Anisotropy-based Analysis for Descriptor Systems with Norm-Bounded Parametric Uncertainties

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***Abstract*—In this paper, linear discrete-time descriptor systems with norm-bounded parametric uncertainties are under consideration. The input signal is supposed to be a "colored" noise with bounded mean anisotropy. Sufficient conditions of anisotropic norm boundedness for such class of systems are given.**

I. Introduction

Robust control has been one of the most popular research areas in control theory in the recent years. Considerable attention is paid to problems of robust stabilization and robust performance of uncertain normal dynamical systems in both continuous and discrete-time cases [1]–[3]. A great number of fundamental results based on the theory of normal state-space systems is successfully extended on descriptor systems [4], [5]. Interest in stability analysis and control of descriptor systems with parametric uncertainties has grown recently due to their frequent presence in dynamical systems, which are often causes for instability and bad performance of control systems. It is known that control of uncertain descriptor systems is much more complicated than that of the normal ones.

In discrete-time $\mathcal{H}_\infty$-approach the input signal is assumed to be square summable, i.e. it has to be with limited power. The so-called bounded real lemma has played an important role in solving this problem. In context of both normal and descriptor systems, a great number of results on $\mathcal{H}_\infty$-control are reported and different approaches are proposed in [3], [4].

Anisotropy-based approach deals with the stationary random Gaussian signals with known mean anisotropy level, which has a sense of "spectral color" of the signal [6], [7]. Anisotropic norm defines a performance index of the system. The feature of anisotropy-based approach is that anisotropic norm of the system lies between the scaled $\mathcal{H}_2$-norm and $\mathcal{H}_\infty$-norm. Hence, solving performance analysis problem for anisotropy-based case automatically solves $\mathcal{H}_2$- and $\mathcal{H}_\infty$-analysis problems. Some results on anisotropy-based robust control design and performance analysis for normal systems with norm-bounded uncertainties are obtained in [8], [9]. These results are based on the solution of algebraic Riccati equations.

The problems of anisotropy-based performance analysis and suboptimal control for descriptor systems are studied in [10]. The obtained result provides numerically effective method of control design for descriptor systems. The aim of this paper is to obtain conditions on anisotropic norm boundedness for descriptor systems with norm bounded uncertainties.

II. Preliminaries

A linear discrete-time stationary descriptor system in the state-space representation is written as

$$Ex(k+1) = Ax(k) + Bw(k), \tag{1}$$
$$y(k) = Cx(k) + Dw(k) \tag{2}$$

where $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^p$, and $W = \{w(k)\}_{k\in\mathbb{Z}}$ is a stationary sequence of square-summable random m-dimensional vectors with a bounded mean anisotropy $\overline{\mathbf{A}}(W) \leqslant a$ ($a \geqslant 0$ is known), E is a singular matrix, i.e. $\operatorname{rank} E = r < n$.

The transfer function of the system (1)–(2) is defined by the expression

$$P(z) = C(zE - A)^{-1}B + D,\ z \in \mathbb{C}.$$

$\mathcal{H}_2$- and $\mathcal{H}_\infty$-norms of the system (1)–(2) are defined as follows

$$\|P\|_2 = \left(\frac{1}{2\pi}\int_0^{2\pi} \operatorname{tr}\big(P^*(e^{i\omega})P(e^{i\omega})\big)d\omega\right)^{\frac{1}{2}},$$

$$\|P\|_\infty = \sup_{\omega\in[0,2\pi]} \overline{\sigma}\left(P(e^{i\omega})\right).$$

Here, $P^*(e^{i\omega}) = P^{\mathrm{T}}(e^{-i\omega})$ is a conjugate system, $\overline{\sigma}(A) = \sqrt{\max_j |\lambda_j(A^{\mathrm{T}}A)|}$ is a maximal singular value of the matrix A.

Definition 1: The system (1) is called admissible if it is

1) regular ($\exists\lambda : \det(\lambda E - A) \neq 0$),
2) causal ($\deg\det(\lambda E - A) = \operatorname{rank} E$),
3) stable ($\rho(E, A) = \max|\lambda|_{\lambda\in z|\{\det(zE-A)=0\}} < 1$).

