Big data and inverse problem for Ekman - Akerblom model

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The Ekman - Akerblom model can be used to describe a velocity in the ocean or atmosphere boundary layer:

$$\begin{matrix} \partial_z k \partial_z u = l\left(v - v_g\right) \\ \partial_z k \partial_z v = -l\left(u - u_g\right) \end{matrix}, \quad z \in [0, H], \qquad u(0) = v(0) = 0,\ u(H) = u_g,\ v(H) = v_g \tag{1}$$

where $\langle u(z), v(z) \rangle$ is the vertical profile of the horizontal flow, $\langle u_g, v_g \rangle$ is the geostrophic wind, l is the Coriolis parameter, and H is the boundary layer height. The turbulent exchange coefficient k in the approximate model of the boundary layer can be defined by various ways. Ekman (1905) and Akerblom (1908) used the simplest version: $k = const > 0$. Many various parameterizations for the coefficient $k(z)$ were considered during the next century assuming that k can depend on the coordinate z, on the temperature stratification $T(z)$, etc. In order to compare the skill of boundary layer parameterizations we could include them to a general atmospheric circulation model (GCM) and analyze its performance. However, finally the comparison results (which require long numerical experiments) may be depending on the GCM applied.

We introduce here an alternative approach: we suggest to use an archive of measurements with a good vertical resolution (obtained e. g. from radiosondes in BUFR code) and determine an "optimal" coefficient $k(z)$ by minimization of the residual of system (1). We integrate (1) with respect to z and introduce its residual. The averaged residual is the integral functional, and we minimize it:

$$I[k(z)] = \sum \frac{1}{H} \int_0^H \left\{ \left[k\partial_z u - l\int \left(v - v_g\right) dz + c_1 \right]^2 + \left[k\partial_z v + l\int \left(u - u_g\right) dz + c_2 \right]^2 \right\} dz \to \min_{k(z)\ge 0, c_1, c_2} . \tag{2}$$

Here we use the summation over a set of measured wind profiles. For the residual of relation (1), the variational approach gives the corresponding Euler equation and the transversality boundary conditions. If we search for $k(z)$ in the form of a piecewise linear function, then the minimization problem (2) is a quadratic programming problem with sparse matrix, and we can use classical numerical methods.

We can use an ansatz from the following: $k = k(z)$, $k = k(z/H)$, $k = k\left[\sqrt{(\partial_z u)^2 + (\partial_z v)^2}\right]$ etc for the minimization of (2). If the inequality $k(z) \ge 0$ is active during the minimization, i.e. the extremal $k(z)$ is not strongly positive, the restriction should be revised to provide a stronger minimization. If we obtain an extremal $k(z) \equiv 0$ that vanishes in some segment $\Delta \subset [0, H]$, and the solution $\langle u(z), v(z) \rangle$ of system (1) does not exist. We should modify system (1):

$$\partial_z \left[A\, \partial_z \begin{pmatrix} u \\ v \end{pmatrix} \right] = l \begin{pmatrix} u - u_g \\ v - v_g \end{pmatrix},\ where\ A = \begin{pmatrix} \gamma(z) & -k(z) \\ k(z) & \gamma(z) \end{pmatrix}. \tag{3}$$

This system has a smooth solution if $k^2 + \gamma^2 \ne 0$, and therefore we do not need the restriction $k(z) \ge 0$ any longer. Now we minimize the functional

$$\frac{1}{H}\sum\int_0^H\left\| A\partial_z\begin{pmatrix}u\\v\end{pmatrix} - l\int\begin{pmatrix}u-u_g\\v-v_g\end{pmatrix}dz + \begin{pmatrix}c_1\\c_2\end{pmatrix}\right\|^2 dz \to \min_{c_1,c_2,\gamma,k} . \tag{4}$$

We defined (see e.g. [Troen and Mahrt 1986]) the height of the boundary layer H as the first root of equation

$$\Theta(H) = \Theta_V(0), \tag{5}$$

where Θ is the potential temperature and Θ_V is the potential virtual temperature. We only considered the profiles, where the root belonged to the interval $2000m > H > 200m$. The condition is fulfilled for 40% of the profiles (2665 radiosondes with mean height $H_{mean} = 534m$). In Fig.1 we show the results of optimization for a) $k(z) \geq 0, \gamma(z) \equiv 0$. and b) k,γ - arbitrary. In case a) the mean error according to formula (4) for this evaluation is equal to $7.3\cdot 10^{-3} m^2/s$, while the correlation between left and right parts of system (3) is 21.6%. In case b) the mean error is $3.5\cdot 10^{-3} m^2/s$, and the correlation is 54.7%.

Results. If we substitute real wind profiles into the Ekman – Akerblom model there is a significant residual. We conjugate information about the real wind profiles with the Ekman – Akerblom model or its generalization. In case b) the optimal coefficient $k = k(z)$ is not strongly positive. The maximum of the new coefficient $\gamma(z)$ is substantially greater than the maximum of $k(z)$.

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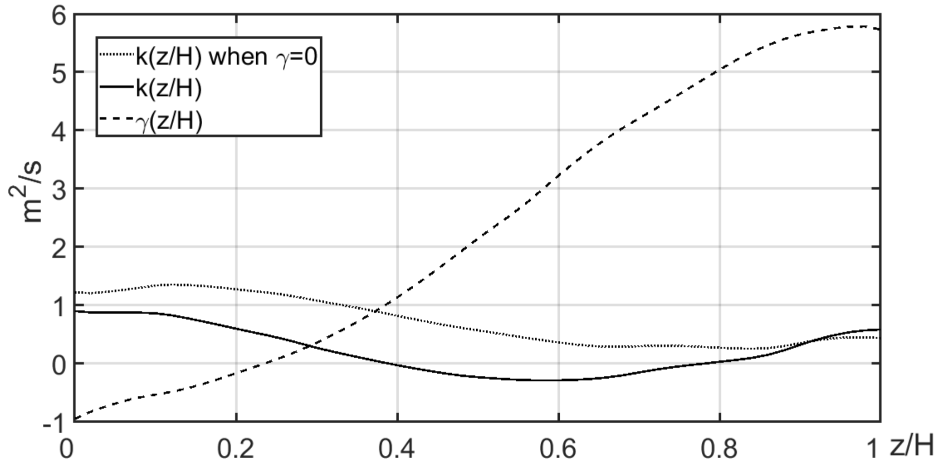

Fig.1. Optimal coefficients (in m^2s^{-1}) a) $\gamma(z/H) \equiv 0$; b) arbitrary functions $k(z/H)$, $\gamma(z/H)$

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