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Sustainable Debt Accumulation in the Logistic Model of Global Leverage

The ILMA Workshop Presentation

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National Research University Higher School of Economics, Moscow

Literature

- **Leverage and Financial Instability**
- *Voprosy Economiki*, #9, 1-27, 2012 (in Russian)
- **Logistic Model of Financial Leverage**
- *HSE Economic Journal,vol.17, #4,*585-616, 2013 (in Russian)
- **Financial Assets Collateralization and Stochastic Leverage**
- *HSE Economic Journal,vol.18, #2,* 183-215, 2014 (in Russian)
- **Stochastic Leverage of the Global Financial System,**
- *Proceedings of XVI International April Conference*, NRU HSE, 732-741, Moscow, 2016
- **Stochastic Logistic Model of the Global Financial Leverage**,
- *The BE Journal of Theoretical Economics*, 2018, issue 1
- **Does the Global Leverage Dynamic Gravitate to an Invariant?**
- report at the Seminar of the Department of Theoretical Economics, NRU HSE, 2018
- **Safe Debt Accumulation in the Logistic Model of Financial Leverage**,
- submitted to the Journal of …

The Problem Formulation

1.This is the 5th report on the theme, so I shall concentrate only

on the new problems and findings. There are three clusters of them:

a) Existence of the leverage values corresponding to safe debt and sound money;

b) Exploring differences between micro- and macro-debt. Transformation of debt redemption into its safe refinancing and rolling over;

c) Deleveraging as an empirical process and its modelling.

2. Basic Definitions

Debt is a contractual obligation, or a"promise" of future payments;

Debt is reimbursed with money, and lenders deserve to be compensated at the face value of their loans;

The problem of money collateralization. The real bills doctrine of *J. Law* and *A. Smith*;

The sound money definition, $M(t) / E(t) = 1$;

Macrofinancial balance and ratios

3. The standard macro-balance of financial assets and liabilities.

No global bankruptcies are allowed, but crises of liquidity are possible

Empirical balances are treated for the fixed time-to-maturity parameter T; the actual time $0 < t \leq T$

$$A(t,T) = [M(t) + B(t,T)] + E(t) = D(t,T) + E(t)$$

In terms of leverage assuming that money is sound, $M(t)/E(t) = 1$:

$B_T(t)/E(t) = l(t) - 2$ or $D_T(t) = l(t) - 1$

Global assets, liabilities and some structural parameters

Years (1)	Assets, \$ tn (2)	Debt, \$ tn (3)	Money \$tn (4)	Bonds, \$tn (5)	Equity \$ tn (6)	Leverage, l_t (7)	Rate of growth $\frac{\Delta l_{t+1}}{l_t}$ (8)	Spread a_t (9)	Spread c_t (10)	Parameter, b_t (11)	Purchasing power, θ (12)	Collateralization ratio, ν (13)
1999	117.1	83.6			33.5	3.51						
2000	116.0	85.0	34.1	50.9	31.0	3.75	0.069	-0.025	-0.087	-0.007	3.4	0.91
2001	112.5	85.6	32.4	53.2	26.9	4.2	0.118	-0.039	-0.144	-0.011	3.47	0.83
2002	118.4	95.6	33.8	61.8	22.8	5.21	0.246	-0.064	-0.268	-0.015	3.5	0.68
2003	142.9	111.7	38.2	73.5	31.2	4.59	-0.121	0.038	0.201	0.007	3.74	0.81
2004	162.5	125.8	42.3	83.5	36.7	4.43	-0.033	0.011	0.051	0.002	3.84	0.87
2005	168.3	127.9	45.2	82.7	40.4	4.17	-0.059	0.019	0.085	0.004	3.72	0.89
2006	194.4	144.4	49.8	94.6	50.0	3.88	-0.067	0.027	0.113	0.007	3.9	1.01
2007	227.1	166.8	56.3	110.5	60.3	3.77	-0.031	0.012	0.046	0.003	4.0	1.07
2008	204.0	171.7	64.1	107.6	32.3	6.33	0.676	-0.13	-0.494	-0.034	3.18	0.51
2009	229.4	184.8	66.8	118.0	44.6	5.15	-0.187	0.048	0.305	0.008	3.43	0.67
2010	243.9	192.4	71.9	120.5	51.5	4.74	-0.078	0.023	0.117	0.005	3.39	0.72
2011	244.6	200.2	80.7	119.5	44.4	5.5	0.171	-0.038	-0.18	-0.008	3.03	0.55
2012	263.0	211.9	84.3	127.6	51.1	5.16	-0.07	0.017	0.091	0.003	3.12	0.61
2013	274.6	214.4	87.2	127.2	60.2	4.55	-0.118	0.033	0.169	0.006	3.15	0.69
2014	275.9	212.4	91.6	120.8	63.5	4.33	-0.048	0.014	0.064	0.003	3.01	0.69
2015	270.8	208.9	90.4	118.5	61.9	4.37	0.009	-0.003	-0.011	-0.001	3.0	0.68
2016	281.3	216.4	101.1	115.3	64.9	4.33	-0.009	0.003	0.014	0.001	2.78	0.64
2017	316.4	237.2			79.2	3.98	0.081	0.029	0.124	0.007		

{Sources: World Bank, Institute of International Finance, Author's estimations}

Zero-coupon riskless macro-bond

4. The basic "building block" of the fixed-income finance is a pure riskless zero-coupon bond:

$$B(t,T)\exp[y(t,T)(T-t)] = M(T);\ 0 \leq t \leq T$$

where $B(t,T)$ is the debt contractual obligation;
$y(t,T)$ is the spot interest rate (yield-to-maturity);
$M(T,T) = M_B(T)$ is the debt principal or its "promised" money equivalent at $t = T$.

The stylized debt reimbursement

5. The stylized process of the macro-debt reimbursement

$$Lim_{T \to t} B(t,T) = B(t,t) \equiv M_B(t)$$

where $M_B(t)$ is the money equivalent of a loan. Hence money is a debt which is repaid instantaneously.

Banknote, digital money. Money is an instantaneous debt.

The actual process of the macro-debt reimbursement is a financial crisis hence it is a highly undesirable event.