For more information, see [4], [11].

The sequence W can be generated from the Gaussian white noise sequence $V = \{v(k)\}_{k\in\mathbb{Z}}$, $v(k) \in \mathbb{R}^m$ with zero mean and identity covariance matrix by an admissible shaping filter with a transfer function $G(z) = C_G(zE_G - A_G)^{-1}B_G + D_G$.

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Mean anisotropy of the signal is Kullback-Leibler information divergence from the probability density function (p.d.f.) of the signal to the p.d.f. of the Gaussian white noise sequence.

Mean anisotropy of the sequence may be defined by the filter's parameters, using the expression

$$\overline{\mathbf{A}}(W) = -\frac{1}{4\pi}\int_{-\pi}^{\pi} \ln\det\frac{mS(\omega)}{\|G\|_2^2}d\omega$$

where $S(\omega) = \widehat{G}(\omega)\widehat{G}^*(\omega)$, $(-\pi \leqslant \omega \leqslant \pi)$, $\widehat{G}(\omega) = \lim_{l\to 1} G(l\mathrm{e}^{i\omega})$ is a boundary value of the transfer function $G(z)$.

Remark 1: Mean anisotropy of the random sequence W, generated by the shaping filter $G(z)$, is fully defined by its parameters, so the notations $\overline{\mathbf{A}}(G)$ and $\overline{\mathbf{A}}(W)$ are equivalent.

Mean anisotropy of the signal characterizes its "spectral color", i.e. the difference between the signal and the Gaussian white noise sequence. For more information, see [6], [7].

Definition 2: For a given constant value $a \geqslant 0$ a-anisotropic norm of the system P is defined as

$$|\!|\!| P |\!|\!|_a = \sup\left\{\|PG\|_2/\|G\|_2 :\; G \in \mathbf{G}_a\right\},$$

i.e. the maximum value of the system's gain with respect to the class of shaping filters

$$\mathbf{G}_a = \left\{G \in \mathcal{H}_2{}^{m\times m} :\; \overline{\mathbf{A}}(G) \leqslant a\right\}.$$

So, a-anisotropic norm $|\!|\!| P |\!|\!|_a$ describes the stochastic gain of the system P with respect to the input sequence W.

For the regular system (1) there exist two nonsingular matrices [11] X and Y such that $E_d = XEY = \mathrm{diag}(I_r, 0)$.

Consider the following change of variables

$$Y^{-1}x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{3}$$

where $x_1(k) \in \mathbb{R}^r$ and $x_2(k) \in \mathbb{R}^{n-r}$.

By left multiplying of the system (1)–(2) on the matrix X and using the change of variables (3), one can rewrite the system (1)–(2) in the form [11]

$$\begin{aligned} x_1(k+1) &= A_{11}x_1(k) + A_{12}x_2(k) + B_1w(k), &(4)\\ 0 &= A_{21}x_1(k) + A_{22}x_2(k) + B_2w(k), &(5)\\ y(k) &= C_1x_1(k) + C_2x_2(k) + Dw(k) &(6) \end{aligned}$$

where

$$A_d = XAY = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\; B_d = XB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$
$$C_d = CY = \begin{bmatrix} C_1 & C_2 \end{bmatrix},\; D_d = D.$$

Matrices X and Y are found from the singular value decomposition (SVD)

$$E = U\,\mathrm{diag}(S, 0)H^{\mathrm{T}}.$$

Here U and H are real orthogonal matrices, S is a diagonal $r\times r$-matrix, it is formed by nonzero singular values of the matrix E

$$X = \mathrm{diag}(S^{-1}, I_{n-r})U^{\mathrm{T}}, \qquad Y = H.$$

Representation (4)–(6) is called *SVD canonical form* [11].

In [10] the condition of anisotropic norm boundedness for descriptor systems SVD canonical form is formulated as follows.

