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THE SHORT ENVELOPE SOLITON DYNAMICS IN INHOMOGENEOUS DISPERSIVE MEDIA WITH ALLOWANCE FOR STIMULATED SCATTERING BY DAMPED LOW-FREQUENCY WAVES

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We consider the soliton dynamics in terms of the extended nonlinear Schrödinger equation taking into account the inhomogeneous linear second-order dispersion (SOD) and stimulated scattering by damped low-frequency waves (SSDW). It is shown that the wave number downshift due to SSDW is compensated by an upshift due to the SOD decrease on the spatial coordinate. A new class of stationary nonlinear localized solutions (solitons) arising as an equilibrium of SSDW and decreasing spatial SOD is found analytically within the framework of the extended inhomogeneous nonlinear Schrödinger equation. A regime of the dynamic equilibrium of SSDW and inhomogeneous dispersive medium with the soliton parameters periodically varied in time is found. Analytical and numerical results are in good agreement for this regime.

1. INTRODUCTION

Interest in the solitons is due to their ability to maintain their shape for a long time. Soliton solutions arise in many problems of modeling the dynamics of intense wave fields in the dispersive media, including optical pulses in fiber-optic communication lines, electromagnetic waves in the plasma, surface waves on deep water, etc. [1–4].

The dynamics of extended high-frequency wave packets is described in the second approximation of the theory of dispersive nonlinear waves allowing for terms of the second order of smallness, including the second-order linear dispersion and the cubic nonlinearity. The basic model equation of this approximation is the nonlinear Schrödinger equation (NSE) [5, 6], in which the soliton solution results from an equilibrium of the dispersion spreading of the wave packet and its nonlinear compression.

The dynamics of sufficiently short high-frequency wave packets is described by the third approximation of this theory, which allows for third-order terms [1], namely, nonlinear dispersion [7], stimulated Raman scattering [8–10], and third-order linear dispersion. The basic model equation in this approximation is the nonlinear Schrödinger equation of the third order (NSE-3) [10–14].

A class of stable short solitons resulting from an equilibrium of the third-order linear dispersion and the nonlinear dispersion was found in [15–17] within the framework of the NSE-3 without the stimulated scattering. It was shown in [18] that an arbitrary initial distribution in terms of the NSE-3 with the stimulated scattering neglected evolves to a system of these short solitons. More recently, similar solutions within the NSE-3 without the stimulated scattering were found in [19–23]. Stationary difference waves resulting from an equilibrium of nonlinear dispesion and stimulated scattering were found in [24, 25] within the NSE-3 without the third-order linear dispersion.

Stimulated Raman scattering is due to excitation of the temporal modes by an external field in the atomic and molecular systems. The temporal-mode frequency Ω is related to the frequencies ω_1 and ω_2 as

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follows: $\omega_1 - \omega_2 = \Omega$. In the extended nonlinear Schrödinger equation, this scattering is described by an additional term with the time delay of a nonlinear Kerr response. For the localized nonlinear wave packets (solitons), taking into account stimulated Raman scattering leads to a downshift of the soliton frequency [8–10] and, as a consequence, loss of its stability and a decay. The influence of stimulated scattering on the dynamics and stability of solitons is described in detail in [3, 26], where the dynamics of short solitons was described by the NSE-3 with allowance for stimulated Raman scattering. The possibility of compensation for the Raman frequency shift in the extended communication lines with variable frequency characteristics was studied in [27]. Compensation for the stimulated Raman scattering by the linear radiation field from the soliton core region was considered in [28]. Compensation for the Raman scattering in the inhomogeneous media was considered in the following cases: in the media with periodic linear dispersion of the second order [29, 30], in the media with biased inflection point of the dispersion characteristic [31], and in the media with decreasing dispersion [32].

In this paper, we study the dynamics of intense high-frequency wave packets in the linear inhomogeneous dispersive media with allowance for stimulated scattering by damped low-frequency waves. In the third approximation of dispersion theory (for sufficiently short wave packets), the initial system of two equations is reduced to an inhomogeneous extended nonlinear Schrödinger equation with a nonlocal antisymmetric Kerr response due to stimulated scattering by damped low-frequency waves. Unlike the stimulated Raman scattering, this scattering leads to a downshift of the spatial spectrum of the soliton wave numbers. Such an effect is due to the excitation of damped spatial modes with the wave number χ by an external field with the wave numbers k_1 and k_2 related by $k_1 - k_2 = \chi$. Essentially, this scattering is a counterpart of stimulated Raman scattering or spatial stimulated Raman scattering. At the same time, spatial inhomogeneity of the dispersion also leads to the wave number variation of a high-frequency wave packet. Equilibrium of the spatial stimulated Raman scattering and the decreasing dispersion leads to the stabilization of the spatial spectrum of the soliton wave numbers.

In this paper, within the extended nonlinear Schrödinger equation with antisymmetric nonlinear Kerr response, we found analytically a new class of solitons resulting from an equilibrium of stimulated scattering by damped low-frequency waves and the dispertion decrease on the spatial coordinate. The regime of the dynamic equilibrium of stimulated scattering and decreasing dispersion, in which the soliton parameters vary periodically with time, is also found.