Rates of return in the fixed-income finance

6. a) the spot interest rate: $y(t,T) = -\ln B(t,T)/(T-t)$;

b) the forward rate of return: $f(t,T) = -\partial \ln B(t,T)/\partial T$;

c) the instantaneous rate of return: $r(t) = y(t,t) = -\partial \ln B(t,t)/\partial t$;

is a price of ultra-short credit like EONIA, SONIA, RUONIA

d) "Money does not beget money" (Aristotle):

$$B(t,t)\exp[y(t,t)(t-t)] = B(t,t);\ 0 < t \le T$$

{Ineffectiveness of monetary stimuli within the QE policy}

The existence of critical leverage values

7. There exist leverage values at which the debt is fully reimbursed with sound money:

$$l(t) = 2;\ B_T(t) / E(t) = 0 \Rightarrow B_T(t) = 0 \approx D_T = M_B(t);$$

$$l(t) = 3;\ B_T(t) / E(t) = 1 \Rightarrow B_T(t) = E(t) \approx B_T(t) = \overset{\frown}{M}(t) = E(t);$$

$$Hence \quad 2 \le l \le 3$$

Feasibility: lower boundary

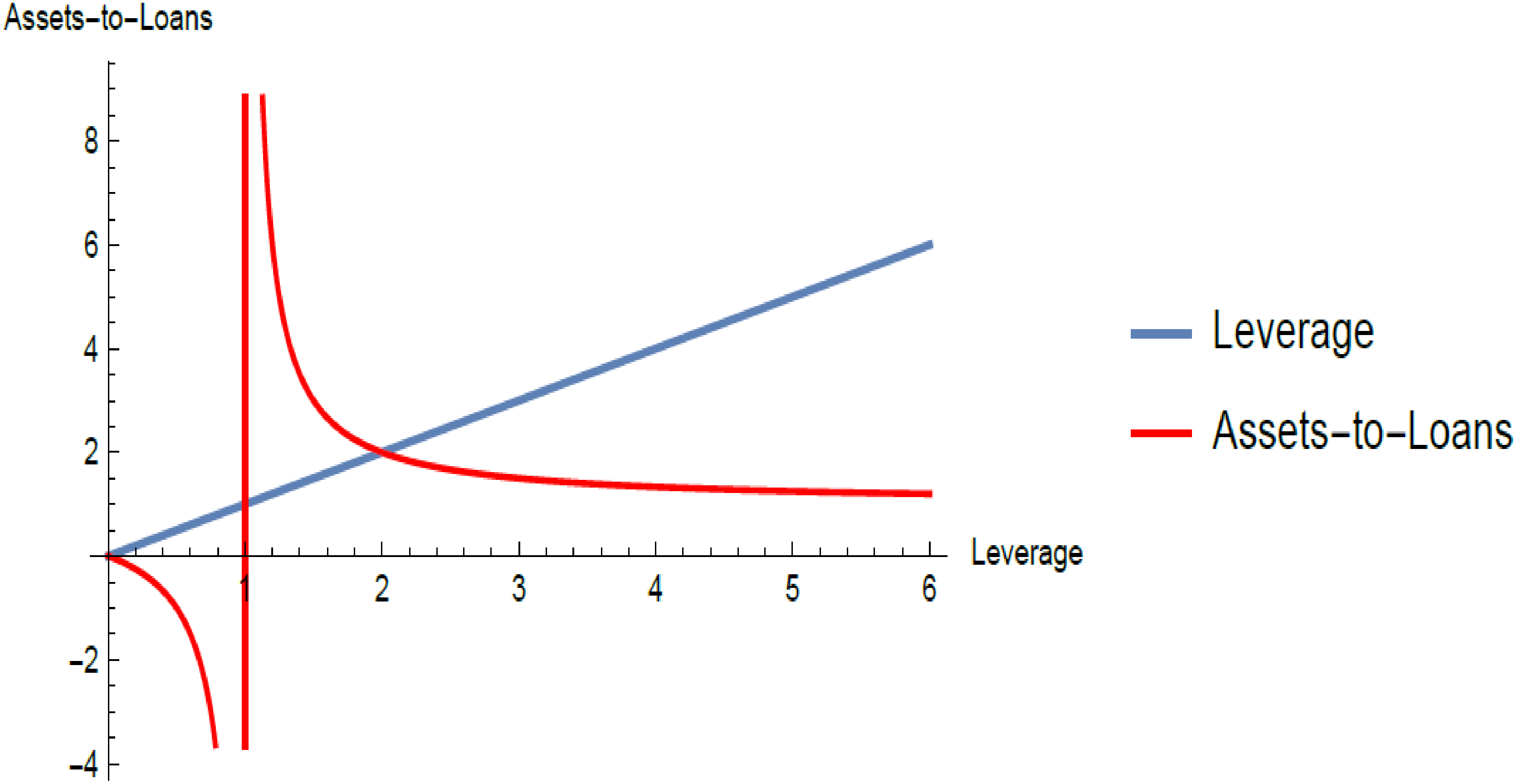
8. The feasibility of leverage values located in the above the interval.

The lower boundary is the (nontrivial) solution to the well-known equation:

$$l = l/(l-1) \quad \text{or} \quad l^2 - 2l = 0.$$

The upper boundary could be reformulated in terms of the long run no-arbitrage relations between aggregates of borrowers and creditors.

Leverage and the Collateral Ratio



The balanced financial market condition

9. Macrofinancial balances of levels and flows

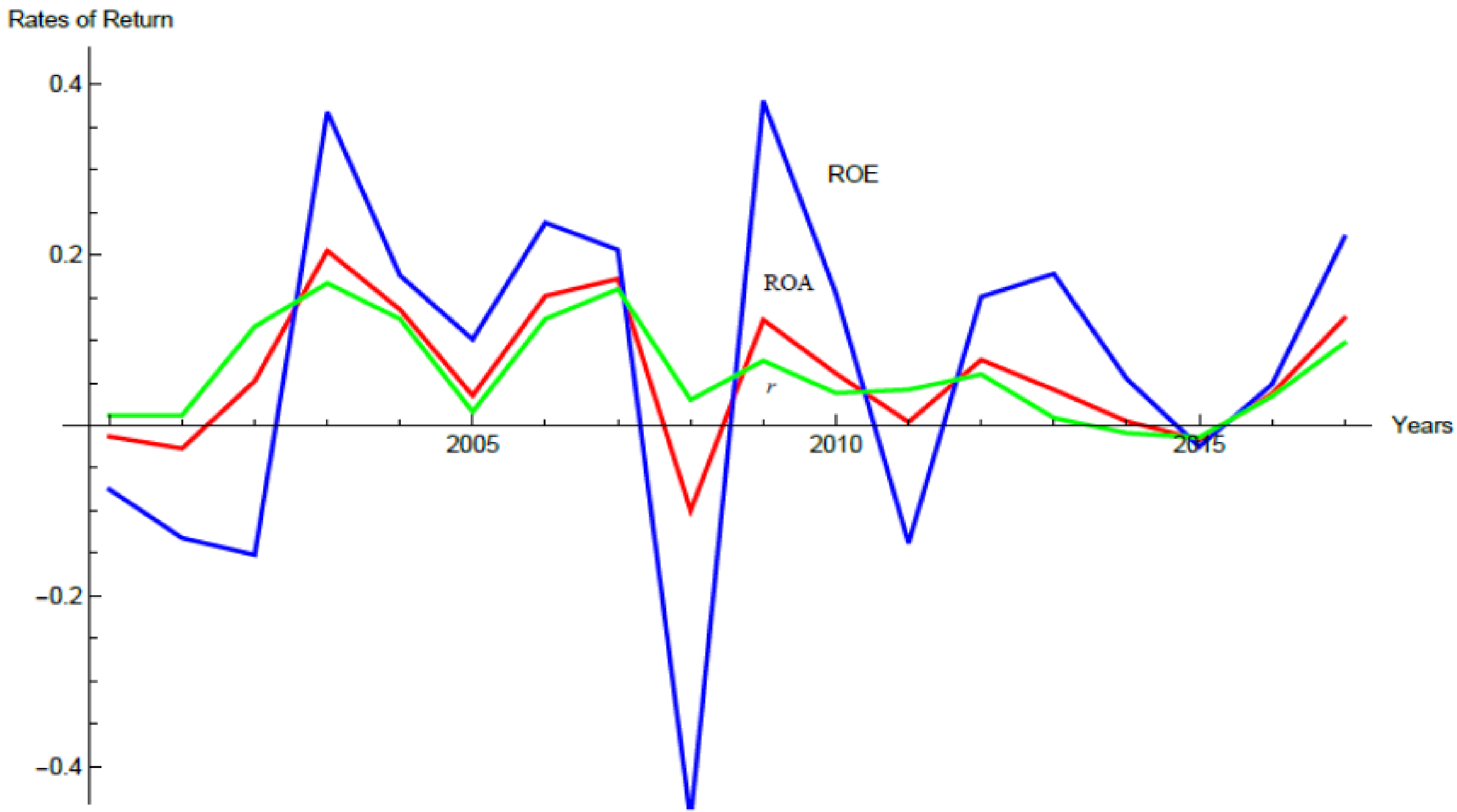
$A(t) = D(t) + E(t)$ and $dA(t) = dD(t) + dE(t)$

