - x^3\right)^m}. \tag{19}$$

Fig. 3 illustrates the change in the steady-state PDF of the RDS (18) with the change of the control parameter for $\alpha \le 0$: $p_{st} \to \infty$ for $x \to 0$. For $\alpha > 0$: PDF has two smooth minima in points $\pm\sqrt[m]{\alpha/3}$, $p_{st} \to \infty$ for $x \to 0$ and $x \to \pm\sqrt{\alpha}$.

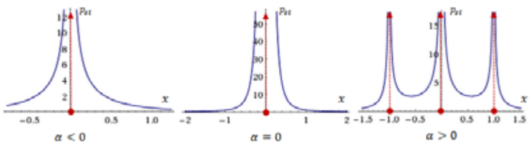

Fig. 3. P-bifurcation in random dynamical system (18)

Thus, if the parameter α passes through zero, then a unimodal PDF with a singularity at zero goes into a multimodal PDF with a singularity at zero and two non-zero points.

3. Stochastic bifurcation and a dot-com bubble

As it has already been noted, the existence of a signal with the PDF (1) indicates that the system, generating this signal, has a potential predisposition to catastrophic events. In this case, the system is in the vicinity of the critical point, possibly in the vicinity of the stochastic bifurcation point α_P . At the point α_P , a slight fluctuation can have a radical effect on RDS. Therefore, it is advisable to consider the effect of the established stochastic bifurcations on the formation of economic bubbles as one of the illustrative examples of the formation of catastrophes in economic systems. An economic bubble is trade in an asset at a price or price range that strongly exceeds the asset's intrinsic value [27]. One of such economic bubbles is the dot-com bubble that existed between 1995 and 2001.The culmination occurred on March 10, 2000, when the NASDAQ index reached 5132.52 points (day high) during trading and fell more than 1.5 times on closing. The bubble was formed as a result of the rise of shares of Internet companies, as well as the appearance of a large number of new Internet companies and the reorientation of old companies to the Internet business in the late XX century. Shares of companies offering to use the Internet to generate income have skyrocketed in price. Such high prices were justified by numerous commentators and economists who claimed that the "new economy" had come, in fact, these new business models were ineffective, and the funds spent mainly on advertising and large loans led to a wave of bankruptcies, and a strong fall of the NASDAQ index.

Let us consider the behavior of a PDF for change of NASDAQ index: $x(t+\tau)-x(t)$. At the same time, as in the previous case, the PDF will be obtained by accumulating NASDAQ values for 1349 index values. In fig. 4 the empirical PDF for NASDAQ time series increments are shown. Fig. 4 (g) corresponds to the culmination of dot-com bubble. The empirical PDF for the entire examined time interval is presented in the fig. 4 (j).

Prior to the culmination of the dot-com bubble, empirical PDFs correspond to symmetric unimodal distributions (fig. 4(a)–4(f)).In this case, PDF has the form of a fat-tailed q-Gaussian distribution [25-27]. PDF data is similar to the theoretical PDF, shown in fig. 1 and fig. 2. At the time of the culmination, a qualitative change in PDF occurs (fig. 4 (g)).In this case, PDF has the form of a bimodal distribution (two columns of the histogram). After passing through the culmination, unimodality again appears in PDF with an increase in the number of index increments (fig. 4(h) – 4(j)), which corresponds to the codimension one stochastic bifurcation –a triple equilibrium point (type I) (fig. 1).Thus, the criterion for achieving the culmination of the dot-com bubble is the formation of a bimodal distribution followed by a transformation into a q-Gaussian distribution as a result of a P-bifurcation of codimension one – a triple equilibrium point (type I). This result can also be used as a basis for building an early detection system for dot-com bubble culminations, and possibly the other economic bubbles.

The obtained result is also of a theoretical importance: the NASDAQ index increment is described with a good degree of accuracy by the RDS (12).