Theorem 1: Let the system (1)–(2) with a transfer function $P(z) = C(zE - A)^{-1}B + D \in \mathcal{H}_\infty{}^{p\times m}$ be admissible. The following rank assumption is true:

$$\mathrm{rank}\begin{bmatrix} E & B \end{bmatrix} = \mathrm{rank}\, E. \tag{7}$$

For given scalar values $a \geqslant 0$ and $\gamma > 0$ a-anisotropic norm of the system is bounded by γ, i.e.

$$|\!|\!| P |\!|\!|_a < \gamma$$

if there exist matrices $L \in \mathbb{R}^{r\times r}$, $L > 0$, $Q \in \mathbb{R}^{r\times r}$, $R \in \mathbb{R}^{r\times(n-r)}$, $S \in \mathbb{R}^{(n-r)\times(n-r)}$,$\Psi \in \mathbb{R}^{m\times m}$, scalar values $\eta > \gamma^2$ and $\alpha > 0$, for which the following inequalities hold true:

$$\eta - (\mathrm{e}^{-2a}\det(\Psi))^{1/m} < \gamma^2, \tag{8}$$

$$\begin{bmatrix} \Psi - \eta I_m + B_d^{\mathrm{T}}\Theta B_d & D_d^{\mathrm{T}} \\ D_d & -I_p \end{bmatrix} < 0 \tag{9}$$

and (10)

where $\Theta = \begin{bmatrix} L & 0 \\ 0 & 0 \end{bmatrix}$, $\Pi = \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}$, $\Gamma = \begin{bmatrix} Q & R \end{bmatrix}$.

The proof of the main result is based on the following lemmas.

Lemma 1: (Petersen [12]) Let matrices $M \in \mathbb{R}^{n\times p}$ and $N \in \mathbb{R}^{q\times n}$ be nonzero, and $G = G^{\mathrm{T}} \in \mathbb{R}^{n\times n}$. The inequality

$$G + M\Delta N + N^{\mathrm{T}}\Delta^{\mathrm{T}}M^{\mathrm{T}} \leqslant 0$$

is true for all $\Delta \in \mathbb{R}^{p\times q}$: $\overline{\sigma}(\Delta) \leqslant 1$ if and only if there exists a scalar value $\varepsilon > 0$ such that

$$G + \varepsilon MM^{\mathrm{T}} + \frac{1}{\varepsilon}N^{\mathrm{T}}N \leqslant 0.$$

Lemma 2: (Schur [13]) Let

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}$$

where X_{11} and X_{22} are square matrices.

If $X_{11} > 0$, then $X > 0$ if and only if

$$X_{22} - X_{12}^{\mathrm{T}}X_{11}^{-1}X_{12} > 0.$$

If $X_{22} > 0$, then $X > 0$ if and only if

$$X_{11} - X_{12}X_{22}^{-1}X_{12}^{\mathrm{T}} > 0.$$

III. Problem statement and main result

A linear discrete-time descriptor system is given by

$$\begin{aligned} Ex(k+1) &= (A + A_\Delta)x(k) + (B + B_\Delta)w(k), &(11)\\ y(k) &= (C + C_\Delta)x(k) + (D + D_\Delta)w(k) &(12) \end{aligned}$$

where $A_\Delta = M_A\Delta N_A$, $B_\Delta = M_B\Delta N_B$, $C_\Delta = M_C\Delta N_C$, $D_\Delta = M_D\Delta N_D$, $x(k) \in \mathbb{R}^n$ is a state vector, $W = \{w(k)\}_{k\in\mathbb{Z}}$ is a stationary sequence of square-summable random m-dimensional vectors with a bounded mean anisotropy $\overline{\mathbf{A}}(W) \leqslant a$ (the level $a \geqslant 0$ is known), $y(k) \in \mathbb{R}^p$ is an