are equivalently represented by their convolution:

$$\mu l(t) = r(l-1) + \rho$$

where $\mu \equiv ROA; \rho \equiv ROE; r = r_s = r_L$. It is the balanced financial market, BFM, condition which is always true in the short run. In the model financial spreads are subject to $c / a = a / b$ where parameter $b = a^2 / c$.

Some global rates of return in 1999-2017



The anchor leverage

10. Natura abhorret vacuum. In the long run aggregates of borrowers and creditors are mutually adjusted so as

$$\rho^e(l) = r + (\mu - r)l;$$
$$\mu^e(l) = r + (\rho - r)l^{-1}$$

and $\rho^e(l) = \mu^e(l)$ or $l^2 - a/b = 0$.

The root of the latter defines the anchor leverage $l_N = (a/b)^{0.5}$.

The safe debt reimbursement in the long run

11. If the macro-debt is reimbursed in the long run then

$$l^2 - 2l = l^2 - a/b$$

with the positive root $\hat{l} = a/2b$. If $\hat{l} \cong 3$ the safe debt interval is

$$2 \le l_N \le a/2b$$

The logistic leverage dynamic

12. The leverage deterministic dynamic is

$$dl(t) = l(t)[a - bl(t)]dt; l(0) = l_0$$

with solution: $$l(t) = K\{1 + (\frac{K}{l_0} - 1)\exp[-at]\}^{-1}$$

where $K = a / b$ is the nontrivial steady state of the logistic equation.

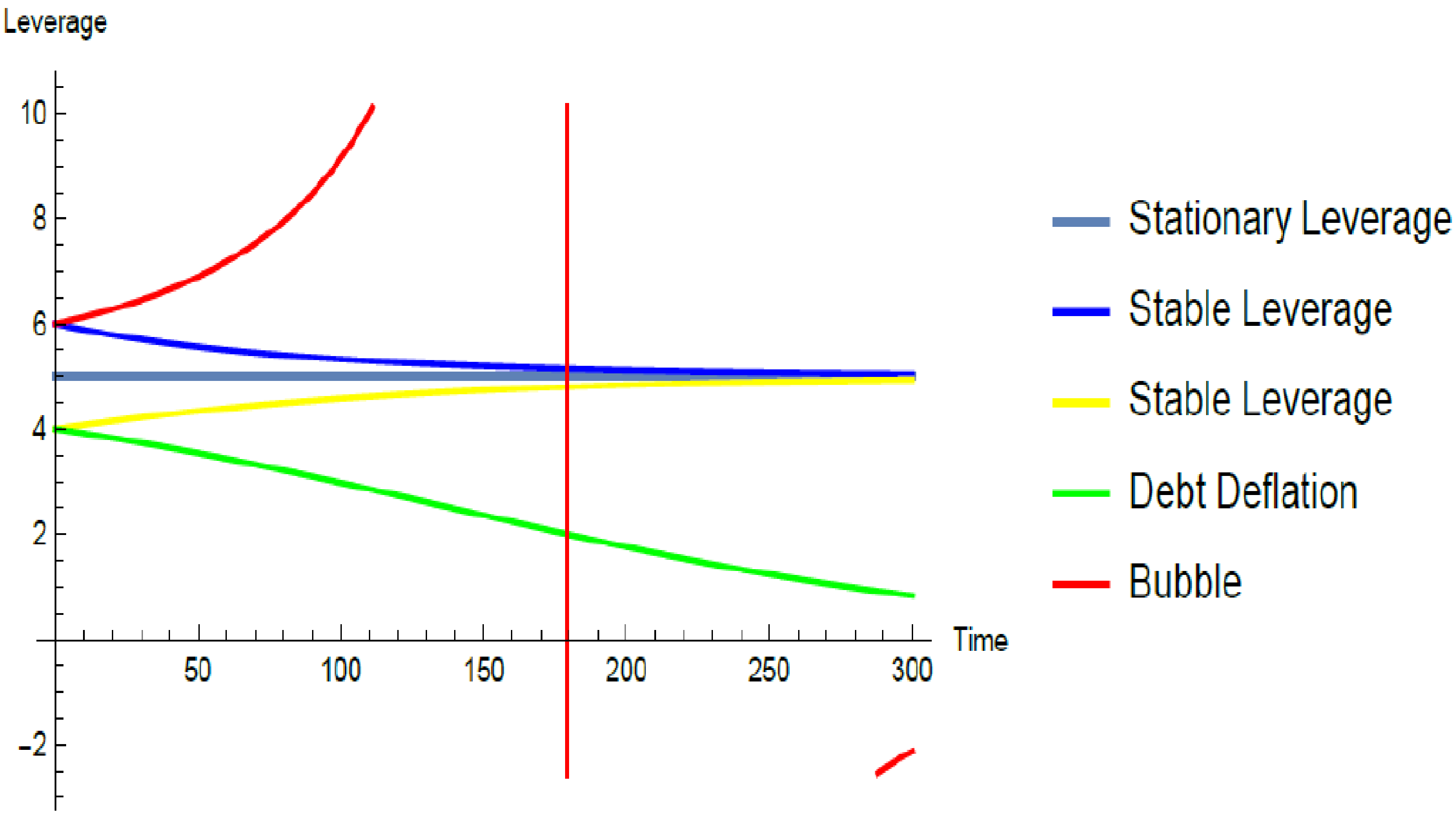
The transition (the pass-through) function:

$$f(l) = al - bl^2$$

measures the debt refinancing and its ability to be rolled over.
Its maximum takes place at $\hat{l} = a / 2b$ where money is sound:

$$f'(l) = a - 2bl = 0 .$$

The logistic leverage trajectories



The micro- vs macro-debt

13. Now, the second strand of the study: micro- vs macro-debt.

Any particular micro-debt has to be repaid in the finite time. It is safe, by definition, being paid off in full.
{Russian rates for "non-callable credits" in 90ties were in vogue due to disordered economic transition}

