A Nonlinear Dynamical Approach to the Interpretation of Microblogging Network Complexity

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Abstract. The present paper is devoted to the research into complexity of microblogging social networks regardless of their internal structure. This approach assumes using the results of nonlinear dynamical analysis of signals generated by the networks. The existence of the main indicators of social network complexity, such as scale invariance, tendency to unexpected and/or extraordinary events, non-equilibrium state and emergent properties, are shown using the example of Twitter. As a result, it is determined that the probability density function for a Twitter time series is a q-exponential (Tsallis) distribution and that the Kaulakys equation is the most adequate nonlinear random dynamical system for modeling of signals in social networks.

1 Introduction

To begin with, let's define system complexity, which is one of the fundamental concepts of the modern science. In synergetics [1], the system complexity is considered as irreducibility of a system to a simple sum of its parts or even existence of such features that haven't appeared in its parts. The system complexity is closely connected with nonlinearity. In fact, the superposition principle is applicable to a linear system. There is also another concept correlated to the first definition of the system complexity regardless of its internal structure. The system is complex when it shows a nontrivial behavior, generating catastrophic (i.e. unexpected or extraordinary) events. This interpretation of complexity is the most relevant to our current research. For example, in case of Twitter it is possible to consider aggregated signals of tweets and retweets – Twitter time series (TTS).

Social networks have the longest history of various studies in comparison to other network types. It is notable, that exactly in social networks D. Price [2] has empirically discovered the power law of distribution of nodes by the number of ties (which is one of the signs of network complexity) for the first time in 1965. In 1999 the physicists from the University of Notre Dame (USA) A.L. Barabasi and R. Albert established [3,4], that in a large number of networks,

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the distribution of nodes tends to obey the power law (instead of the expected probabilistic distribution of nodes by the Poisson's law). Among the latest papers related to our research, papers [5,6] can be mentioned. The comparison of the results obtained in the above-mentioned papers to our research will be shown later as required in the corresponding sections of the present paper. Some other relevant works in this area are those of Refs. [7–11].

These papers are devoted to studies of network complexity, that were based on their structural features (for example, see [8]): non-trivial topological features, such as the lattices or random graphs. In our paper, we use simpler (considering that it doesn't require information about the network structure), and even more substantial (in some sense) approach to the study of complexity. This approach is based on the study of the above-mentioned features of system complexity by means of research into the results of nonlinear-dynamical, statistical and spectral analysis of signals (eg.: TTS for Twitter) in microblogging networks. In other words, the key question of our study is the following one: Are the microblogging networks complex in terms of the above-mentioned definition of complexity? By "more substantial" we consider the possibility of building the qualitative nonlinear-dynamical models of social networks, that have solutions in a form of TTS.

Therefore, this paper is organized as follows. The key features of complexity are considered in the 2^{nd} section. The results of the nonlinear dynamical analysis of the empirical TTS, probability density functions (PDFs) and periodograms for the empirical TTS are presented in the 3^{rd} section. In the 4^{th} section we show the results of nonlinear analysis and PDFs for a sample of 3-dimensional nonlinear dynamical model of Twitter network as an open nonequilibrium system [12], as well as comparison with empirical results. We provide the results of nonlinear analysis and PDFs for the model of Twitter network as nonlinear random dynamical system comparing them with empirical results and describe the possibilities of applying the Tsallis entropy for analysis of TTS. The 5^{th} section includes the conclusions of this paper.

2 Key Features of Complexity

Complexity is closely connected with nonlinearity of a system from the mathematical point of view, but its description from the physical point of view is usually possible just in such statistical terms as PDF, autocorrelation, power spectral density (PSD), invariant measures of dynamical chaos, etc.

There are currently 3 paradigms in nonlinear dynamics. The first paradigm is related to the research into the dissipative structures [13]; the second one is based on studies of deterministic chaos [14]; the third one is devoted to the analysis of complexity. Per Bak et al. pioneered the use of this paradigm [15–17].

The key features defining the system complexity are scale invariance, accident proneness (the possibility of the catastrophic events), nonequilibrium and emergent behaviors [17].

The scale invariance means that events or objects do not have their own characteristic dimensions, duration, energy, etc. Such systems usually have a power spectral density (PSD) of the following type:

$$S(f) \sim f^{-\beta}. \tag{1}$$

The case of $\beta = 1$, or pink noise, is both the canonical case, and the one of most interest, but the more general form, where $0 < \beta \leq 3$, is sometimes referred to simply as $1/f$. $1/f^{\beta}$ noise is of interest because it occurs in many different systems, both in the natural world and in man-made processes. This PSD is an evidence of a potential proneness of systems to the appearance of huge fluctuations, i.e. the internal susceptibility to accidents. This allows to assume that the system is in the neighborhood of a critical point. Mechanisms of self-organized criticality in social knowledge creation process are presented in the paper [5]. It's shown that the long-range correlations and the event clustering are primarily determined by the universal social dynamics, providing the external driving of the system by the arrival of new users. Authors [5] compare the social avalanches to the avalanche sequences occurring in the field-driven physical model of disordered solids, where the factors contributing to the collective dynamics are better understood.