$$\begin{bmatrix} -\frac{1}{2}Q-\frac{1}{2}Q^{\mathrm{T}} & \Gamma A_d & \Gamma B_d & L^{\mathrm{T}}-Q^{\mathrm{T}}-\frac{1}{2}Q & 0 \\ A_d^{\mathrm{T}}\Gamma^{\mathrm{T}} & \Pi A_d + A_d^{\mathrm{T}}\Pi^{\mathrm{T}} - \Theta & \Pi B_d & A_d^{\mathrm{T}}\Gamma^{\mathrm{T}} & C_d^{\mathrm{T}} + \alpha A_d^{\mathrm{T}}\Pi^{\mathrm{T}}C_d^{\mathrm{T}} \\ B_d^{\mathrm{T}}\Gamma^{\mathrm{T}} & B_d^{\mathrm{T}}\Pi^{\mathrm{T}} & -\eta I_m & B_d^{\mathrm{T}}\Gamma^{\mathrm{T}} & D_d^{\mathrm{T}} + \alpha B_d^{\mathrm{T}}\Pi^{\mathrm{T}}C_d^{\mathrm{T}} \\ L-Q-\frac{1}{2}Q^{\mathrm{T}} & \Gamma A_d & \Gamma B_d & -Q-Q^{\mathrm{T}} & 0 \\ 0 & C_d+\alpha C_d\Pi A_d & D_d+\alpha C_d \Pi B_d & 0 & -I_p \end{bmatrix} < 0 \qquad (10)$$

output, $\Delta \in \mathbb{R}^{q\times q}$ is an unknown spectral norm-bounded matrix: $\|\Delta\|_2 := \overline{\sigma}(\Delta) \leqslant 1$ iff $\Delta^{\mathrm{T}}\Delta \leqslant I_q$ (or Frobenius norm-bounded matrix as $\|\Delta\|_2 \leqslant \|\Delta\|_F$). Matrices E, A, B, C, D, M_A, N_A, M_B, N_B, M_C, N_C, M_D and N_D are constant real of appropriate dimensions. The matrix E is singular, i.e. $\operatorname{rank} E = r < n$. The system (11)–(12) is supposed to be admissible for all Δ from the given set.

The transfer function of the system (11)–(12) is given by $P_\Delta(z) = (C+C_\Delta)(zE-(A+A_\Delta))^{-1}(B+B_\Delta)+(D+D_\Delta)$.

Hereinafter, we suppose that the system (11)–(12) satisfies the following rank conditions:

$$\operatorname{rank} E^{\mathrm{T}} = \operatorname{rank}\left[E^{\mathrm{T}}, C^{\mathrm{T}}, N_C^{\mathrm{T}}\right], \quad \operatorname{rank} E = \operatorname{rank}\left[E, B, M_B\right].$$

For the regular system (11) there exist two nonsingular matrices X and Y, using which we can transform the system (11)–(12) to SVD canonical form (4)–(6). Introduce the following denotations:

$$M_B^d = XM_B = \begin{bmatrix} M_{B1}^d \\ M_{B2}^d \end{bmatrix}, \; N_B^d = N_B, \; M_A^d = XM_A, \; N_A^d = N_A Y, \; M_C^d = M_C, \; N_C^d = N_C Y.$$

The problem is to find for known values $a \geqslant 0$ and $\gamma > 0$ the conditions, which guarantee, that the following inequality is true:

$$\|P_\Delta\|_a < \gamma.$$

Theorem 2: Let the system (11)–(12) in SVD canonical form be admissible. If for known scalars $a \geqslant 0$ and $\gamma > 0$ there exist such scalars $\eta > \gamma^2$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and matrices $Q \in \mathbb{R}^{r\times r}$, $R \in \mathbb{R}^{r\times(n-r)}$, $S \in \mathbb{R}^{(n-r)\times(n-r)}$, $\Psi \in \mathbb{R}^{m\times m}$, $L \in \mathbb{R}^{r\times r}$, $L > 0$, $\Upsilon \in \mathbb{R}^{r\times r}$, $\Upsilon > 0 : \Upsilon L = I_r$, for which the following inequalities hold true:

$$\eta - (\mathrm{e}^{-2a}\det(\Psi))^{1/m} < \gamma^2, \qquad (13)$$

$$\begin{bmatrix} \$ + \varepsilon_1 M_1 M_1^{\mathrm{T}} & N_1^{\mathrm{T}} \\ N_1 & -\varepsilon_1 I \end{bmatrix} < 0, \qquad (14)$$

$$\begin{bmatrix} \Xi + \varepsilon_2 N_2^{\mathrm{T}} N_2 & M_2 \\ M_2^{\mathrm{T}} & -\varepsilon_2 I \end{bmatrix} < 0, \qquad (15)$$

then $\|P_\Delta\|_a < \gamma$.

