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Macroeconomic Concentration Index of Corporate Sector Companies

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Abstract: *The high concentration of assets and revenues of corporations or large enterprises on a national scale represents a risk that can hinder sustainable development. For instance, in terms of a bank's loan portfolio concentration risk is so crucial that it is a separate significant type of risk in addition to credit risk. In the proposed article a corporate sector concentration index was constructed for countries for which revenue data of TOP-50 or TOP-100 largest enterprises is accessible from open sources. The index is based on the revealed pattern of decrease in the volume indicator of the company (revenue, assets) of the order of 1/n^z, where n is the serial number of the companies ranked in descending order of the indicator, and z is a country-specific parameter. Based on all possibly available open statistics, providing data on 30 countries, the consistency of the statistical hypothesis about the dependence of the corporate sector concentration index on the Human Development Index and Ease of doing business is substantiated.*

1. INTRODUCTION

There are several generally accepted metrics for estimating the concentration of assets, markets, etc. These metrics are actively used to study the issue of asset concentrations in industries, in various businesses, in matters of uneven distribution of benefits, incl. salaries. The first metric is the simplest, it estimates what proportion of the volume is occupied by the largest objects in the amount of N pieces, for example, 10 or 20. Top N or N% companies concentration (CR):

$$CR_N = \sum_{i=1}^{N} MS_i,$$

with MS_i being the market share for each of the N largest manufacturers, in terms of market share.

The second metric is defined as the Gini index and shows the degree of unevenness in the distribution of objects, ordered from large to small. For the first time, this metric was introduced as an economic scientific index (Gini, 1921).

$$Gini = \frac{1}{n(n-1)} \sum_{i=1}^{n} (2i - n - 1) \cdot MS_i,$$

with n being the number of manufacturers in the market, and MS_i being the market share of the manufacturer ordered by increasing size.

The third metric is the Herfindahl–Hirschman index, introduced into scientific use in (Herfindahl, 1950), which has been widely used in theoretical literature.

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$$HHI = \sum_{i=1}^{n} (MS_i)^2 \tag{1}$$

with n being the number of manufacturers in the market, and MS_i being the market share. We will use this index to assess the concentration of companies in a particular country. The reciprocal of this index has direct economic meaning as the Effective Number of the Largest Companies (ENLC):

$$ENLC = 1 \,/\, HHI$$

If MS_i (1) is equal, then $ENLC = n$. The concentration index indicates the degree of monopolization of the economy, showing the risk of concentration, which may impede sustainable development. Banks pay special attention to concentration risk when assessing the required capital to cover credit losses. It is accounted for as a capital penalty (Statement of Policy, 2021) or Granularity adjustment and directly depends on the Herfindahl–Hirschman index calculated from the distribution of loans by exposure volume (Gordy & Luetkebohmert, 2007).

The main problem in assessing the country concentration index is that we do not know the volume of assets of all companies in the country for which we want to calculate this index. The number of legal entities can be measured in hundreds of thousands and even millions. However, the good news is that the Herfindahl-Hirschman index is primarily determined by the largest companies (or corporations) operating in the economy. But how many should be taken into consideration, 100, 1000, 10000 or more? It can be assumed that there is an empirical law of the distribution of assets of companies, sorted in descending order. This hypothetical law is given by the formula

$$A_n \cong \frac{\hat{a}}{n^z} \tag{2}$$

It turns out that it is performed for almost all countries with a high degree of accuracy (Figure 1). The coordinates are taken as the logarithm of revenue and the logarithm of the company number in a descending sorted list.

It can be seen that in the vast majority of observations, the coefficient of determination exceeds 90%, and for some, it is even higher than 99% for the sample of the TOP-100 largest companies in the country. The interpolation dependence is determined by only one exponent z, which is individual for each of the countries. This indicator will allow us to mathematically extrapolate the Herfindahl-Hirschman index to an unspecified number of companies with numbers higher than 100.