Contrary to that, the macro-debt is a perpetual aggregate of promises; separate tranches have to be repaid, and their simultaneous reimbursement is tantamount to a financial crisis.

The proof of the macro-debt safety is in its ability to be refinanced and rolled over. If every market participant is convinced in the safety of her/his money the debt as a whole is safely rolled over.

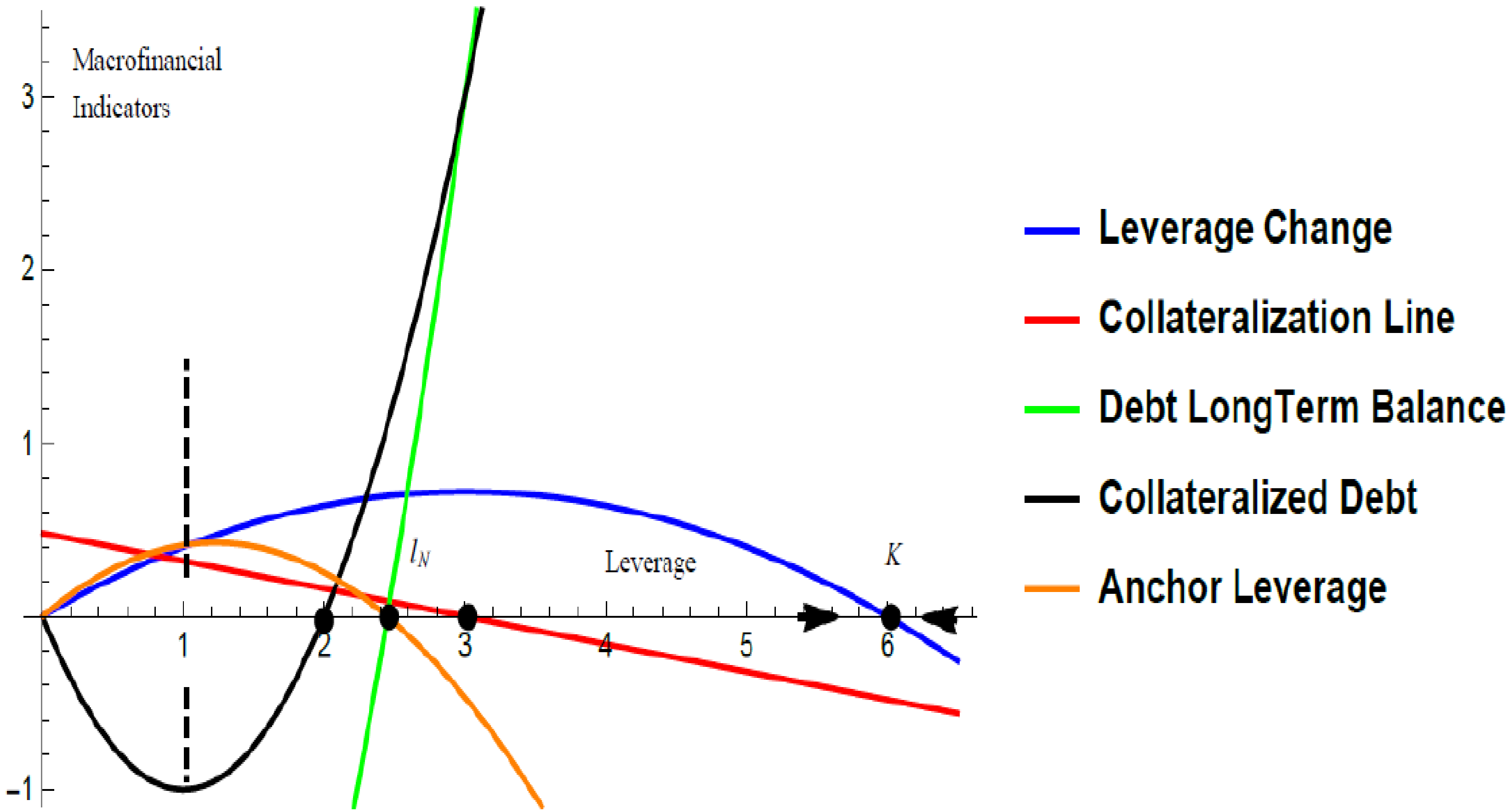
The self-negation mechanism

14. The mechanism of self-negation: when everybody knows that her/his money is callable on short notice nobody would claim it back without particular personal reasons.

Debt reimbursement is transformed into its safe rolling over at the leverage value $\hat{l} = a / 2b$.

Thus, the problem seems to be solved; it is, but not completely because the macrofinancial system is unstable (semi-stable) at the safe debt roll-over point.