The statistical form of the scale invariance is a PDF of the following kind at $x \to \infty$:

$$p(x) \sim x^{-(1+\gamma)}, \gamma > 0. \tag{2}$$

The PDF (2) belongs to the class of the power law PDFs or the fat-tailed PDFs. The fundamental difference of the PDF (2) from the compact distributions is the fact, that those events, which fall on the distribution tail area, take place not so rarely to be neglected.

The fractal structure of a system is another indicator of a scale invariance. According to one of the definitions, a fractal is an object, where the Huasdorff–Besicovitch dimension is higher than its topological dimension [18]. The studies of Mandelbrot [18] have proved, that the fractals are the systems with the scale invariance.

If a system is prone to accidents, then the small causes can lead to the big consequences. This fact assumes the considerable nonequilibrium. Equilibrium systems can't be complex. In addition, the appearance of the catastrophes is possible just in case of the emergent behavior of different system elements, i.e. when the system has emergent features. The ability of the system to have a "long" memory for its past, and the ability of the system elements to "feel" each other wide-apart, can be considered by the emergent behavior. At the statistical level the emergent behaviors are usually related to the long-range time and space correlations. The matter concerns the time-series with the long memory, or those time-series, autocorrelation of which decreases slowly. The expression "a long-range dependence", which is sometimes used to refer to $1/f^{\beta}$ noise, has also been used in the other contexts with somewhat different meanings. "Long memory" and other variants are also sometimes used in the same way.

3 Analysis of Empirical Twitter Time Series

For analysis of empirical TTS, we chose the following time series obtained from the resource Mozdeh "Big Data Text Analysis" (http://mozdeh.wlv.ac.uk/): bbc_breaking, from 16/05/29 to 17/05/26, with the 1 h step; cnn_breaking, from 16/07/12 to 17/01/11, with the 1 h step; nasa, from 16/09/26 to 17/05/26, with the 1 h step. It is clear, that these time series represent impulse-type signals with integer values. TTS have non-continuous values from 0 to 15 for bbc_breaking, from 0 to 28 for cnn_breaking and from 0 to 17 for nasa. Figure 1 provides an example of empirical TTS.

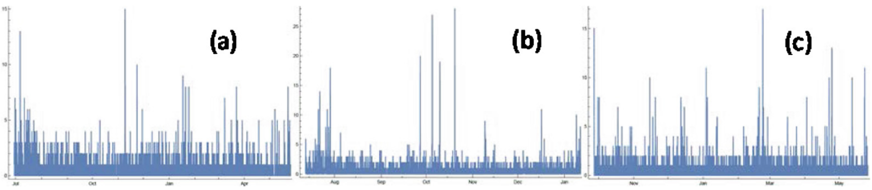

Fig. 1. Twitter time series: (a) bbc_breaking, (b) cnn_breaking, (c) nasa

The nonlinear analysis was conducted for all chosen TTS. Such measures as correlation dimension (D_2), embedding dimension (m), Hurst exponents (H), power of PSD (β) and fractal dimension (D_F) were calculated (Table 1).

Table 1. Results of nonlinear TTS analysis

User	D_2	m	H	β	D_F
cnn_breaking	3.732	6	0.6648	2.329	1.3352
bbc_breaking	3.984	6	0.6165	2.233	1.3835
nasa	4.202	6	0.6833	2.367	1.3167

Determination of the correlation dimension [19] for a supposed chaotic process directly from experimental time series is often used to get information about the nature of the underlying dynamics (see, for example, contributions in Ref. [20]). In particular, such analysis has been made to support the hypothesis that the time series are generated from the inherently low-dimensional chaotic process [20]. The geometry of chaotic attractors can be complex and difficult to describe. It is therefore useful to understand quantitative characterizations of such geometrical objects. One of these characterizations is D_2. D_2 of the attractor of dynamical system can be estimated using the Grassberger–Procaccia algorithm [19].

D_2 has several advantages in comparison to the other dimensional measures:

- if D_2 is finite, then a TTS is a chaotic time series (generated by a dynamical system);
- if D_2 is infinite, then a TTS is a stochastic time series (generated by a purely random process).