2. INITIAL SYSTEM AND THE BASIC EQUATION

Consider the dynamics of the intense high-frequency wave packet $U(\xi, t)\exp(i\omega t - ik\xi)$ in a nonlinear, inhomogeneously dispersing medium with allowance for the interaction with low-frequency damped waves. As the initial system, we consider a system of two model unidirectional nonlinear equations of the Zakharov type [33–35]:

$$2i\frac{\partial U}{\partial t} + \frac{\partial}{\partial \xi}\left[q(\xi)\frac{\partial U}{\partial \xi}\right] - \rho U = 0; \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial \xi} - \mu\frac{\partial^2 \rho}{\partial \xi^2} = -\frac{\partial |U|^2}{\partial \xi}. \tag{2}$$

Here, ρ is the low-frequency disturbance of the medium parameters, $q(\xi)$ is the second-order linear dispersion coefficient, and μ is the high-frequency loss coefficient for low-frequency waves. In particular, this system describes the dynamics of intense electromagnetic or Langmuir waves in an isotropic plasma with allowance for their interaction with damped ion-acoustic waves.

In the second approximation of the nonlinear-wave dispersion theory, the nonlinear response of the medium is local, $\rho = -|U|^2$, and the envelope of the high-frequency wave packet is described by a nonlinear Schrödinger equation. In the third approximation of the nonlinear-wave dispersion theory in describing short high-frequency wave packets ($k\Delta \ll \mu$, where Δ and k are the length and the additional wave number of the wave packet, respectively), the nonlinear response of the medium contains a nonlocal nonsymmetric term stipulated by the decay of low-frequency wave packets due to low-frequency waves: $\rho = -|U|^2 - \mu\,\partial|U|^2/\partial\xi$. In this approximation, the model equation for the wave packet envelope is as follows:

$$2i\frac{\partial U}{\partial t} + \frac{\partial}{\partial \xi}\left[q(\xi)\frac{\partial U}{\partial \xi}\right] + 2\alpha U\,|U|^2 + \mu U\frac{\partial(|U|^2)}{\partial \xi} = 0. \tag{3}$$

The last term in Eq. (3) describes the stimulated scattering of a high-frequency wave field by damped low-frequency waves and is a spatial counterpart of the stimulated Raman scattering (spatial stimulated Raman scattering).

3. ANALYTICAL RESULTS

We now apply the method of integrals for solution of Eq. (3). Under zero conditions on the infinity, $U\big|_{|\xi|\to\infty} \to 0$, Eq. (3) has the following moments of the distribution:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{-\infty}^{+\infty} |U|^2\,\mathrm{d}\xi = 0; \tag{4}$$

$$2\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{-\infty}^{+\infty} K\,|U|^2\,\mathrm{d}\xi = -\mu\int\limits_{-\infty}^{+\infty}\left(\frac{\partial|U|^2}{\partial\xi}\right)^2\mathrm{d}\xi - \int\limits_{-\infty}^{+\infty}\frac{\partial q}{\partial\xi}\left|\frac{\partial U}{\partial\xi}\right|^2\mathrm{d}\xi; \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{-\infty}^{+\infty}\left|\frac{\partial U}{\partial\xi}\right|^2\mathrm{d}\xi = -\mu\int\limits_{-\infty}^{+\infty} K\left(\frac{\partial|U|^2}{\partial\xi}\right)^2\mathrm{d}\xi + \alpha\int\limits_{-\infty}^{+\infty} K\frac{\partial|U|^4}{\partial\xi}\mathrm{d}\xi$$
$$- \int\limits_{-\infty}^{+\infty}\frac{\partial q}{\partial\xi}K\left(3\left|\frac{\partial U}{\partial\xi}\right|^2 - \frac{1}{2}\frac{\partial^2|U|^2}{\partial\xi^2} - 2K^2\,|U|^2\right)\mathrm{d}\xi; \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{-\infty}^{+\infty}\left(\frac{\partial|U|^2}{\partial\xi}\right)^2\mathrm{d}\xi = -2\int\ limits_{-\infty}^{+\infty}\frac{\partial^2|U|^2}{\partial\xi^2}\frac{\partial(qK\,|U|^2)}{\partial\xi}\mathrm{d}\xi; \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{-\infty}^{+\infty}\xi\,|U|^2\,\mathrm{d}\xi = \int\limits_{-\infty}^{+\infty} qK\,|U|^2\,\mathrm{d}\xi. \tag{8}$$

Here, $U = |U|\exp(i\varphi)$, and $K = \partial\varphi/\partial\xi$ is the additional wave number of the wave packet. To close up Eqs. (4)–(8), the spatial distribution of the wave number K should be related to the parameters of the wave packet envelope and the inhomogeneity of the medium. For this we assume that the scales of the dispersion inhomogeneity and variation in the local wave number K are much greater than the scale of inhomogeneity of the wave packet envelope, i. e., $D_{q,K} \gg D_{|U|}$. Thus, the spatial distribution of the wave number K in the vicinity of the "center of mass" of the wave packet $\bar{\xi} = N^{-1}\int_{-\infty}^{+\infty}\xi\,|U|^2\,\mathrm{d}\xi$, where $N = \int_{-\infty}^{+\infty}|U|^2\,\mathrm{d}\xi$, can be approximated by the relationship $K(\xi) = K(\bar{\xi}) + (\partial K/\partial\xi)_{\bar{\xi}}\,(\xi - \bar{\xi})$. The gradient of the wave number in the "center of mass" of the wave packet can be found from the imaginary part of Eq. (3) under the condition $(\partial|U|/\partial\xi)_{\bar{\xi}} = 0$:

$$\left(\frac{\partial K}{\partial\xi}\right)_{\bar{\xi}} = -\left(\frac{2}{q\,|U|}\frac{\partial|U|}{\partial t} + \frac{1}{q}\frac{\mathrm{d}q}{\mathrm{d}\xi}K\right)_{\bar{\xi}}. \tag{9}$$