It is worth noting that pattern (2) is very reminiscent of Power-Law Distributions in Empirical Data, which is widely observed in empirical data in a diverse range of natural and man-made Phenomena (Clauset et al., 2009) since it was confirmed for the distribution of income and wealth (Benhabib et al., 2011; Champernowne, 1953; Klass et al., 2006; Pareto, 1896; Singh & Maddala, 1976; Toda, 2012; Wold & Whittle, 1957), consumption (Toda, 2017; Toda & Walsh, 2015), firm size (Axtell, 2001; Luttmer, 2007; Stanley et al., 1995), farmland size (Akhundjanov & Chamberlain, 2019), city size (Berliant & Watanabe, 2015; Devadoss et al., 2016; Gabaix, 1999; Ioannides & Overman, 2003; Krugman, 1996), natural gas and oil production (Balthrop, 2016). However, it is important for our study that (2) is not a probabilistic law, but an empirical fitting of pre-ordered fractions. Therefore, the method for estimating the power component of z should not rely on the maximum likelihood method, which, as shown in many works (see, for example, Deng & Wang, 2023), is the most accurate in estimating the distribution parameter. The exponent z is easily calculated as a regression parameter and tests show that it is statistically significant and has a small statistical error.

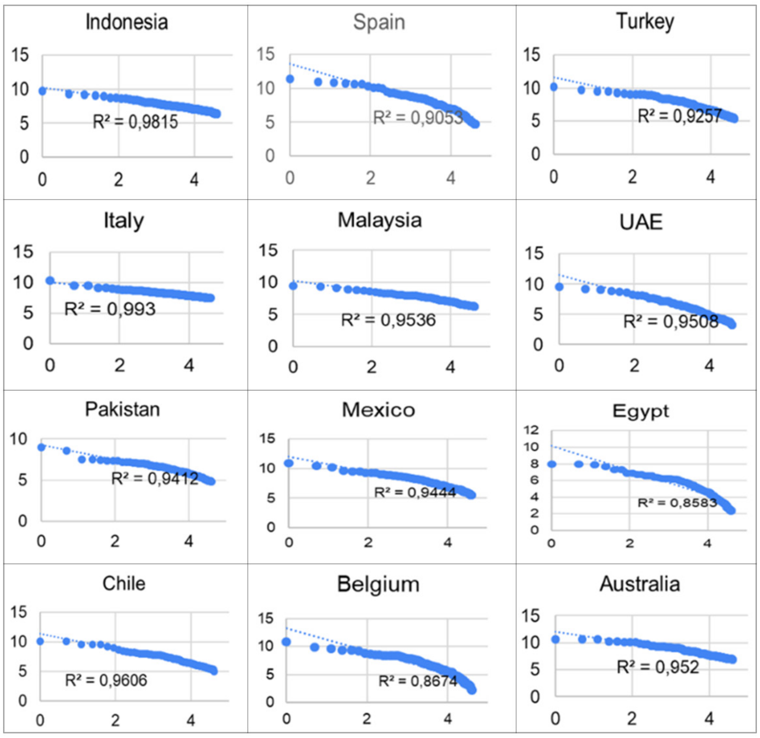

Figure 1. Demonstration of the relevance of the dependence hypothesis (2) for the largest companies sorted in descending order of revenue for a random sample of countries

Source: Own calculations

Thus, each country under study has its exponent in the interpolation function of the distribution in descending order of the assets of the largest companies. As an indicator characterizing the assets, we take the company's annual revenue. The total revenue limiter we offer is the country's Gross Domestic Product for the same period. At this point it is worth making a certain disclaimer, anticipating criticism of the approach. Since the Gross Domestic Product is rather an indicator of the total profit of the economy and not the total revenue of all its subjects. In fact, you need to put a limiter as a multiplicative GDP, but we do not know the rate of return by country; the limiter needed is proportional to the volume of corporations, and the largest contribution to the concentration index is made by the largest ones. Based on these economic assumptions, we believe it is reasonable to set a cap on the volume of Gross Domestic Product since we are examining the index, which is not necessarily equal to the exact value.

2. METHODOLOGY FOR EVALUATION OF THE CONCENTRATION INDEX

Model assumptions:

1. Each country has its own z parameter for the ordered concentration function (2).
2. The assets of the TOP m=100 companies in the country are taken as the annual revenues.
3. The country's GDP is the limiter of total revenue included in the calculation of the concentration index.