The self-negation mechanism



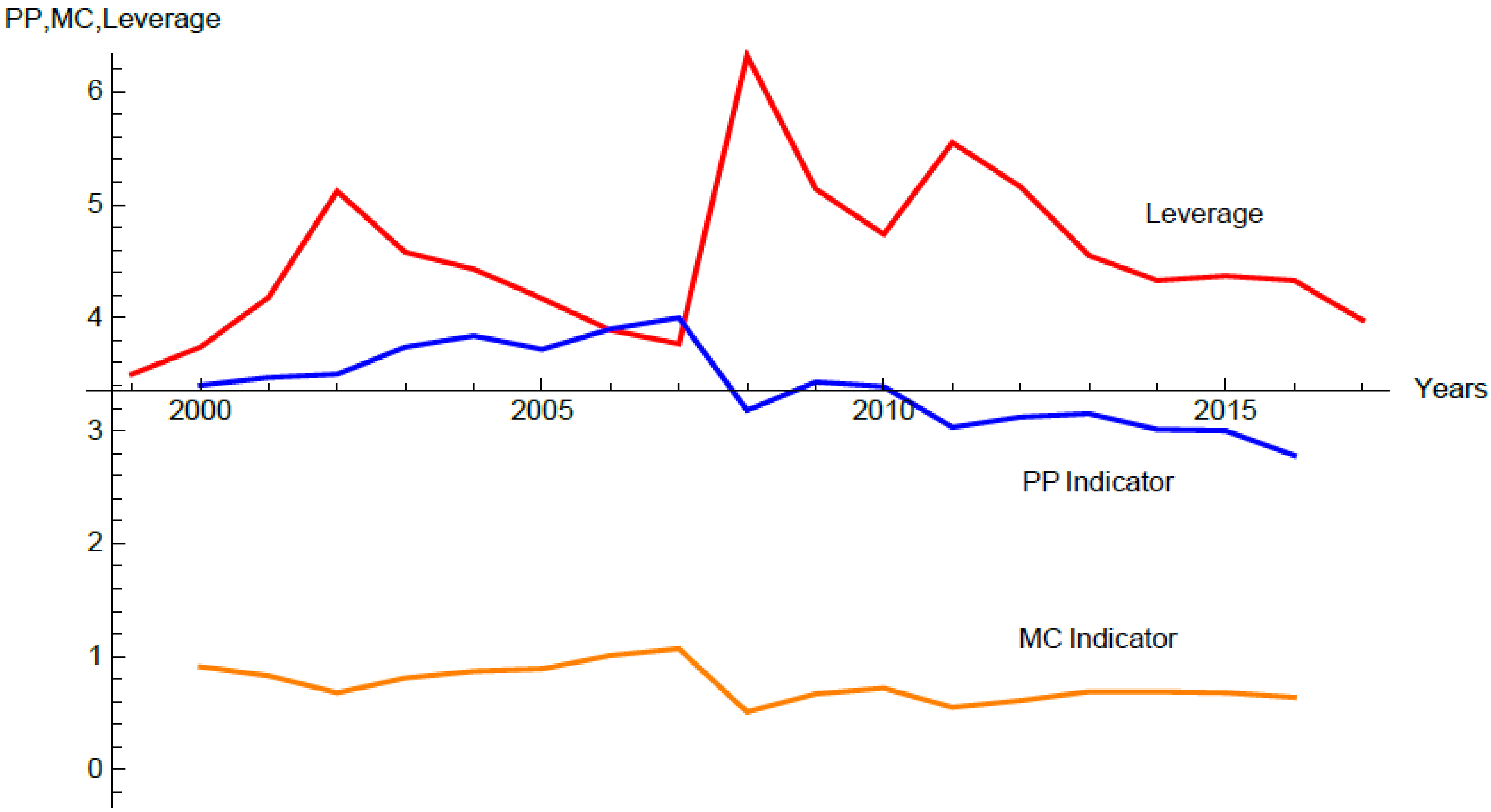
15. Let us return to the empirical data in 1999-2017.
The asset/equity elasticity in 1999-2017 is 1.25, but the leverage process is very different: it is, in fact, a process of deleveraging.

16. Deleveraging: debts outstanding are netted and compressed;
{mutual indebtness offsetting in the Soviet economy, and a lucrative multibillion business now, especially on markets of derivatives}

central banks purchases of large chunks of public debt;
{BoJ holds 43 percent of government debt);

world equity markets tend to be overvalued for many years
{August, 2019 correction and the YC inversion on the major markets}

Two phases of deleveraging in 1999-2017



The process of deleveraging

17. The standard logistic equation is modified into the following model:

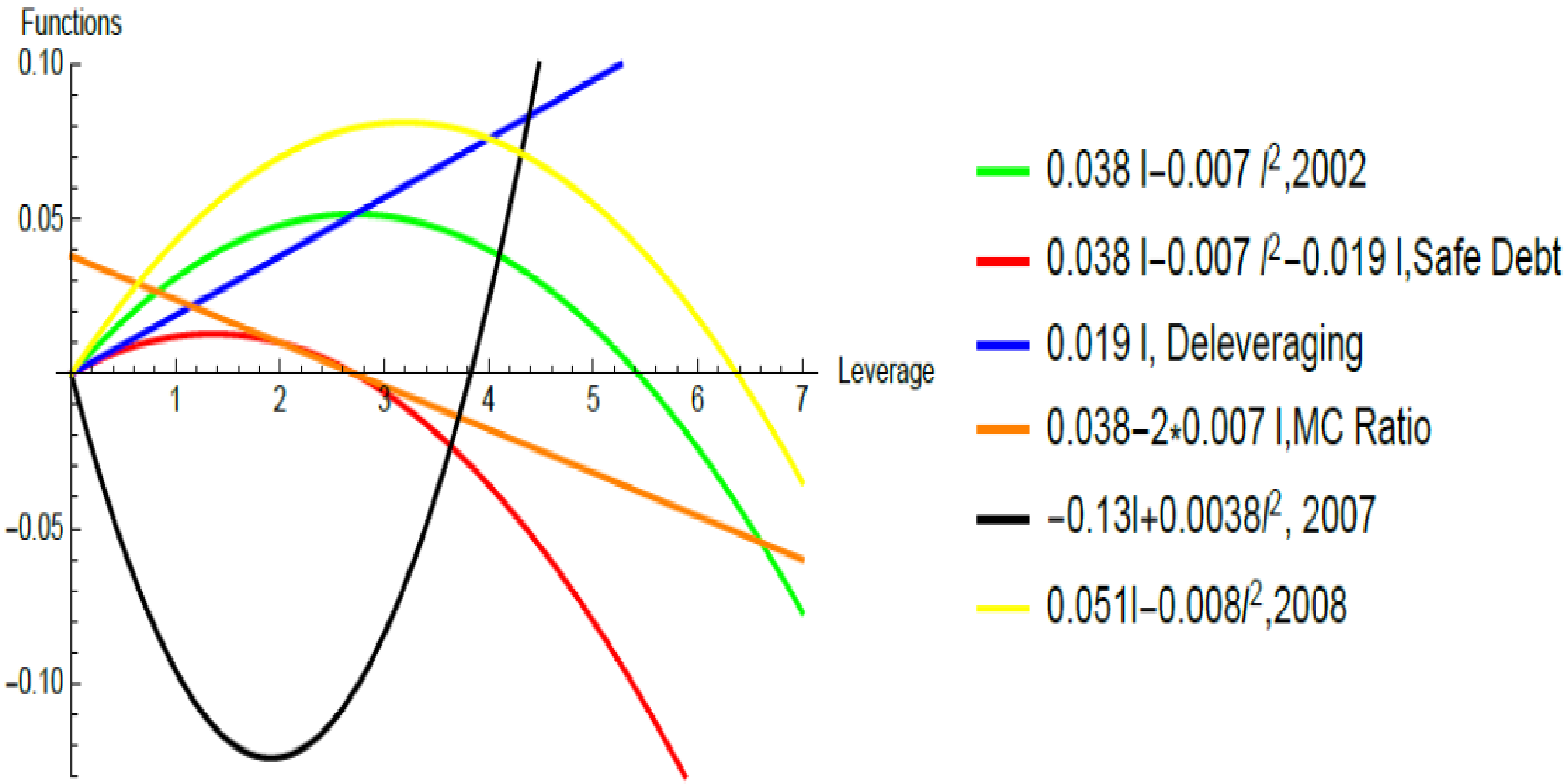
$dl(t) = [(a-\delta)l(t) - bl^2(t)]dt$

with pass-through function: $f(l,\delta) = al - bl^2 - \delta l$, and stable steady state $K(\delta) = K(1-\delta/a)$.