For the wave packets whose amplitude and length are connected by a soliton-like relationship, we obtain, in view of Eqs. (4) and (9), that $K(\xi,t) = K(\bar{\xi},t) \equiv k(t)$. The system of equations (4)–(8) in this case becomes closed.

$$2\frac{\mathrm{d}k}{\mathrm{d}t} = -\mu\frac{L_0}{N_0}l - q'(\bar{\xi})z; \tag{10}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\mu\frac{L_0}{N_0}kl - 3kq'(\bar{\xi})z + 2k^3q'(\bar{\xi}); \tag{11}$$

$$\frac{\mathrm{d}l}{\mathrm{d}t} = -3kq'(\bar{\xi})l; \tag{12}$$

$$\frac{\mathrm{d}\bar{\xi}}{\mathrm{d}t} = kq(\bar{\xi}). \tag{13}$$

Here, $q'(\bar{\xi}) = (\partial q/\partial\xi)_{\bar{\xi}}$ is the dispersion gradient in the "center of mass" of the wave packet, $n = N/N_0$, $l = L/L_0$, $z = Z/N_0$, $Z \equiv \int_{-\infty}^{+\infty} |\partial U/\partial\xi|^2\,\mathrm{d}\xi$ and $L \equiv \int_{-\infty}^{+\infty} (\partial|U|^2/\partial\xi)^2\,\mathrm{d}\xi$ are the integrals of the wave packet and $N_0 = N(0)$, $Z_0 = Z(0)$ and $L_0 = L(0)$ are the integrals of the wave packets at a zero time. The equilibrium state of system (10)–(13) is reached under the condition

$$k = 0, \qquad \mu L_0 = -q'(\bar{\xi}_0)Z_0. \tag{14}$$

In the equilibrium state, the spatial stimulated Raman scattering is compensated by the dispersion decrease. For analysis of system (10)–(13) outside the equilibrium state, we consider the case of exponential spatial dependence of the dispersion $q = q_0\exp(-\xi/D)$. Using the replacements $\theta = tq_0/D$ and $\eta = \bar{\xi}/D$, we reduce system (10)–(13) to the following form:

$$2\frac{\mathrm{d}k}{\mathrm{d}\theta} = z\exp(-\eta) - rl; \tag{15}$$

$$\frac{\mathrm{d}z}{\mathrm{d}\theta} = [3z\exp(-\eta) - rl - 2k^2\exp(-\eta)]\,k; \tag{16}$$

$$\frac{\mathrm{d}l}{\mathrm{d}\theta} = 3kl\exp(-\eta); \tag{17}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}\theta} = k\exp(-\eta), \tag{18}$$

where $r \equiv \mu L_0 D/(q_0N_0)$. With allowance for the integral $l = \exp(3\eta)$, system (15)–(18) reduces to

$$2\frac{\mathrm{d}k}{\mathrm{d}\theta} = z\exp(-\eta) - r\exp(3\eta); \tag{19}$$

$$\frac{\mathrm{d}z}{\mathrm{d}\theta} = [3z\exp(-\eta) - r\exp(3\eta) - 2k^2\exp(-\eta)]\,k; \tag{20}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}\theta} = k\exp(-\eta). \tag{21}$$

System (19)–(21), in turn, has the integral

$$3\frac{k^2}{z_0}\exp(-\eta) - \lambda\,[1 - \exp(3\eta)] + 3\left(1 - \frac{k_0^2}{z_0}\right)[1 - \exp(\eta)] = 3\frac{k_0^2}{z_0}, \tag{22}$$

where $k_0 = k(0)$, $\lambda \equiv r/z_0$, and $z_0 \equiv Z_0/N_0$. The curves in the parameter plane $(k/\sqrt{z_0}, \eta)$, which are described by Eq. (22) for $k_0 = 0$ and different values of λ, are given in Fig. 1.