Within the framework of the proposed assumptions, formula (1) can be adjusted, having data on the revenue of TOP-m companies, taking into account the GDP limit and the concentration model (2) (see, Appendix). If *HHI* is the quasi Herfindahl-Hirschman Index for a limited number of observations of top companies, for $z > 1/2$ the evaluation formula is used

$$HHI = \left(\frac{\sum_{k=1}^{m} A_k}{\mathrm{GDP}}\right)^2 \cdot \left(\widehat{HHI}_m + \begin{vmatrix} \left[(2z-1)\cdot m^{2z-1}\cdot\left(\zeta(z)+\frac{m^{1-z}}{1-z}\right)^2\right]^{-1} & , if\ |1-z| > \varepsilon \\ \frac{1}{m\cdot(\gamma + ln\,(m))^2} & , if\ |1-z| \le \varepsilon \end{vmatrix} \right) \qquad (3)$$

where

$$\widehat{HHI}_m = \frac{\sum_{k=1}^{m} {A_k}^2}{\left(\sum_{k=1}^{m} A_k\right)^2},\ \zeta(z)$$

– Riemann zeta function, ε is chosen according to the accuracy of calculations ($\varepsilon = 0.002$ is recomended), γ denotes Euler’s constant from Laurent decomposition of the Riemann zeta function (Candelpergher & Coppo, 2020).

$$\gamma = \lim_{N\to\infty}\left(\sum_{n=1}^{N}\frac{1}{n} - \ln\,(N)\right) = 0.577215665,\ \sum_{\mathrm{k}=1}^{\mathrm{m}} \mathrm{A_k}$$

– the sum of TOP-m assets of companies from the sample.

The next task is to build a macroeconomic model for the dependence of the concentration index (3) on macroeconomic country factors.

3. MACROECONOMIC CONCENTRATION INDEX MODEL

Macroeconomic indices, characterizing the development of the country, were studied by researchers multiple times, and we carefully studied some of the most outstanding ones. For instance, the paper “Corruption and Growth” (Mauro, 1995) illustrates the devastating impact of corruption on the economic growth, prosperity, and wellness of the country. Fighting this phenomenon can be difficult as corruption can be deeply rooted in society and institutional structures. Nevertheless, improving the institutional work, increasing transparency, and strengthening the rule of law can help fight corruption and improve economic growth.

Furthermore, the article “Unemployment in the OECD Since the 1960s: What Do We Know?” (Nickell et al., 2005) contains many important findings on unemployment in Organization for Economic Co-operation and Development (OECD) countries since the 1960s.

The study found that during periods of high unemployment in OECD countries, there was an increase in income inequality, indicating that poverty may be associated with high unemployment. The analysis showed that high unemployment is often associated with policies aimed at reducing poverty, such as lowering the minimum wage or raising taxes, and the decline in unemployment may be due to improved education and training, as well as the development of technology and innovation. The authors of the study conclude that policies to reduce unemployment must take into account both economic and social aspects, including income inequality and social problems associated with poverty.

In addition, there was a research "Institutions and Economic Performance: Cross-Country Tests Using Alternative Institutional Measures" (Knack & Keefer, 1995) exploring the impact of the institutional environment on economic performance across countries. The article presents the results of empirical studies that show that the institutional environment can have a significant impact on economic growth and development. The authors conclude that different institutional arrangements can have different impacts on economic productivity. Some measures, such as the quality of the legal system, the efficiency of the judiciary, and the level of corruption, may be more important for economic development than others, such as the level of taxes and the degree of market freedom. Countries with more developed institutional arrangements, such as an efficient legal system and low corruption, can have a higher level of economic development and a higher standard of living for their citizens. However, the way certain factors correlate with monopolization is still not obvious, and this is the reason for our research.

4. DATA DESCRIPTION

In order to calculate the Effective Number of Largest Companies, it was necessary to use revenues of N largest companies. Among all publicly available data, we could find information on top-600 largest companies for Russia (RAEX-RR, 2021), top-150 for the USA, Canada, Germany, the UK, China, India, Japan, top-100 for 20 countries (France, Norway, Israel, Hong Kong, Egypt, Indonesia, Spain, Turkey, Malaysia, the UAE, the Netherlands, Thailand, Chile, Mexico, Peru, Italy, Australia, Nigeria, Pakistan, Belgium, Sweden) (Statista, 2023) and top-70 for Brazil and South Korea for 2021 (market) as the latest available period. There was also available data for the Russian largest companies from 1995 to 2021 which allowed us to look at ENLC dynamics during 25 years (RAEX-RR, 1995).