Here

$$\$ = \begin{bmatrix} \Psi - \eta I_m & D_d^{\mathrm{T}} & B_1^{\mathrm{T}} \\ D_d & -I_p & 0 \\ B_1 & 0 & -\Upsilon \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0 & 0 \\ M_D & 0 \\ 0 & M_{B1}^d \end{bmatrix}, \; N_1 = \begin{bmatrix} N_D & 0 & 0 \\ N_B^d & 0 & 0 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} \Gamma M_A^d & \Gamma M_B^d & 0 & 0 \\ \Pi M_A^d & \Pi M_B^d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Gamma M_A^d & \Gamma M_B^d & 0 & 0 \\ 0 & 0 & M_C^d & M_D \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 0 & N_A^d & 0 & 0 & 0 \\ 0 & 0 & N_B^d & 0 & 0 \\ 0 & N_C^d & 0 & 0 & 0 \\ 0 & 0 & N_D & 0 & 0 \end{bmatrix},$$

$$\Theta = \begin{bmatrix} L & 0 \\ 0 & 0 \end{bmatrix}, \; \Pi = \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}, \; \Gamma = \begin{bmatrix} Q & R \end{bmatrix},$$

Ξ is defined by (16).

Proof: Consider the inequality (9) from the theorem 1. Taking into account the condition (7), we get $B_2 = 0$. Transform the expression $B_d^{\mathrm{T}}\Theta B_d = \begin{bmatrix} B_1^{\mathrm{T}} & 0 \end{bmatrix}\begin{bmatrix} L & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} B_1 \\ 0 \end{bmatrix} = B_1^{\mathrm{T}} L B_1 > 0$. So the inequality (9) is equal to

$$\begin{bmatrix} \Psi - \eta I_m + B_1^{\mathrm{T}} L B_1 & D_d^{\mathrm{T}} \\ D_d & -I_p \end{bmatrix} < 0,$$

using the conditions of lemma 2, we have

$$\begin{bmatrix} \Psi - \eta I_m & D_d^{\mathrm{T}} & B_1^{\mathrm{T}} \\ D_d & -I_p & 0 \\ B_1 & 0 & -\Upsilon \end{bmatrix} < 0. \qquad (17)$$

Now we write the inequality of the form (17) for the admissible system (11)–(12)

$$\begin{bmatrix} \Psi - \eta I_m & (D_d + M_D\Delta N_D)^{\mathrm{T}} & (B_1 + M_{B1}^d \Delta N_B^d)^{\mathrm{T}} \\ D_d + M_D \Delta N_D & -I_p & 0 \\ B_1 + M_{B1}^d \Delta N_B^d & 0 & -\Upsilon \end{bmatrix} < 0 \qquad (18)$$

or

$$\begin{bmatrix} \Psi - \eta I_m & D_d^{\mathrm{T}} & B_1^{\mathrm{T}} \\ D_d & -I_p & 0 \\ B_1 & 0 & -\Upsilon \end{bmatrix} + \operatorname{sym}\left(\begin{bmatrix} 0 & 0 \\ M_D & 0 \\ 0 & M_{B1}^d \end{bmatrix} \Delta \begin{bmatrix} N_D & 0 & 0 \\ N_B^d & 0 & 0 \end{bmatrix}\right) < 0. \qquad (19)$$