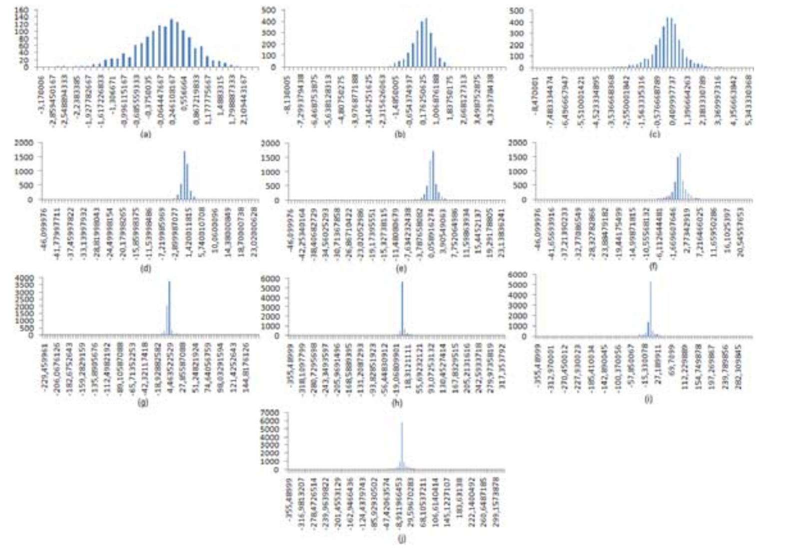

Fig. 4. Empirical probability density function for change of NASDAQ time series (accumulation of index change is 1349)

In more detail, the formation of the bimodal distribution is shown in fig. 5. The PDF data is obtained by accumulating NASDAQ increments over 100 index increments: (a) from 1971/02/05 to 1998/12/02, (b) from 1971/02/05 to 1999/05/02, (c) from 1971/02/05 to 1999/10/18, (d) from 1971/02/05 to 2000/03/10, (e) from 1971/02/05 to 2000/08/02, (f) from 1971/02/05 to 2000/12/22. Fig. 5 (d) corresponds to the culmination of the dot-com bubble. In this case, the transition from the unimodal distribution (fig.5 (a) - 5 (c)) to the bimodal distribution is seen. Thus, the birth of the point of culmination of the dot-com bubble corresponds to a P-bifurcation in nonlinear RDS. This stochastic bifurcation can correspond to one of the two above-mentioned bifurcations: a stochastic bifurcation in the RDS (12) by changing the control parameter from 0 to a negative value (fig. 1) or a stochastic bifurcation in the RDS (15) by changing the control parameter from 0 to a positive value (fig. 2).

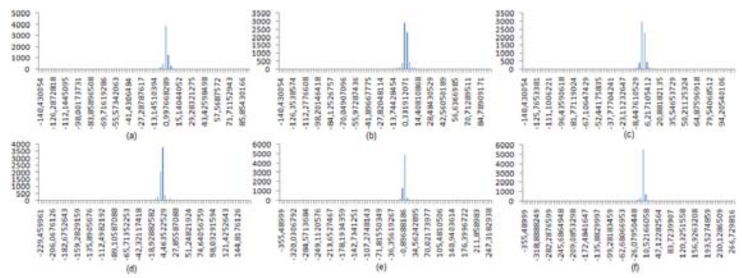

Fig. 5. Empirical probability density function for change of NASDAQ time series (accumulation of index change is 100)

4. Conclusion

The main contributions of the present paper look as follows:

A nonlinear RDS (6) is obtained, which determines the Ito process driven by the standard Wiener process with a fat-tailed distribution. The existence of the fat-tailed distribution is confirmed by the Fokker-Planck equation for a steady-state PDF (9) under zero boundary conditions for the probability current.

An important feature of the proposed RDS is the existence of stochastic P-bifurcations. There are two types of stochastic bifurcations: codimension one bifurcation – a triple equilibrium point and codimension one bifurcation – a five-fold equilibrium point. In the case of codimension one bifurcation – a triple equilibrium point, when passing control parameter through zero, the bimodal fat-tailed PDF with singularities at nonzero points becomes the unimodal fat-tailed PDF. In the case of codimension one bifurcation – a five-fold equilibrium point, when passing control parameter through zero, a unimodal fat-tailed PDF with a singularity at zero goes into a multimodal fat-tailed PDF with a singularity at zero and two nonzero points.

By the example of a dot-com bubble, the possibilities of applying the stochastic bifurcation theory to the early detection of culminations of economic bubbles are explored. For the increments of NASDAQ it is established, that the criterion of reaching the culmination for the dot-com bubble is a formation of a bimodal distribution with the subsequent conversion to a unimodal distribution as a result of a P-bifurcation of codimension one – a triple equilibrium point (type I). This result can be used as a basis for building an early detection system for dot-com bubble culminations, and possibly the other economic bubbles.

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