For calculation of D_F we used the algorithm described in a paper [21]. If $D_F > D_T$ (D_T is a topological dimension of the TTS, that equals 1 for all time series), then the TTS is a random fractal.

A value of $H = 2 - D_F$ allows to give a noise classification ($1/f$-classification, where f is a signal frequency) of the TTS [22]:

- if $0 < H < 0.5$, then the TTS is characterized by anti-persistence (the time series changes the tendency more often, than a series of independent random variables) and represents a process with $1/f$ noise or a pink noise;
- if $0.5 < H < 1$, then the TTS is characterized by persistence (the time series is characterized by the effect of the long memory and has an inclination to follow the trends) and represents a process with $1/f^\beta$ ($\beta > 2$) noise or a black noise;
- if $H = 0.5$, then TTS represents a process with the absence of memory or a white noise.

Figure 2 provides periodograms as estimates of the spectral density of TTS.

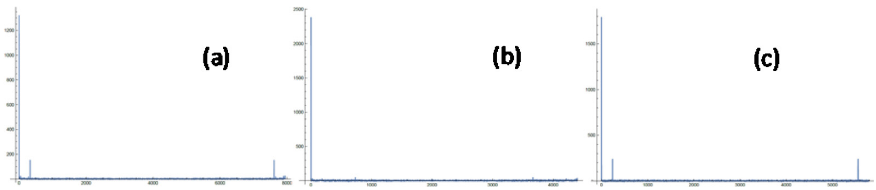

Fig. 2. Periodograms of TTS: (a) bbc_breaking, (b) cnn_breaking, (c) nasa

Empirical TTS is a black random process. As it is shown at the Fig. 2, it is clear that the spectral density predominantly has zero power excluding some spikes. Most of the time series, which can be observed in existence, can usually be related to one of the above-mentioned classes [23,24]. Thus, the time series observed in turbulence processes, show the best correlation with the pink noise. The black noises can be registered in floods, a solar activity, statistics of the natural and induced catastrophes. The black noise indicates long term persistence and long memory.

There is a simple scaling relation, connecting β and H [22]: $\beta = 2H + 1$. The results for β are shown in a Table 1.

Apart from some features and a general form of PDF, nonlinear TTS analysis allows to define the main features of system complexity. Point and interval estimates of the parameters of empirical PDF are the questions of statistical analysis.

Figure 3 provides PDFs for empirical TTS. Visually, these PDFs correspond to the exponential (compact distribution) or to the generalized exponential fat-tailed PDF (2), for example, the q-exponential distribution [25–27]:

$$p(x) = (2 - q)\,\lambda \exp_q(-\lambda x), \tag{3}$$

where $\exp_q(x) \equiv [1 + (1 - q)\,x]^{\frac{1}{1-q}}$.

The distribution (3) is a two-parameter generalization (q <2 is a shape parameter, λ>0 is a rate parameter) of the one-parameter exponential distribution. Table 2 contains the estimated values for parameters of PDF (3) obtained by the maximum likelihood method [28].

Table 2. Point and interval estimations of the PDF (4) parameters and γ-parameters of the PDF (2)

User	q	λ	γ
cnn_breaking	1.505 ± 0.005	1.980 ± 0.074	0.980
bbc_breaking	1.675 ± 0.025	1.482 ± 0.086	0.481
Nasa	1.734 ± 0.038	1.362 ± 0.069	0.362

From the Table 2 we conclude that empirical PDF corresponds to q-exponential distribution. Besides, the values of γ-parameters, calculated with consideration of (2) and (3) are presented: $\gamma = (2 - q)/(q - 1)$.

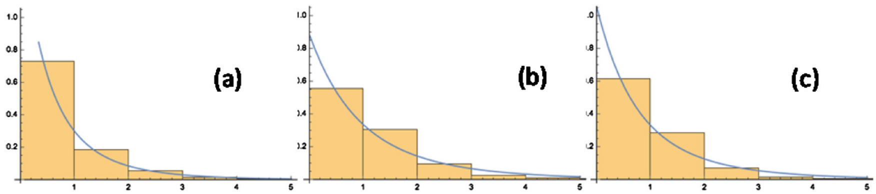

Fig. 3. Histograms of TTS: (a) bbc_breaking, (b) cnn_breaking, (c) nasa

Thus, according to the point values of measures, shown in the Table 1 and the Table 2, the following conclusions can be made:

- TTS is a chaotic time series, i.e. it is generated by dynamical systems in a phase space dimension that equals 6;
- TTS has a fractal structure ($D_F > D_T$);
- TTS represents processes with the long memory or long-range persistence ($0.5 < H < 1$);
- TTS is a signal with $1/f^{\beta}$ noise (β-values vary from 2.233 to 2.367);
- PDF is a fat-tailed PDF ($(1 + \gamma)$-values vary from 1.362 to 1.980).