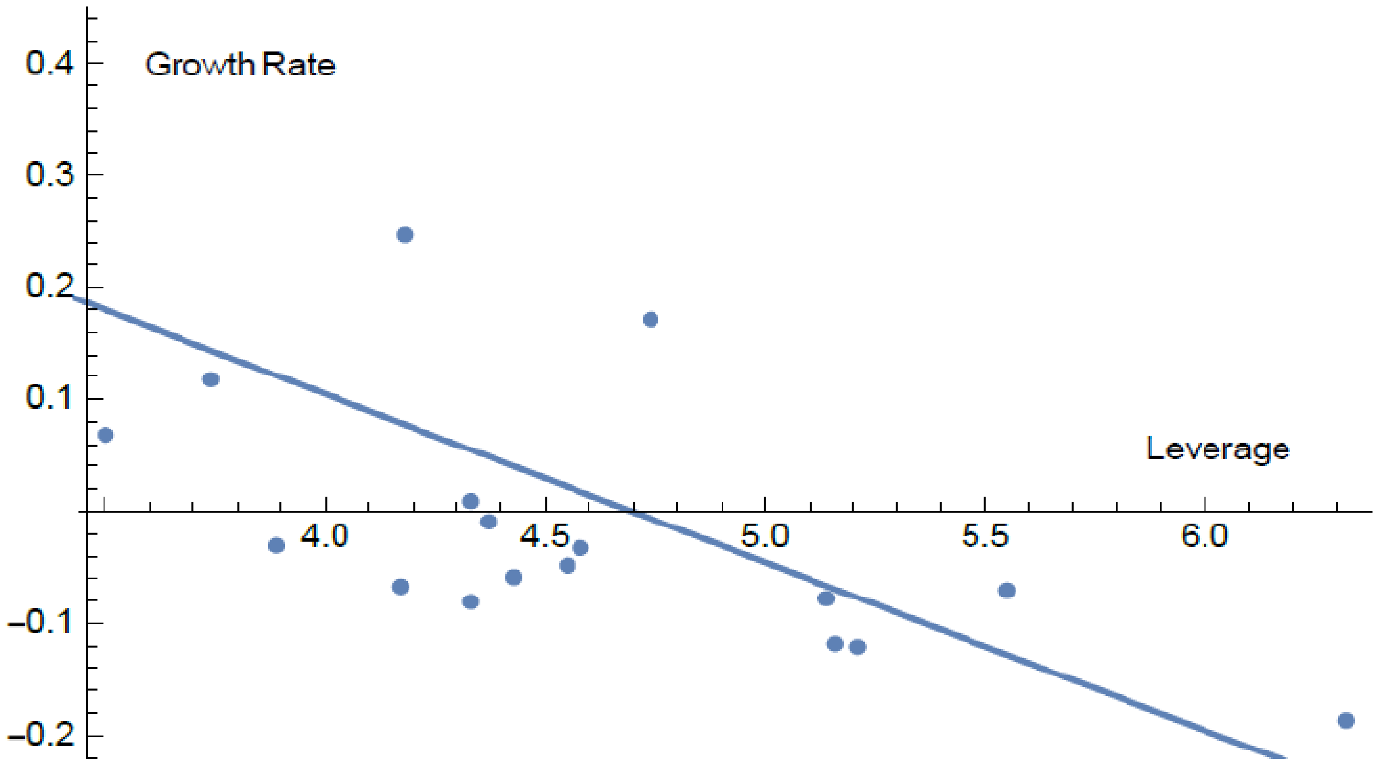
The maximal effect of deleveraging, $\max_\delta[\delta K(\delta)]$ takes place at $\hat{\delta} = 0.5a$, that is, precisely, at the safe debt roll-over point, $\hat{l} = a/2b$.

The solution of $K_{17}(\delta) = K_{12}(1-\delta/a)$ defines $a_{17} = 0.0029; \delta = 0.0004$ hence the prolong and uncertain travel between different steady states.

Deleveraging in 2002-2007 reconstruction



The global leverage regression



The stochastic logistic diffusion

18. The new factor governing the process appears in the logistic stochastic diffusion:

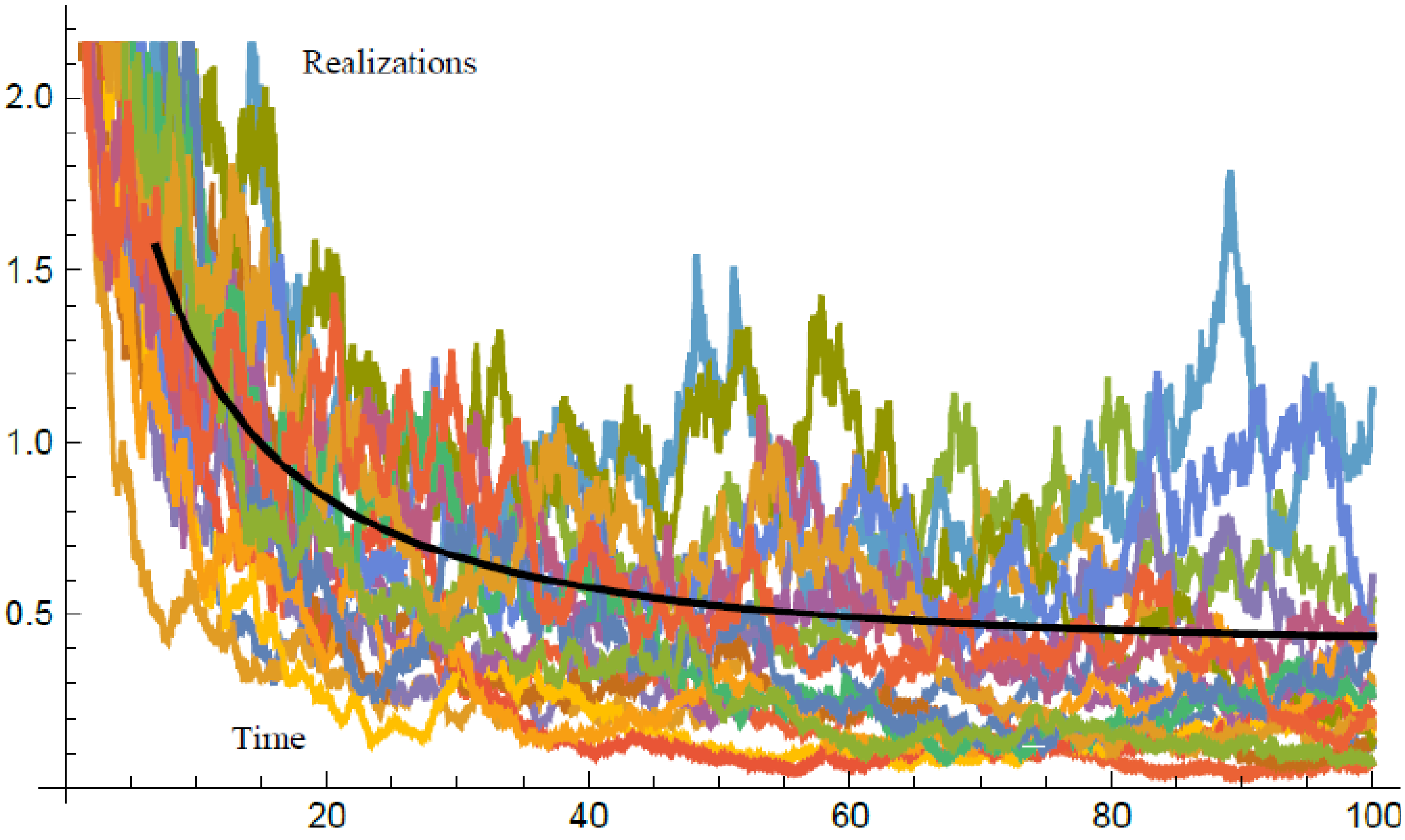
$$dl(t) = l(t)[a - bl(t)]dt + \sigma l(t)dZ(t)$$