For correlation analysis indexes from World Bank Database (2021) were considered. We examined indexes that could be connected with ENLC and used their values for 2021. They can be grouped into political, economic, agricultural, and labour factors.

The full list consists of GDP, Human Development Index (HDI, 2022), Environmental Performance Index (EPI, 2022), Economic Freedom, Share of SME in GDP, Corruption perceptions, Total natural resources rents (% of GDP), Military expenditure (% GDP), the Poverty rate (% population), Gini index, Ores and metals import, Land area (sq. km), Rural population (% of the total population), Agriculture, forestry, and fishing, value added (% of GDP), CO2 emissions (metric tons per capita), GDP growth (annual %), Imports of goods and services (% of GDP), Exports of goods and services (% of GDP), Unemployment (% of the total labour force), Foreign direct investment, net inflows (BoP, current US$), Market capitalization of listed domestic companies (% of GDP), the Business extent of disclosure, Ease of doing business rank, Researchers in R&D (per million people), Logistics performance index.

5. ENLC CALCULATIONS

The above-mentioned formula of ENLC performed the following results. If sorted in ascending order, countries are distributed in the following way (Figure 2). Thus, countries can be divided into 3 groups with Russia and China on the borderlines. The higher the ENLC, the lower the monopolization, which means Nigeria has the lowest level of capital concentration, while South Korea – has the highest.

As mentioned earlier, historical data on the Russian largest companies allowed us to calculate ENLC for a variety of periods (Figure 3). There is a slightly declining trend, which can be interpreted as an increase in corporate sector monopolization. It corresponds to the reality – market leaders enlarge their capital, making smaller competitors abandon the market and preventing potential ones from appearing. The fact that ENLC dynamics match the actual situation proves the efficiency and reliability of the suggested instrument.

