Absorbing Boundary Conditions for Finite-Difference Approximations of Basic Mathematical Physics Equation

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In most mathematical models, boundary conditions for differential or finite-difference equations are inalienable. The Dirichlet boundary conditions are the most common: they describe solution fixation on the segment’s ends. When we use a difference scheme, limitation of memory and computational time is strong obstacle for consideration of the Cauchy problem and only mixed boundary problems can be solved. In many problems there are no special physical processes happening on the boundary of the computational area. Popular boundary conditions (Dirichlet, Neuman, etc.) do not provide the real solution. In this case, the boundary conditions that imitate the Cauchy problem (ICP) are needed [1,2].

IPC boundary conditions were developed for most famous differential equations. They are not local; they are integro-differential. They include convolution with respect to time and along the boundary. They allow solving the problem without any area's extension, but realization of these convolutions is expensive. We need to localize the boundary conditions.

In present work, several ICP boundary conditions for finite-difference approximations of three basic PDE are submitted: wave equation, diffusion equation, Schrödinger equation. We determine Hermite - Pade for approximations of the symbols of the finite-difference operators, which are used in the ICP boundary conditions. The exact solutions of Cauchy problem were compared with solutions that were obtained for the mixed initial-boundary problems. We evaluated the norms of the errors.

Example. Let us approximate the one-dimensional wave equation $\partial_t^2 u = c^2 \partial_x^2 u$ by the leap-frog scheme with a space step h and a time step τ. Figure 1 shows logarithm of absolute difference between an exact solution u_{true} and the obtained one u at different slices of time ($\nu = \frac{c\tau}{h}$).

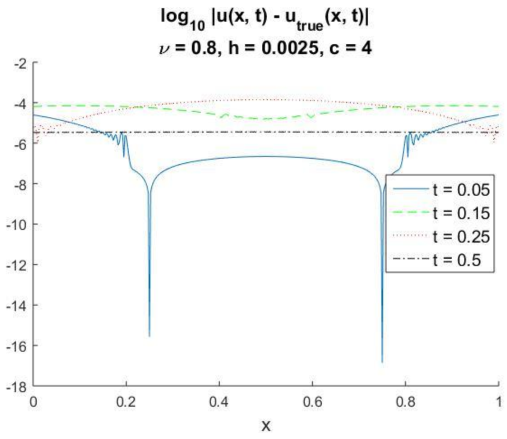

Figure 1

References

[1] V. Gordin, Mathematics, computer, weather forecast and other mathematical physics scenarios, Moscow: FIZMATLIT, 2013.

[2] V. Gordin, "About Mixed Boundary Problem that Imitates Cauchy's Problem," *Uspekhi Matematicheskikh Nauk*, vol. 33, no. 5, pp. 189-190, 1978.

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