Consider the solution of Eq. (3) in the form of a stationary wave $U(\xi,t) = \psi(\xi)\exp(i\Omega t)$ for the exponental profile $q(\xi) = q_0\exp(-\xi/D)$:

$$q_0\exp(-\xi/D)\frac{\mathrm{d}^2\psi}{\mathrm{d}\xi^2} - \frac{q_0}{D}\exp(-\xi/D)\frac{\mathrm{d}\psi}{\mathrm{d}\xi} + 2\alpha\psi^3 - 2\Omega\psi + \mu\psi\frac{\mathrm{d}(\psi^2)}{\mathrm{d}\xi} = 0. \tag{23}$$

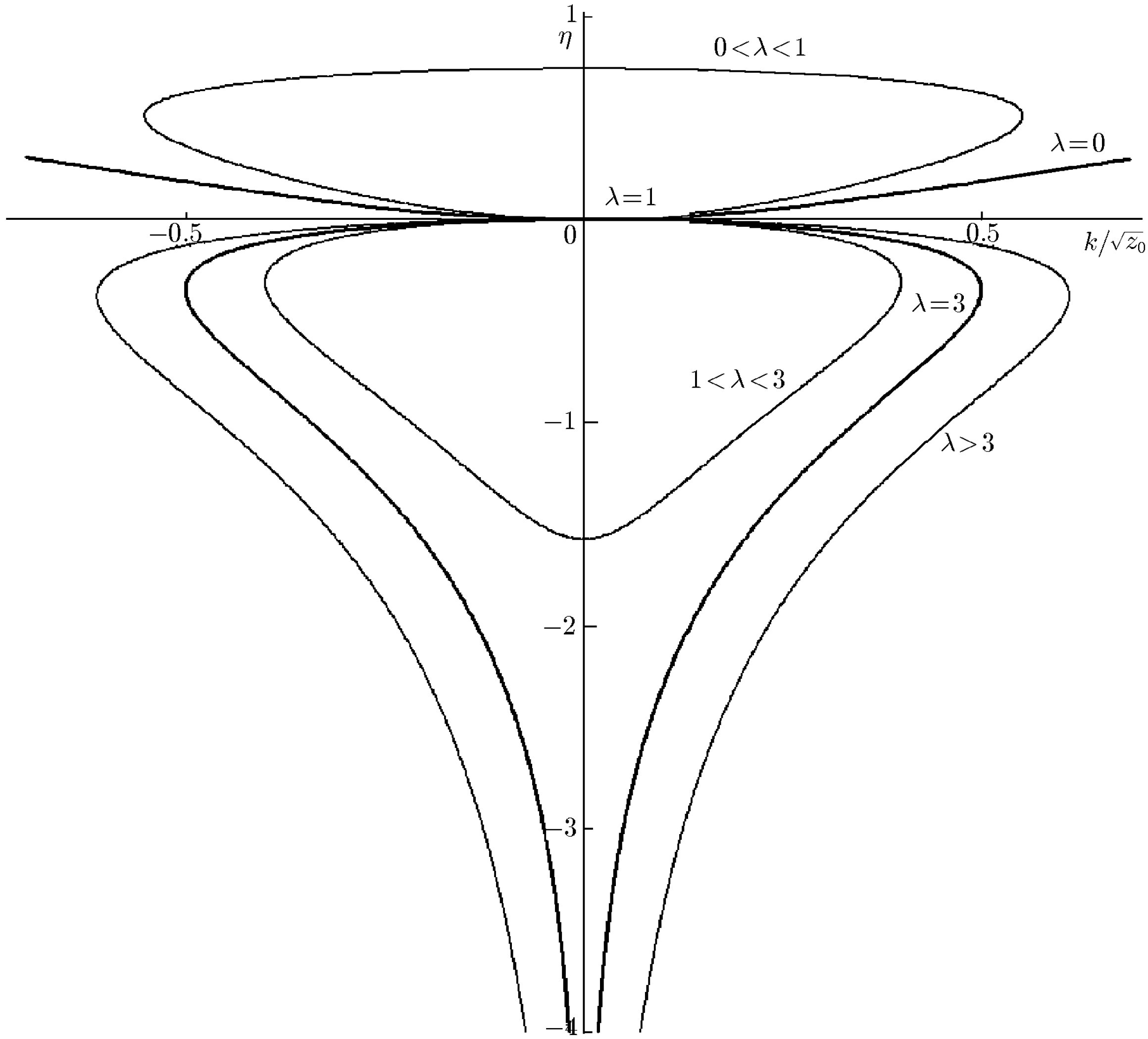

Fig. 1. The curves described by Eq. (22) for $k_0 = 0$ and different values of λ.

Assume that the scale of the dispersion inhomogeneity is much greater than the scale of inhomogeneity of the wave packet envelope, $D \gg L_\psi$. With allowance for the smallness of the parameter $\varepsilon \approx L_\psi/D \approx \mu \ll \{\alpha, q_0\}$, we will seek the solution of Eq. (23) in the form $\psi = \psi_0 + \psi_1$, where $\psi_1 \sim \varepsilon\psi_0 \ll \psi_0$. Keeping terms of the order of ε, we have

$$q_0 \frac{\mathrm{d}^2\psi_0}{\mathrm{d}\xi^2} + 2\alpha\psi_0^3 - 2\Omega\psi_0 = 0; \tag{24}$$

$$q_0 \frac{\mathrm{d}^2\psi_1}{\mathrm{d}\xi^2} + \left(6\alpha\psi_0^2 - 2\Omega\right)\psi_1 = \frac{q_0}{D}\frac{\mathrm{d}^2\psi_0}{\mathrm{d}\xi^2}\xi - \frac{2}{3}\mu\frac{\mathrm{d}(\psi_0^3)}{\mathrm{d}\xi} + \frac{q_0}{D}\frac{\mathrm{d}\psi_0}{\mathrm{d}\xi}. \tag{25}$$