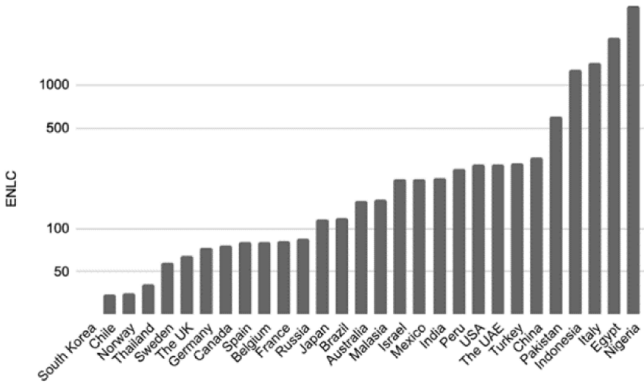

Figure 2. Distribution of countries' ENLC in a logarithmic scale

Source: Own calculations

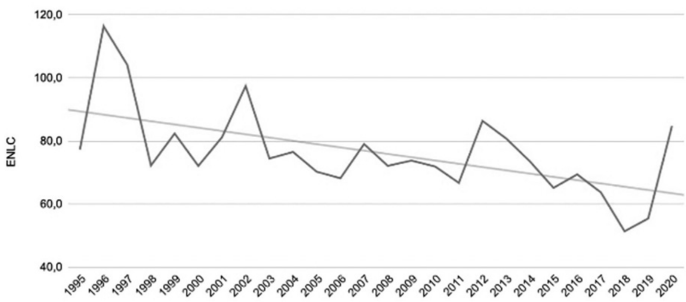

Figure 3. ENLC dynamics in Russia

Source: Own calculations

Table 1. Correlation analysis of ENLC and macroeconomic country indices

World Bank indices	**Correlation**	**Standard Error**	**Significant at 5% significance level**	**Significant at 1% significance level**
Ease of doing business rank	66%	14%	TRUE	TRUE
Human Development Index (HDI)	-61%	15%	TRUE	TRUE
Agriculture, forestry, and fishing, value added (% of GDP)	61%	15%	TRUE	TRUE
Poverty rate, %population	52%	16%	TRUE	TRUE
Rural population (% of total population)	52%	16%	TRUE	TRUE
Market capitalization of listed domestic companies (% of GDP)	-51%	16%	TRUE	TRUE
Corruption perceptions	-48%	17%	TRUE	TRUE
Logistics performance index: Overall (1=low to 5=high)	-48%	17%	TRUE	TRUE
Economic Freedom	-45%	17%	TRUE	FALSE
Researchers in R&D (per million people)	-45%	17%	TRUE	FALSE
EPI	-38%	18%	TRUE	FALSE
CO2 emissions (metric tons per capita)	-36%	18%	TRUE	FALSE
Exports of goods and services (% of GDP)	-35%	18%	FALSE	FALSE
Imports of goods and services (% of GDP)	-29%	18%	FALSE	FALSE
Total natural resources rents (% of GDP)	26%	18%	FALSE	FALSE

Unemployment, total (% of total labour force)	24%	18%	FALSE	FALSE
Military expenditure, %GDP	-23%	18%	FALSE	FALSE
ln(GDP)	-21%	18%	FALSE	FALSE
Share of SME in GDP	-18%	19%	FALSE	FALSE
Land area (sq. km)	-16%	19%	FALSE	FALSE
GDP	-10%	19%	FALSE	FALSE
GDP growth (annual %)	-10%	19%	FALSE	FALSE
Foreign direct investment, net inflows	-9%	19%	FALSE	FALSE
Ores and metals import	-8%	19%	FALSE	FALSE
Gini index	-5%	19%	FALSE	FALSE
Business extent of disclosure index	0%	19%	FALSE	FALSE

Source: Own calculations

As for the next step of the research, the correlation between ENLC and World Bank indices was calculated (Table 1). Among 26 factors only 12 of them showed statistical significance at the level of 5%, and 8 factors were significant at the level of 1%. It was decided to use 12 factors in the following models.

6. MODEL CONSTRUCTION

The main goal of model construction was to find factors that can explain ENLC most accurately and precisely.

First, we built linear regressions on a different number of factors (from 2 to 6) with the highest correlation, making a restriction on coefficient signs that should be of the same sign as the correlation between ENLC and each index. Secondly, AIC was calculated for each model (Table 2), showing the least number for the 2-factor model. Thus, only 2 factors (HDI and Ease of doing business rank) were included in the model.

Table 2. AIC for linear regressions

№ factors	AIC	R_sq
2	-116,64	0,46
3	-115,55	0,48
4	-114,93	0,50
5	-113,24	0,51
6	-111,91	0,52

Source: Own calculations

However, when examining a simple linear regression, we found heteroscedasticity via the predicted values – residuals plot and White's test and needed to change the functional form of the model. The best result was achieved with the semi-log model.

Thus, the final model was the following:

$$\text{Ln}(ENLC) = -4{,}72 \times HDI + 0{,}01 \times \textit{Ease of doing business rank} \quad (4)$$

It had characteristics shown in Table 3.

We conducted tests, checking the correctness of the model, such as White's test, VIF, F-test, Ramsey's test, and correlation analysis. The model showed the absence of multicorrelation and heteroscedasticity, general significance, and correct specification.

Table 3. Final model characteristics

Dep. Variable	Ln_ELNC					
Model	OLS					
Method	Least Squares					
No. Observations	30					
Df Residuals	27					
Df Model	2					
Covariance Type	Nonrobust					
R_squared	0.459					
Adjusted R_Squared	0.418					
F_statistic	11.43					
Prob (F-static)	0.000253					
Log-Likelihood	-40.574					
AIC	87.15					
BIC	91.35					
	coef	**Std err**	**t**	**P > |t|**	**[0.025**	**0.975]**
Intercept	8.56	2.42	3.54	0.001	3.6	13.52
Ease_business	0.01	0.008	1.3	0.2	-0.006	0.027
HDI	-4.72	2.53	-1.87	0.073	-9.92	0.474
Omnibus	0.265					
Prob(Omnibus)	0.88					
Skew	0.19					
Kurtosis	2.73					
Durbin-Watson	1.36					
Jarque-Bera (JB)	0.28					
Prob(JB)	0.87					
Cond. No.	1.10e+03					