In a certain way similar results of analysis for the time series in ddDiggs and Ubuntu chats data are shown in the paper [6].

4 Twitter Time Series as a Realization of the Nonlinear Dynamical System

The results of nonlinear and statistical analysis of TTS, which are shown in the Sect. 3, allows to construct a qualitative model of Twitter. The solutions of these models are time series with the established features.

Paper [12] proposes a model of Twitter social network as an open nonequilibrium system. Omitting the detailed construction of dynamical system, the model of Twitter is described by the well-known Lorenz–Haken equation:

$$\dot{x}_t = -\alpha x_t + \beta y_t, \dot{y}_t = -\gamma y_t + c y_t z_t, \dot{z}_t = \varepsilon \left(I_0 - z_t\right) + k x_t y_t. \tag{4}$$

In Eq. (4) $x_t = TR_t - TR_{eq}$ represents the scaled deviation of number of tweets and retweets (TR_t) from the equilibrium value TR_{eq}; $y_t = I_t - I_{eq}$ is the scaled deviation of aggregated internal amount of information (I_t) from equilibrium value I_{eq}; $z_t = N_t^{|u\rangle} - N_t^{|l\rangle}$ is instantaneous difference in a number of users between the state $|u\rangle$ and the state $|l\rangle$. According to the model, a particular user, being in the$|u\rangle$-state, has enough information for sending tweet or retweet. If the user is in a $|l\rangle$-state (so, he or she does not have enough amount of information), then he or she will not send any tweets or retweets. Control parameter I_0 shows the intensity of external information flow.

The most important conclusions from the model implementation are: impossibility of social network to be in an equilibrium state and occurrence of low-dimensional chaos in social network for the significant intensity of external information flow I_0. Except for values of higher Lyapunov exponent [29] as one of the measures of low-dimensional chaos, paper [12] does not contain calculated fractal dimensions for observed TTS.

3-dimensional dynamical model (4) explains some properties of social network functioning such as fractality ($D_F = 1.4972$), chaotic nature ($D_2 = 1.896$) and absence of memory ($H = 0.5028$) of TTS. The weakness of this model lies in significant discrepancy between empirical and theoretical trajectories of TTS. Moreover, it is impossible to fit theoretical trajectories to observed data by varying control parameters (in a range of chaotic state) of the dynamical system. The dynamical system (4) has 3 equilibrium points for any values of control parameters in a range of chaotic state. Therefore, theoretical PDF is a three-modal distribution (three maxima of the PDF) and x_t is a white random process. This PDF is not fat-tailed distribution. The white noise signal is not a signal of catastrophic events.

There are at least two possible ways to achieve the accordance between empirical and theoretical TTS: by adding specific noise to dynamical system (4) or by using a one-dimensional nonlinear random dynamical system [30] as a model of Twitter network. According to the Table 1 at $m = 6$ the estimated value of correlation dimension reaches its “saturation point” and stops changing significantly. Because of that, the actual number of variables for constructing an adequate model is 6, but not 3 as it is for model (4).

In 1998 Bronislovas Kaulakys et al. [31] proposed a nonlinear random dynamical system (RDS) as a basic model generating signals with $1/f^{\beta}$ noise and fat-tailed PDF:

$$dx_t = \sigma^2 \left(\eta - \frac{1}{2}\lambda \right) (x_t + x_0)^{2\eta - 1} \, dt + \sigma \, (x_t + x_0)^{\eta} \, dW_t, \tag{5}$$

where $x_t \geq 0$ is a signal, $\eta \neq 1$ is a power-law exponent of the multiplicative noise, $\lambda > 0$ is a parameter defining the behavior of the stationary PDF, W is a standard Wiener process, σ is a parameter of the multiplicative noise. Parameter x_0 limits the divergence of the power-series distribution x_t by $x_t \to 0$. If $x << x_0$, then (5) generates a linear additive stochastic process (Brownian movement with the stable drift); if $x >> x_0$, then (5) generates a multiplicative process [32]. If $x_0 = 1$, then the stationary solution of the equation (5) takes the form of a q-exponential distribution (3) by $q = 1 + 1/\lambda$ [33]. Besides, some realizations of the process (5) give a power spectral density in a form of $1/f^{\beta}$. Therefore, the random process (5) generates the time series with the long memory [34].