Equation (24) has a classical soliton solution $\psi_0 = A_0/\cosh(\xi/\Delta)$, where $\Delta = \sqrt{q_0/\alpha}$, $\Omega = \alpha A_0^2/2$. Equation (25) after the replacements $\eta = \xi/\Delta$ and $\Psi = \psi_1 D/(A_0\Delta)$ takes the form

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}\eta^2} + \left(\frac{6}{\cosh^2\eta} - 1\right)\Psi = \frac{2\eta}{\cosh^3\eta} - \frac{\eta}{\cosh\eta} + \frac{5}{4}\frac{\mu}{\mu_*}\frac{\sinh\eta}{\cosh^4\eta} + \frac{\sinh\eta}{\cosh^2\eta}, \tag{26}$$

where $\mu_* = -5q_0/(8A_0^2 D)$ is the high-frequency loss coefficient corresponding to the equilibrium state of

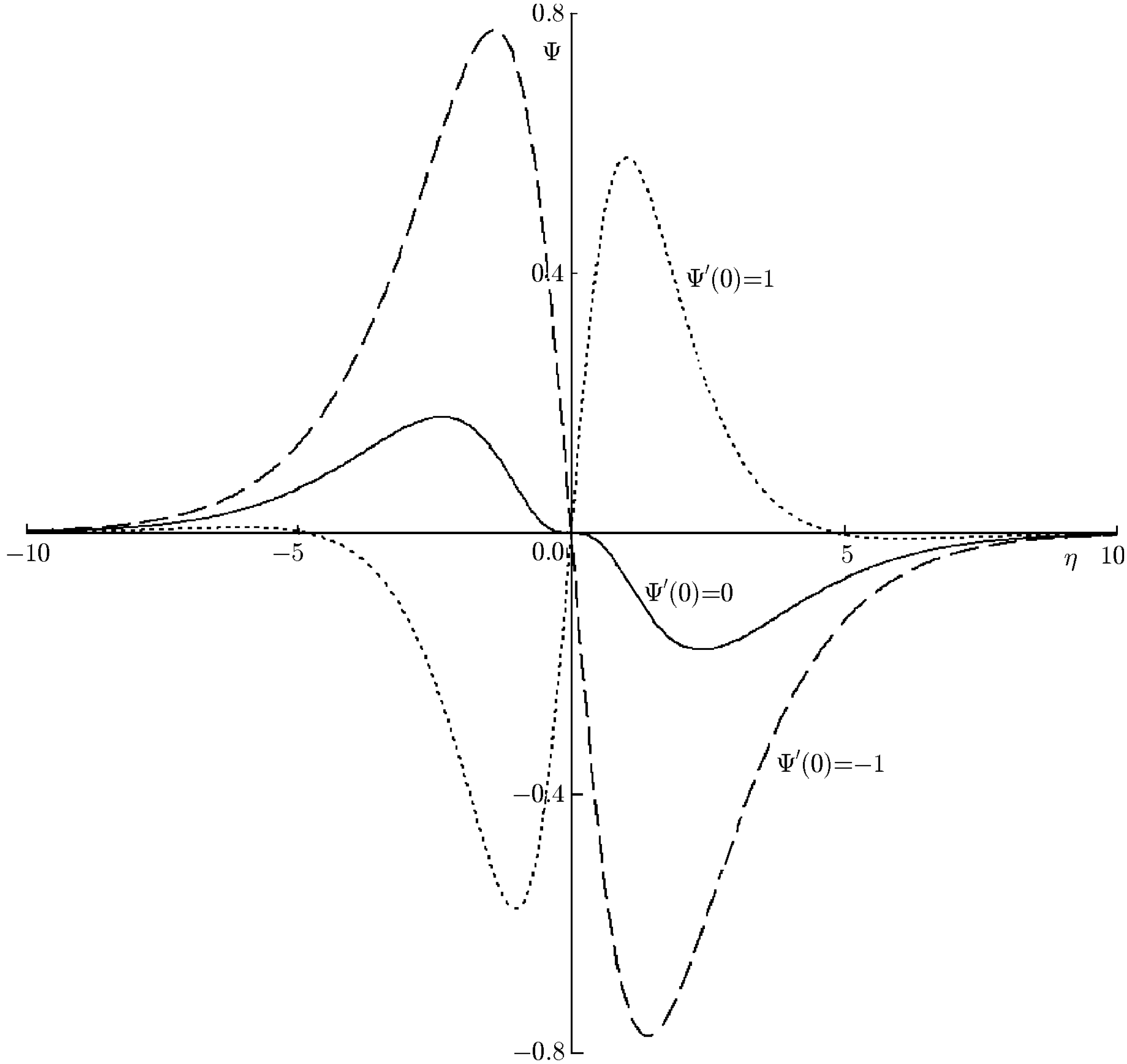

Fig. 2. The distribution $\Psi(\eta)$ for $\mu = \mu_*$ and different $\Psi'(0)$.