Source: Own calculations

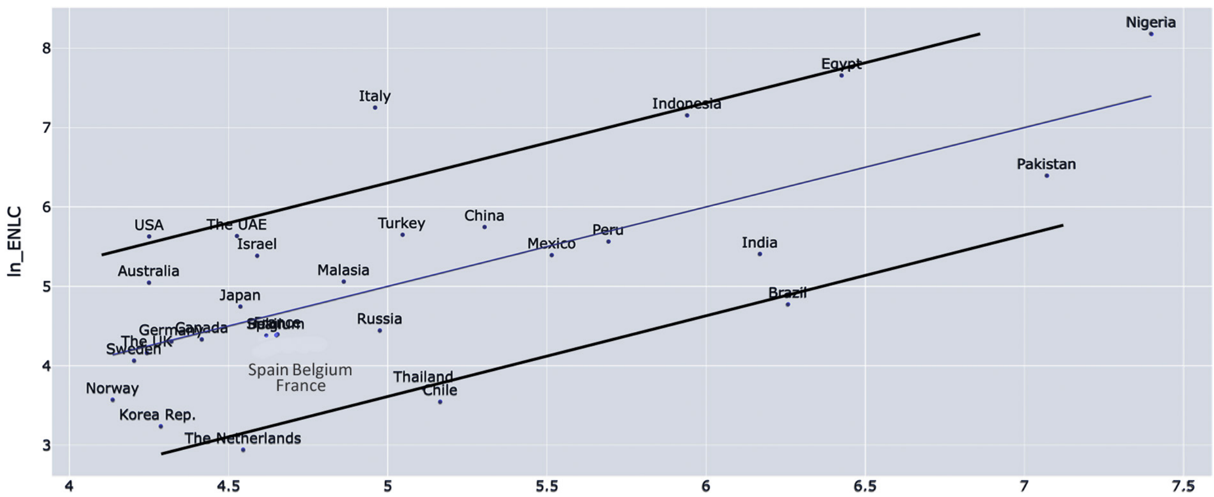

Figure 4. Distribution of countries relative to the predictor of the proposed model. The dotted line marks the deviation at the level of 30%

Source: Own calculations

7. SOCIAL-ECONOMIC HYPOTHESES ABOUT EMITTING COUNTRIES

In our research emitting countries are the countries that appeared to be statistical outliers, thus not being able to be explained by the given model.

The regression was applied to the given data in order to find outliers and therefore suggest possible explanations for the observable result. For most countries predicted values are in adequate intervals, while there are 4 outliers, which are Italy, the Netherlands, Chile, and Brazil (Figure 4). The former has a rather low level of monopolization, while the others vice versa.

After conducting a qualitative country analysis, the following hypotheses were put forward:

1. **Italy** has a very low level of monopolization due to strict antitrust laws, and high involvement of the government and organizations that protect consumers' interests and control the prices of goods and services.
2. **The Netherlands** has a high level of monopolization because the economy is largely focused on highly monopolized industries such as oil and gas, chemical, and metallurgy and the country has complicated legislation for small enterprises.
3. **Chile** has a high level of monopolization because of extremely high poverty and few highly developed industries that are likely to be monopolistic.
4. **Brazil** has a high level of monopolization because of uneven development of regions, high concentration of commodity markets, corruption, low investment, and immense public sector (the largest in Latin America).

Overall, the countries have common reasons to be outliers: there is high involvement of the government, which often suppresses economic development and growth unless key industries are natural monopolies.