Using the conditions of lemmas 1 and 2, we can rewrite the inequality (19) as (14). Now we write the expression (10) for the system (11)–(12) in the form

$$\Xi + \operatorname{sym}(M_2 \Delta N_2) < 0. \qquad (20)$$

Transform the inequality (20), applying lemmas 1 and 2:

$$\Xi + \frac{1}{\varepsilon_2} M_2 M_2^{\mathrm{T}} + \varepsilon_2 N_2^{\mathrm{T}} N_2 < 0,$$

$$\Xi = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^{\mathrm{T}} & \Gamma A_d & \Gamma B_d & L^{\mathrm{T}} - Q^{\mathrm{T}} - \frac{1}{2}Q & 0 \\ A_d^{\mathrm{T}}\Gamma^{\mathrm{T}} & \Pi A_d + A_d^{\mathrm{T}}\Pi^{\mathrm{T}} - \Theta & \Pi B_d & A_d^{\mathrm{T}}\Gamma^{\mathrm{T}} & C_d^{\mathrm{T}} \\ B_d^{\mathrm{T}}\Gamma^{\mathrm{T}} & B_d^{\mathrm{T}}\Pi^{\mathrm{T}} & -\eta I_m & B_d^{\mathrm{T}}\Gamma^{\mathrm{T}} & D_d^{\mathrm{T}} \\ L - Q - \frac{1}{2}Q^{\mathrm{T}} & \Gamma A_d & \Gamma B_d & -Q - Q^{\mathrm{T}} & 0 \\ 0 & C_d & D_d & 0 & -I_p \end{bmatrix} \tag{16}$$

$$\Xi + \varepsilon_2 N_2^{\mathrm{T}} N_2 - M_2(-\varepsilon_2 I)^{-1} M_2^{\mathrm{T}} < 0,$$

$$\begin{bmatrix} \Xi + \varepsilon_2 N_2^{\mathrm{T}} N_2 & M_2 \\ M_2^{\mathrm{T}} & -\varepsilon_2 I \end{bmatrix} < 0.$$

The last inequality coincides with (15). The expression (13) is equal to (8). Consequently, the conditions of the theorem 1 hold true for the system (11)–(12), it means that its anisotropic norm is bounded by a positive scalar value: $\|P_\Delta\|_a < \gamma$. ■

Remark 2: If mean anisotropy level goes to infinity, i.e. $a \to +\infty$, then the inequality (13) holds true for any Ψ, and (14) has no sense. The expression (15) is a sufficient condition of $\mathcal{H}_\infty$-norm boundedness for the system (11)–(12).

Remark 3: Let the system be given by (11)–(12). If $M_B = 0$ and $N_B = 0$, then the conditions of the theorem 2 become simpler

$$\eta - (\mathrm{e}^{-2a} \det(\Psi))^{1/m} < \gamma^2,$$

$$\begin{bmatrix} \$ + \varepsilon_1 M_1 M_1^{\mathrm{T}} & N_1^{\mathrm{T}} \\ N_1 & -\varepsilon_1 I \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Xi + \varepsilon_2 N_2^{\mathrm{T}} N_2 & M_2 \\ M_2^{\mathrm{T}} & -\varepsilon_2 I \end{bmatrix} < 0.$$

Here $\$ = \begin{bmatrix} \Psi - \eta I_m + B_1^{\mathrm{T}} L B_1 & D_d^{\mathrm{T}} \\ D_d & -I_p \end{bmatrix}$,

$$M_1 = \begin{bmatrix} 0 \\ M_D \end{bmatrix}, \; N_1 = \begin{bmatrix} N_D & 0 \end{bmatrix}.$$

M_2, N_2 and Ξ are defined in the theorem 2.

In this case, we do not use the algorithm of mutually inverse matrices computation in order to find Υ.

IV. Conclusion

This paper studies the problem of anisotropy-based performance analysis for the class of uncertain discrete-time linear descriptor systems with norm-bounded parametric uncertainties in all matrices of the system and output equations. The derived above conditions require mutual inverse matrix searching procedure, which can be found, for example, in [14]. It is shown that the stated problem can be solved via matrix inequality approach involving no parametric uncertainties. Thus, the obtained result can be applied to anisotropy-based robust controllers design problem for descriptor systems.

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