The random multiplicative process for the interevent time $\tau_k = t_{k+1} - t_k$ [35] is used as a basis for (5):

$$\tau_{k+1} = \tau_k + \gamma \tau_k^{2\mu - 1} + \sigma \tau_k^{\mu} \varepsilon_k. \tag{6}$$

In the model the interevent time τ_k fluctuates due to the random perturbations by a sequence of uncorrelated normally distributed random variables $\{\varepsilon_k\}$ with zero expectation and unit variance; σ is the standard deviation of the white noise and $\gamma << 1$ is a coefficient of the nonlinear damping [35]. The existence of the random fluctuating interevent time of TTS is determined by the fact, that the empirical values of TTS take the non-continuous values from 0 to some finite natural number in a random way. Therefore, despite the fact that empirical values of TTS have a definite constant step (1 hour), appearance of the random interevent time is determined by the random appearance of the zero-values of TTS.

Thus, the random dynamical system (5) is the most adequate model of Twitter.

q-exponential distribution takes place by the maximization of the Tsallis entropy [36] considering definite limitations. Tsallis entropy as a non-additive generalization of the Boltzmann–Gibbs entropy has the following form:

$$T_q = \frac{1}{q - 1} \left(1 - \sum_{i=1}^{N} p_i^q \right) \tag{7}$$

The probability $p_i = N_i / N(\varepsilon)$ can be estimated in much the same way as that one used in the Renyi entropy: N_i is a number of system elements for the i-element of the ε-partition; $N(\varepsilon)$ – is a full the given ε-cover. If $q \to 1$, then the entropy (6) transforms into the well-known Shannon entropy. In contrast to all entropy types, the Tsallis entropy is nonadditive. Being applied to the microblogging network (such as, for example, Twitter) it gives a possibility to correctly describe a social network, where any user interacts not only with the nearest user or several nearest users, but also with the whole network or some of its parts. Besides, from (6) it follows that T_q is concave by $q > 0$ and convex by $q < 0$. For example, the non-additive entropy describes emotion dynamics that is confirmed by computing the q-generalized Kolmogorov–Sinai entropy rate in the empirical data of chats as well as in the simulations of interacting emotional agents and Bots [6].

Thus, the entropy description of Twitter, based on Tsallis statistics is appropriate for studying of evolution of a social network that contains a large number of users who interact with each other in a particular way and specifically every user can interact not only with his or her nearest neighbors but also with remote users.

5 Conclusions

As a result of the present study it can be noted, that the microblogging social networks, and Twitter in particular, are the complex networks. The social networks are characterized by the non-trivial behavior, i.e. they are able to generate the "catastrophic" (unexpected and/or extraordinary) cases. An example of such an "accident" in case of Twitter is a transition of the network from the egocentric into the polycentric state [37].

Microblogging social networks have all features of system complexity [15–17]:

- The scale invariance of the microblogging social networks. TTS represent the random processes with $1/f^{\beta}$noise, where β varies from 2.233 to 2.367 (Table 1), fat-tailed PDF with γ from 0.362 to 0.980 (Table 2) and a fractal structure ($D_F > D_T$, see Table 1).
- Microblogging social networks can be considered as nonequilibrium open systems. The proof of this statement is shown in a paper [12]. In brief, the social network includes users, who can have just two states: a ground state and an excited state. Those users, who didn't get sufficient amount of information from the mass media and other sources to be able to send tweets, stay in the ground state. Those users, who got sufficient amount of external information to be able to send tweets, are in the excited state. By sending tweets the network users transfer from the excited state to the ground state. This information flow, in some sense, "pump up" the social network, making inverse population of users. Taking into account continuous information pumping, social network is always functioning in nonequilibrium state, making "avalanches" of tweet and retweet. Due to information pumping, the equilibrium state is almost unreachable. It is crucially important, that existence of chaotic states is a fundamental property of nonequilibrium open systems [38,39]. Indeed, TTS is a chaotic time series: D_2 is finite (Table 1).
- Microblogging social networks have emergent properties. Indeed, TTS represents processes with the long memory or long-range persistence: $0.5 < H < 1$ (Table 1).

We have conducted a research into the empirical PDF of some TTS to construct a model of the microblogging network in a form of one-dimensional non-linear RDS (5). As a result, it has been recognized that the observable PDF has a q-exponential distribution (Table 2). For such distribution, the one-dimensional nonlinear RDS has been suggested. It has been shown, that in contrast to all entropy types, the Tsallis entropy gives a possibility to correctly describe a network, where any user interacts not only with the nearest user or several nearest users, but also with the whole network or some of its parts. Use of the Tsallis entropy allows to describe the macroscopic stability of a microblogging network as well.

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