8. CONCLUSION

The paper proposes a new country index that measures the concentration of the largest manufacturing companies based on open statistics TOP-70/100 in terms of revenue and GDP of the country. The limited access of the authors to the full statistics of the countries did not allow for calculating the concentration index for most countries; however, it made it possible to make a calculation for 30 countries that happened to be in the available list. Based on this list, a macroeconomic statistical two-dimensional model of the concentration index dependence on macroeconomic indices was built. The most powerful two-dimensional model in terms of the R-square metric was obtained for the Ease of doing business rank and Human Development Index, which makes the expected economic sense. The correlation analysis of the concentration index with other macro-indices was carried out and a significant (at the level of 99%) correlation with eight indices was found. A brief analysis of several countries on the possible reasons for their concentration index deviation from the two-dimensional model is presented. A mathematical derivation of the HHI concentration index formula based on the power law distribution hypothesis, which is substantiated by observations, is presented. The proposed concentration index may be included in the World Bank Database list of macroeconomic country indices as a new reasonable indicator. After expanding the calculation base for a larger number of countries, macroeconomic models and the conclusions of the correlation analysis can be refined.

9. APPENDIX

In this paragraph, the derivation of formula (3) is given. It is well known, see for example theorem 4.11 (Titchmarsh, 1986), that the Riemann zeta function ζ(z), z ∈ R, z > 0, z ≠ 1 satisfies the equation

$$\sum_{k=1}^{n} \frac{1}{k^z} = \zeta(z) + \frac{1}{1-z} n^{1-z} + o(n^{-z}). \tag{1.A}$$

The following constraint is assumed: $\sum_{k=1}^{n} A_k = G$.

It is possible to calculate the «zero approximation» for Herfindahl–Hirschman index

$$\widehat{HHI}_m = \frac{\sum_{k=1}^{m} {A_k}^2}{A^2},$$

formulated for renowned companies TOP-m, $m \ll n, A = \sum_{k=1}^{m} A_k$.

It is necessary to estimate the index $HHI = \frac{1}{G^2} \sum_{k=1}^{n} {A_k}^2$. Obviously,

$$HHI = \frac{1}{G^2}\left(\widehat{HHI}_m \cdot A^2 + \sum_{k=m+1}^{n} {A_k}^2\right) \tag{2.A}$$

Using hypothesis (2) as well as (1. A) we have

$$\sum_{k=m+1}^{n} {A_k}^2 \cong \sum_{k=m+1}^{n} \left(\frac{\hat{a}}{k^z}\right)^2 = \hat{a}^2 \cdot \left(\sum_{k=1}^{n} \frac{1}{k^{2z}} - \sum_{k=1}^{m} \frac{1}{k^{2z}}\right) \cong$$
$$\cong \frac{\hat{a}^2}{1-2z}(n^{1-2z} - m^{1-2z}) \cong \frac{\hat{a}^2}{2z-1} m^{1-2z}, \text{ since } \ m \ll n \text{ and } z > \frac{1}{2}.$$

Further, taking into account (1. A), is $A = \sum_{k=1}^{m} A_k \cong \hat{a} \cdot \sum_{k=1}^{m} \frac{1}{k^z} \cong \hat{a} \cdot \left(\zeta(z) + \frac{1}{1-z} m^{1-z}\right)$, then

$$\hat{a} = A \cdot \left(\zeta(z) + \frac{1}{1-z} m^{1-z}\right)^{-1} \tag{3.A}$$

Substituting (3. A) in (2. A), taking into account the estimation for $\sum_{k=m+1}^{n} {A_k}^2$, the sought formula is obtained

$$HHI = \frac{A^2}{G^2}\left(\widehat{HHI}_m + \left[(2z-1) \cdot m^{2z-1} \cdot \left(\zeta(z) + \frac{m^{1-z}}{1-z}\right)^2\right]^{-1}\right) \tag{4.A}$$

where z > 1/2, z ≠ 1. The last condition can be violated at z ≈ 1, so it is necessary to determine the asymptotics of (4. A) by removing the singularity. The Laurent expansion of the Riemann zeta function about z = 1 is

$$\zeta(z) = \frac{1}{z-1} + \gamma + O(z-1),$$

then

$$\lim_{z \to 1}\left[(2z-1) \cdot m^{2z-1} \cdot \left(\zeta(z) + \frac{m^{1-z}}{1-z}\right)^2\right] = m \cdot \lim_{z \to 1}\left(\gamma + \frac{m^{1-z}-1}{1-z}\right)^2 = m \cdot (\gamma + \ln(m))^2.$$

From the last equation follows the second part of the formula (3) for values of z close to unity. The parameter ε in (3) was recommended from the practice of calculations for single computer precision of variables.

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