system (19)–(21). For $\Psi(0) = 0$, Eq. (26) has an exact solution

$$\Psi(\eta) = \left[\Psi'(0)\eta - \frac{\eta^2}{4}\tanh\eta + \frac{\mu}{4\mu_*}(\tanh\eta)\,\ln(\cosh\eta)\right]\mathrm{sh}\,\eta + \frac{1}{12}\left(\frac{\mu}{\mu_*} - 1\right)(\tanh^2\eta)\sinh\eta. \tag{27}$$

For $\mu = \mu_*$ corresponding to an equilibrium of stimulated scattering and decreasing dispersion, solution (27) is localized. In this case, asymptotic form (27) for a large argument is as follows: $\Psi(\eta \to \pm\infty) \approx \pm\eta^2 \exp(-|\eta|)$. The wave field distribution $\Psi(\eta)$ for $\mu = \mu_*$ and different values of $\Psi'(0)$ is shown in Fig. 2. The solution $\Psi(\eta)$ is antisymmetric. The solitons with antisymmetric "tails" also arise in the well-known system of linearly coupled nonlinear Schrödinger equations [36].

For $\mu \neq \mu_*$, solution (27) is not localized, $\Psi(\eta \to \pm\infty) \to \infty$. Such a distribution of the function $\Psi(\eta)$ for the initial conditions $\Psi'(0) = 0$ and different values of μ is given in Fig. 3.

4. NUMERICAL SIMULATION

Consider the initial problem of the dynamics of a soliton-like wave packet $U(\xi, t = 0) = \mathrm{sech}\,\xi$ within the framework of Eq. (3) ($q(\xi) = \exp(-\xi/10)$ and $\alpha = 1$) from the viewpoint of numerical simulation. From

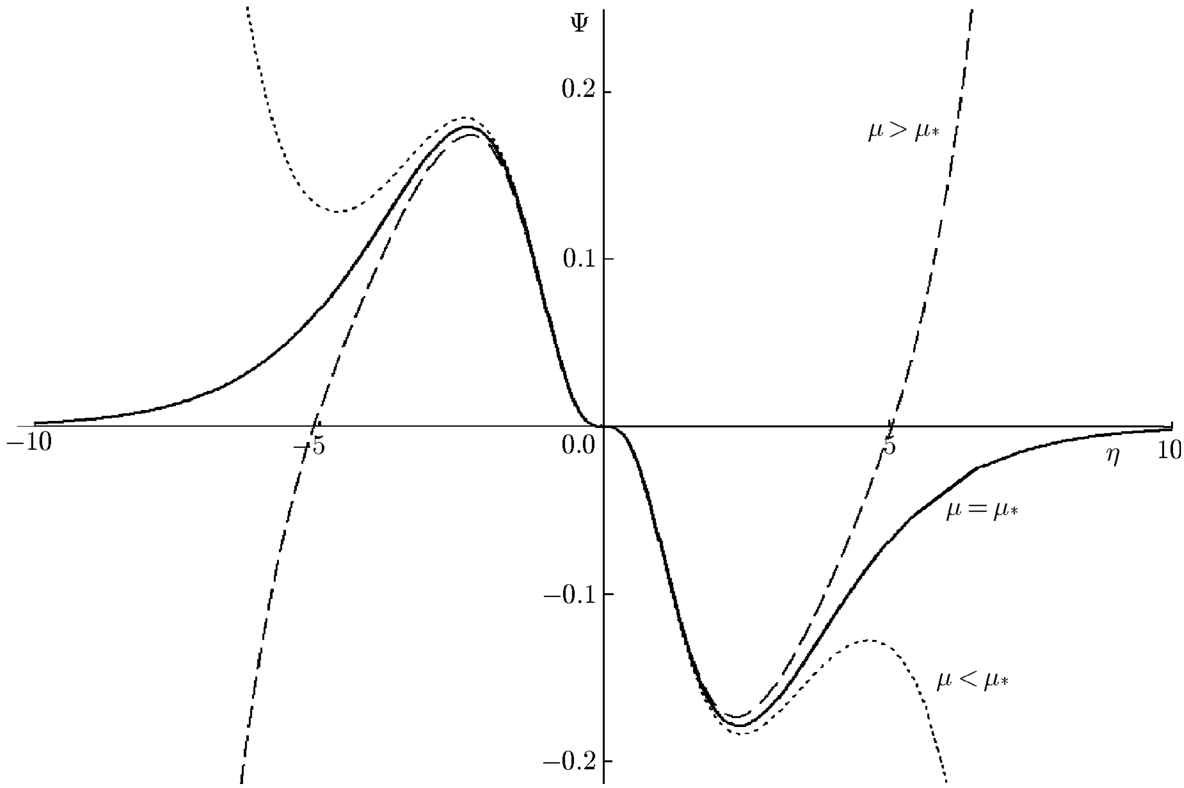

Fig. 3. The distribution $\Psi(\eta)$ for $\Psi'(0) = 0$ and different values of μ.

the analysis of system (19)–(21) one can find the equilibrium value of the coefficient μ for a given initial pulse, namely, $\mu_* = 1/16$.