where $\sigma = 0.16$ is the empirical volatility; and $Z(t) = \int_0^t dZ(u)$ is

a standard Brownian motion with independent and self-similar
(the Hurst exponent, $H = 0.5$) increments $dZ(t)$.
Parameter a was used *per se* or including the deleveraging intensity δ.

Due to high empirical volatility, $\sigma = 0.16$, the future leverage realizations dramatically decreased: from $l_{17} = 4.14$ to $l^* = 0.416$, that is, to values smaller than one.

Realizations of stochastic leverage



A random leverage equivalent representations

19. It is known that a random process $x(t)$ with coefficients of drift and noise given by the general Ito processes $P[x(t),t]$ and $\sqrt{Q[x(t),t]}$ can be equivalently represented either by its SDE:

$$d\,x(t) = P[x(t),t]\,dt + \sqrt{Q[x(t),t]}\,d\,Z(t)\,,$$

or by the Kolmogorov-Fokker-Plank equation:

$$\frac{\partial}{\partial t}\,p[x(t),t] = -\frac{\partial}{\partial l}\{P[x(t),y]\,p[l(t),t]\} + \frac{1}{2}\frac{\partial^2}{\partial l^2}\{Q[x(t),t]\,p[x(t),t]\}$$

where $p[x(t),t]$ is the “transition” probability density function. Boundaries and initial conditions of the KPF equation are specified by the actual process under consideration.

The KFP stationary equation and its solution

20. A non-trivial solution to the ordinary differential KFP equation:

$$-\frac{d}{dl}[l(a-bl)p(l)]+\frac{1}{2}\frac{d^2}{dl^2}[\sigma^2 l^2 p(l)]=0$$

is the stationary pdf, $p(l)$, of a random leverage process

$$p(l;\alpha,\beta)=[\beta^\alpha/\Gamma(\alpha)]l^{\alpha-1}\exp[-\beta l].$$

It defines the gamma distribution with parameters $\alpha=(2a/\sigma^2)-1$

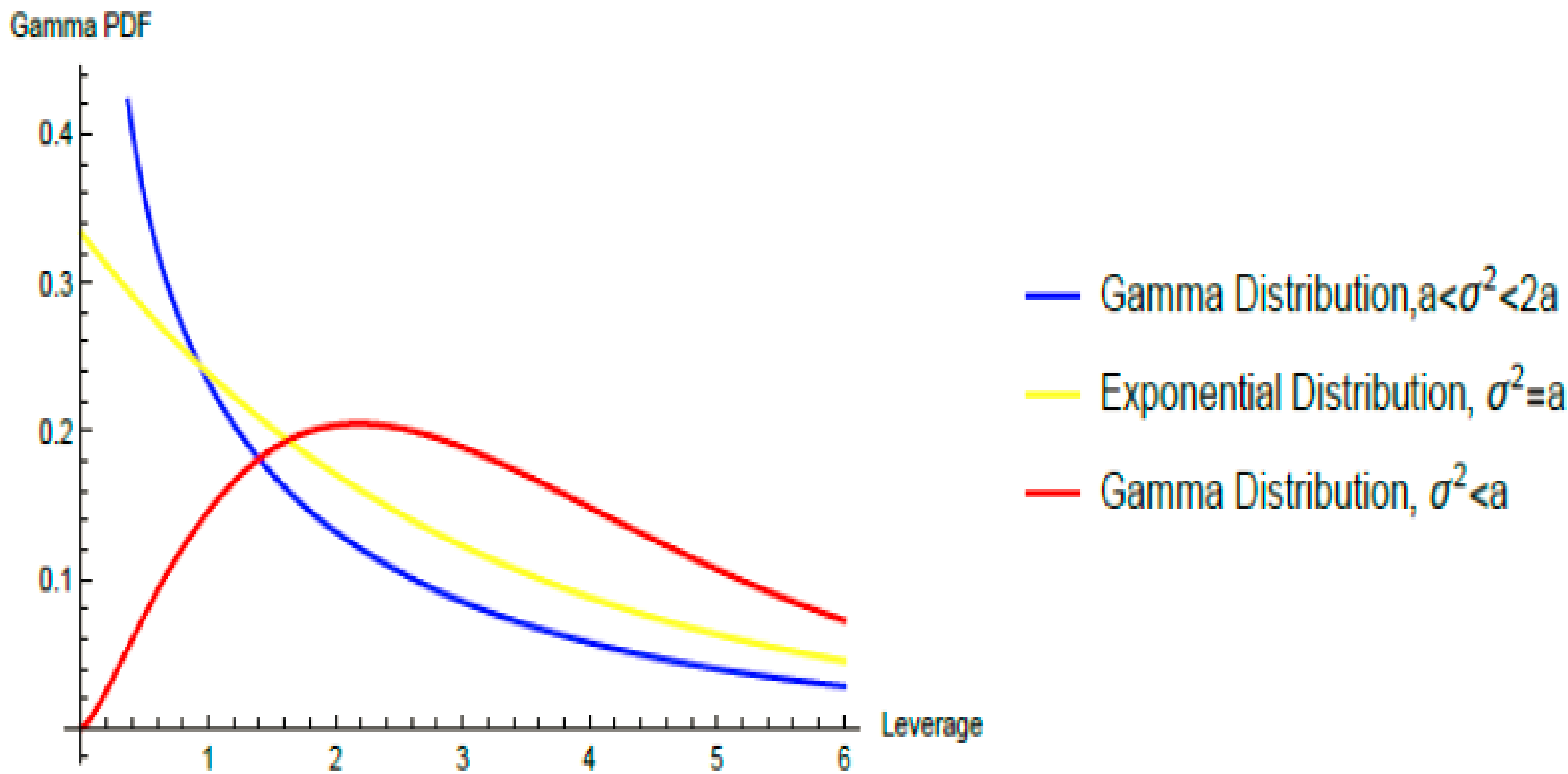
and $\beta=2b/\sigma^2$ (the parameter of scale $1/\beta$ is used in *Mathematica 10*).