In the numerical calculations for $\mu = 1/16$, the initial wave packet evolves to a stationary localized distribution (solid curve in Fig. 4) with a zero wave number. This distribution coincides with the analytically obtained solution of the system of equations (24) and (25) for $q_0 = \alpha = A_0 = 1$, $D = 10$, and $\mu = \mu_*$:

$$|U| = \left\{1 + \frac{1}{40}(\tanh\xi)\ \ln(\cosh\xi) - \xi^2(\tanh\xi)\Big]\right\} \mathrm{sh}\,\xi.$$

For comparison, Fig. 4 shows a distribution of the soliton solution envelope for the nonlinear Schrödinger equation $|U| = 1/\cosh\xi$ (dotted curve). A small antisymmetric deviation of the profile of the envelope from the profile of a classical soliton is seen in the figure.

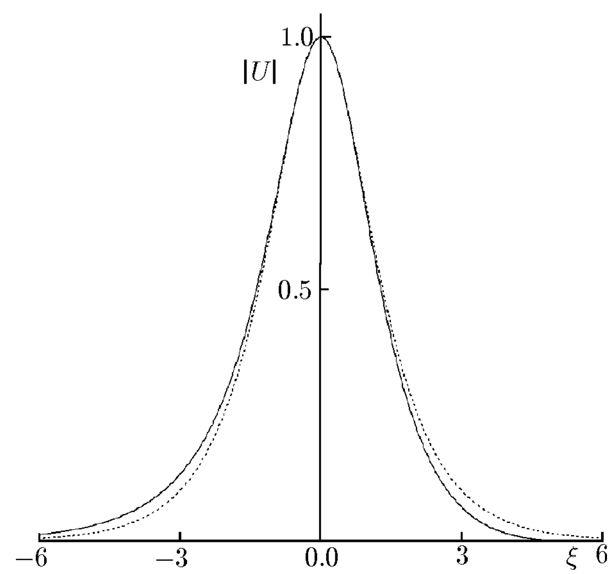

Fig. 4. The numerically obtained distribution of $|U|$ over $|\xi|$ for $5 < t \leq 150$ at $q(\xi) = \exp(-\xi/10)$ and $\mu = 1/16$ (solid curve) and the soliton profile of the nonlinear Schrödinger equation $|U| = 1/\cosh\xi$ (dotted line).

Deviation of the parameter μ from the equilibrium value of μ_* leads to a temporal variation in the soliton parameters (wave number and amplitude). Figure 5 shows the spatial distribution of the absolute value of the envelope $|U|$ and the local wave number K for $\mu = 1/32$ at various time points. The distribution of the wave number in the soliton core region conforms to the statements adopted in the analytical part of this study.

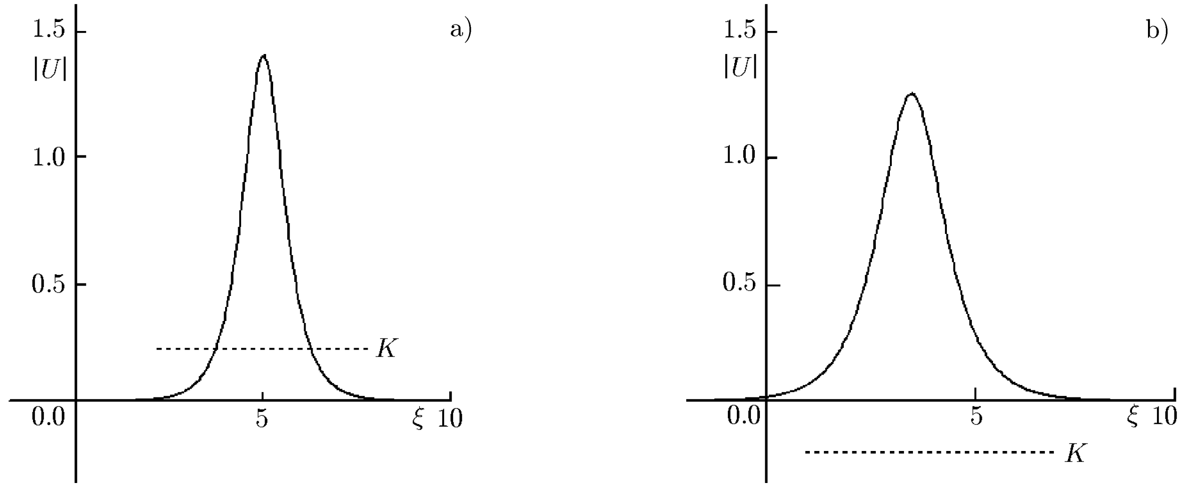

Fig. 5. Spatial distribution of $|U|$ (solid line) and the local wave number K (dotted line) for $\mu = 1/32$ at the times $t = 40$ (*a*) and $t = 70$ (*b*).

Figure 6 shows the temporal dynamics of the local wave number at the point of the maximum absolute value of the wave packet envelope for $q(\xi) = \exp(-\xi/10)$ and different values of μ. The solid and dotted curves correspond to the results of numerical calculations and the analytical solution of system (21)–(23), respectively.

For $\mu < 3/16$, which corresponds to the regime of the dynamic equilibrium of stimulated scattering and decreasing dispersion, the numerical calculation results and the analytical consideration are well correlated. For $\mu \geq 1/16$, the results do not coincide. This noncoincidence is due to the fact that the radiation fields from the soliton core, which appear in the numerical calculations, were not taken into account in the analytical consideration. Radiation fields lead to a decrease in the soliton core energy and, as a consequence, a deviation of the numerically obtained wave numbers of the wave packet from the values obtained by analytical solution of Eqs. (26) and (27).