The normalization constant $\beta^\alpha/\Gamma(\alpha)$ is found from a gamma function:

$$\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}\exp[-x]dx.$$

Figure IX

Three forms of a gamma distribution



Moments of the stationary gamma distribution

22. The PDF of a gamma distribution is skewed to the right and its expectation:

$$\langle L \rangle = \frac{\alpha}{\beta} = K - \frac{\sigma^2}{2b}$$

is larger than its mode (the most probable long-term leverage):

$$Mo = \frac{\alpha - 1}{\beta} = K - \frac{\sigma^2}{b};\ \alpha \geq 1.$$

The random leverage convergence criterion

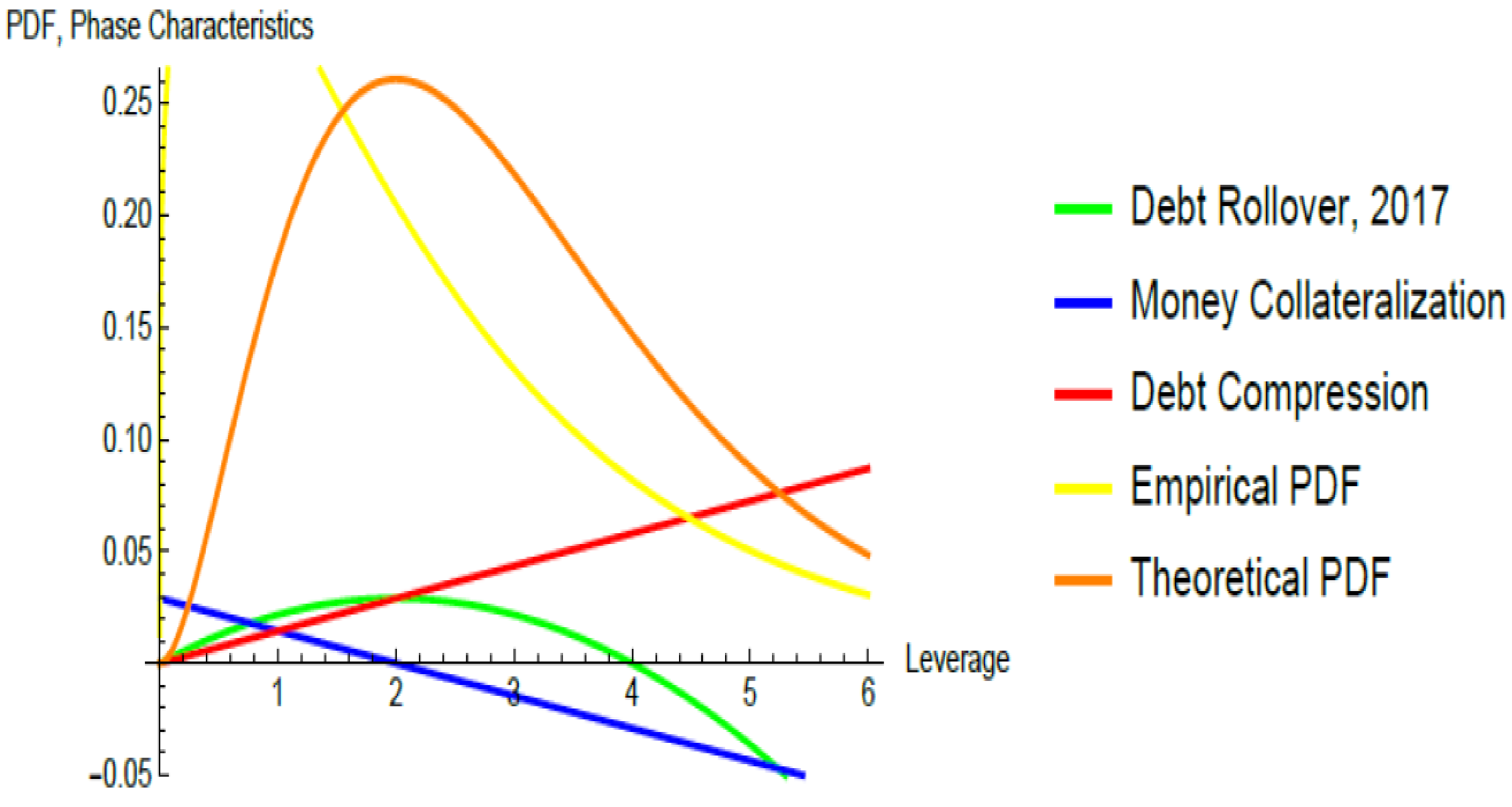
23. If the mode is the same as the anchor leverage l_N the above formula could be transformed into:

$$(a/b)^{0.5} = (a - \sigma_N^2)/b \quad \text{or} \quad a/2b = (a - {\sigma_N}^2)/b$$

where σ_N^2 is the implied variance of the gamma distributed stationary leverage. It is an indicator of the long-term default risk. For the positive parameters $\{a, b\}$ the implied variance σ_N^2:

$$\sigma_N^2 = a - \sqrt{a\,b} \quad \text{or} \quad \sigma_N^2 = 0.5\,a\,.$$

The long-term leverage scenarios



Conclusions

24. The global leverage evolution is modelled as a generalized process of parametric deleveraging; the safe debt is rolled-over within particular interval of leverage values.

The model outlined alternatives: either deleveraging is kept under control and debt is safely accumulated, or devastating consequences of a bursting equity bubble are inevitable.

Without excessive global equity valuation the first alternative is realized at $\widehat{\delta} = 0.5a$; its stochastic analogue $\sigma_N^2 \leq 0.5\,a$ provides convergence to the unimodal gamma distribution.

The sustained deleveraging could be accomplished due to coordinated efforts of financial regulators and central banks.