5. CONCLUSIONS

In this paper, we consider the soliton dynamics within the framework of the extended Schrödinger equation with inhomogeneous dispersion taking into account stimulated scattering by damped low-frequency waves. The study was both analytical and numerical. It is shown that the spatial stimulated scattering by damped low-frequency waves, which leads to a downshift of the spatial spectrum of the soliton wave numbers, and the dispersion decrease on the spatial coordinate, which leads to an upshift of the soliton spectrum, can be counterbalanced. A soliton solution resulting from this equilibrium is obtained in explicit form. The regime of the dynamic equilibrium of stimulated scattering and inhomogeneous dispersion, in which the soliton parameters vary in time periodically, is also found.

The soliton dynamics was considered neglecting the inhomogeneity of the cubic nonlinearity, group velocity of the high-frequency component, nonlinear dispersion, and third-order linear dispersion. The soliton dynamics with allowance for the effects stipulated by these summands will be examined elsewhere.

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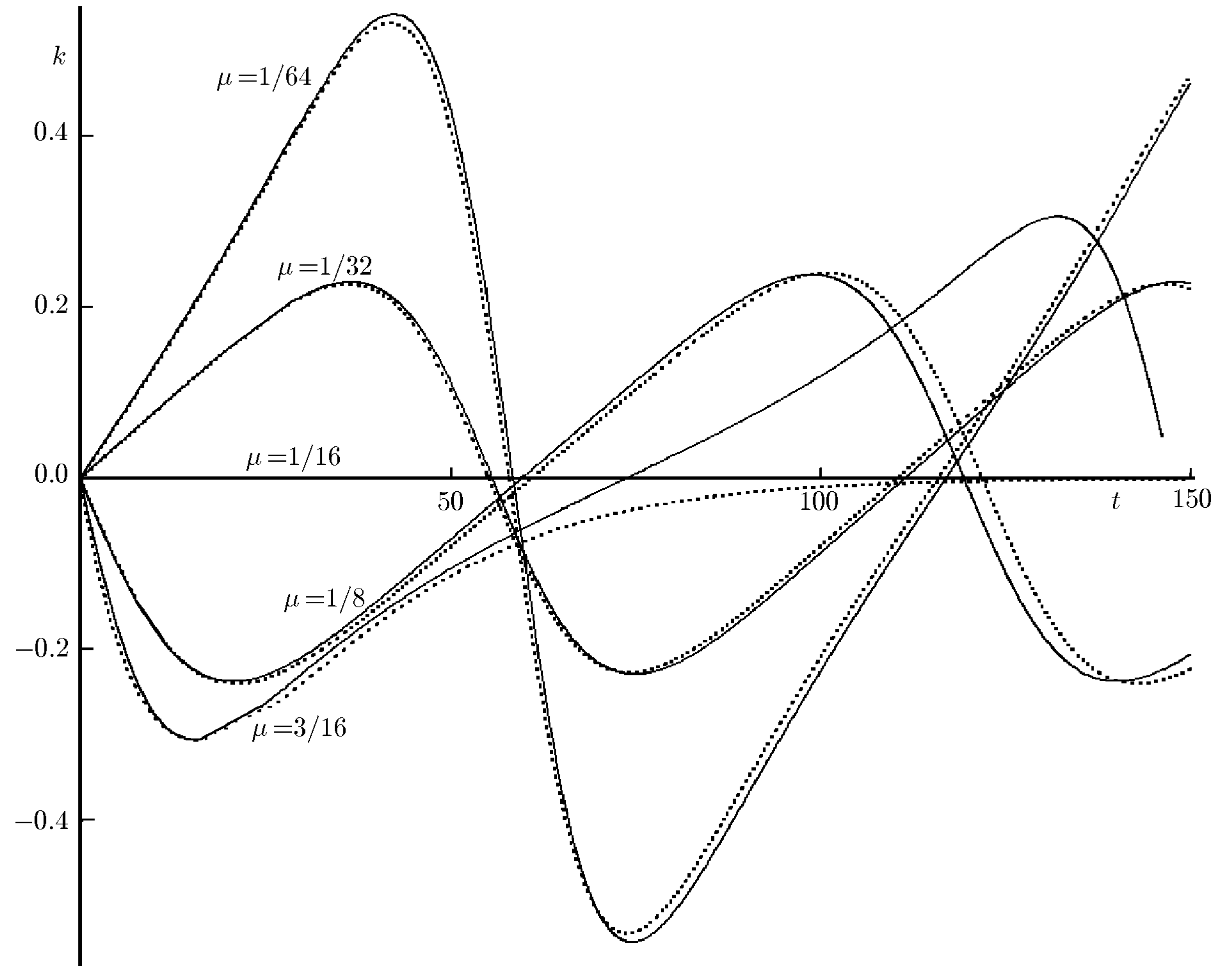

Fig. 6. Temporal dynamics of the wave number k of a soliton at the center of the maximum of the wave field for $q(\xi) = \exp(-\xi/10)$ and different values of μ. The dotted and solid lines correspond to the analytical and numerical results, respectively.

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