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Application of the Nested Convex Programming to the Optimal Power Flow in MT-HVDC Grids.

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Abstract: This paper deals with an application of the nested convex programming to the optimal power flow (OPF) in multi-terminal high-voltage direct-current grids (MT-HVDC). The real-world optimization problem under consideration is non-convex. This fact implies some possible inconsistencies of the conventional numerical minimization algorithms (such as interior point method). Moreover, the constructive numerical treatment of this problem is usually based on some approximative approaches, namely, on the suitable linearizations and problem relaxations. The resulting convex programming model constitutes an approximated model and can naturally involve the significant (approximation) errors. In difference to the strongly approximate computational approaches mentioned above, the numerical scheme we propose takes into account the specific bi-linear structure of the problem and operates with the originally given non-convex formulation of the problem. We implement the proposed nested optimization approach and study the numerical consistency of the resulting optimal design. The Python based numerical experiments demonstrate the imlementability of the proposed methodology. Optimization problem of the modified version of the CIGRE MT-HVDC is next used as a benchmark test for the approach we developed.

Keywords: Multi-terminal HVDC transmission, optimal power flow, supergrids, advanced convex programming, numerical optimization

1. INTRODUCTION

Multi-terminal high-voltage direct current (MT-HVDC) has becoming an important part of the modern power systems applications such as offshore wind farms, supergrids and urban distribution. It is well-known that the practical operation of these MT-HVDC grids need to be optimized over the set of optimal dynamic models commonly called as “optimal power flow” (OPF). Note that there are various practically motivated classes of these applied models for both AC and DC grids (see Capitanescu [2016] and the reference therein). However, the usual optimization involved approach requires to minimize losses, subject to the energy balance and some natural capability constrains. The resulting mathematical model is non-linear and non-convex. Moreover, it requires to be executed constantly and autonomously during the prescribed system operation time. Therefore, the convergence requirement of the implemented system optimization algorithm constitutes a vital condition for the general functionality of the system.

The OPF is usually integrated into the hierarchical control of MT-HVDC grids which consists in at least of four stages. These stages are called: the level-zero control, the primary control, secondary and tertiary control. Level-zero controls include the necessary vector oriented control that maintains the required voltage in each terminal. This local control needs to be fast with the concrete time constants below to 1.6 ms. It acts at the dynamics of the converters and the passive components of the grid. The primary control usually constitutes a droop that guarantee stability by the local actions on the voltages. However, these local actions can involve some operations points that are strongly inefficient. Therefore, the centralized controls, called secondary and tertiary controls are additionally required in order to carry out the system to a feasible and optimal operation point. The OPF itself constitutes the last control stage. Hence, it needs to be executed in a real time, namely, for the time intervals of 1 to 15 minutes.

Recent investigations on the subject under consideration have focused on convex approximations such as McCormick envelops, linearizations, second order relaxation schemes, conic and semidefinite relaxation. We refer to Montoya et al. [2018] and Gan and Low [2014] for some generic approximation approaches. Despite the increasing interest about the theoretical characteristics of these approximations, non of them are used by power systems operators. A method that deals with the non-convex nature of the problem is clearly required.

In this paper, we propose a conceptually different optimization approach that guarantees a (weak) convergence of the solution procedure applied to the initially given nonlinear and non-convex model. Our system optimization approach is finally applied to the convex objective function in

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the presence of the practically motivated restrictions with the bi-linear structure. These specific problem characterization makes it possible to solve the original problem using the so called nested optimization approach. This framework also allows to establish the numerical convergence of the resulting iterations. Theoretical approach developed in this paper was finally applied to some numerical experiments simulated in Python for the generic CIGRE MT-HVDC benchmark test system. The obtained numerical results show the expected convergence of the method and establish implementability of the proposed algorithm.

The reminder of the paper is organized as follows. Section 2 contains a formal description of the engineering problem and its mathematical representation. The necessary mathematical tool related to the nested convex optimization is collected in Section 3. The main optimization algorithm we propose is finally described in Section 4. Some applied results are next given that section. Section 5 summarizes our contribution.

2. ON THE POWER FLOW IN MT-HVDC GRIDS AND ITS MATHEMATICAL MODEL

A generic multi-terminal HVDC network is formally described by an oriented graph

$$\mathcal{G} = \{\mathcal{N}, \mathcal{E}\},$$

with $\mathcal{N} = \{0, 1, \ldots, n-1\}$ representing the set of nodes. [1] Moreover, $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ denotes the set of branches. Parameters of the electrical grid under consideration are given in per-unit and the generic nodal admittance matrix is calculated with entries g_{km}. Note that we does not include here the neutral point.

We next assume that the graph under consideration is connected and hence, the nodal admittance matrix is positive definite. A primary control is considered at each terminal for droop constant h_k and with the reference voltage 1pu as depicted in Figure 1. The secondary control is not considered here and the corresponding dynamics of level-zero controls is neglected. The nodal power constitutes a decision variable p_k as well as the nodal voltage v_k.

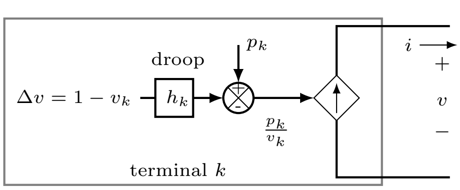

Fig. 1. Representation of a generic terminal in a MT-HVDC grid

Finally we can conclude that the current in each node is given by

$$i_k = \frac{p_k - h_k(1 - v_k)}{v_k}, \tag{1}$$

where

$$v_{\min} \leq v_k \leq v_{\max}.$$

[1] We put the starting count in zero in order to maintain the same representation also in the further Python codes.

Let us note that usually one has

$$v_{\min} = 0.90, \; v_{\max} = 1.1$$

or the corresponding values are taken according to the grid codes.

The main optimization model we consider consists in minimizing the energy losses P_L, subject to the global energy balance at each node. Additionally we incorporate the typical box constraints for each nodal power, branch flow and nodal voltage into the resulting optimization problem. This main optimization model can be formalized as follows

$$\begin{aligned}
&\text{minimize } P_L(p, h, v) = \sum_{k=0}^{n-1}\sum_{m=0}^{n-1} g_{km} v_k v_m \\
&p_k - h_k(1 - v_k) = \sum_{m=0}^{n-1} g_{km} v_k v_m \;\forall k \in \mathcal{N} \\
&p_{k(\min)} \leq p_k - h_k(1 - v_k) \leq p_{k(\max)} \;\forall k \in \mathcal{N} \\
&-f_{km(\max)} \leq \frac{v_k - v_m}{r_{km}} \leq f_{km(\max)} \forall km \in \mathcal{E} \\
&v_{\min} \leq v_k \leq v_{\max} \;\forall k \in \mathcal{N}
\end{aligned} \tag{2}$$

The objective function P_L in (2)evidently has a bilinear structure. This is a simple consequence of the positive definiteness of the given quadratic form. Equation from 2 is in fact a set equality bi-linear forms and this fact causes the non-convexity of the main optimization problem. Note that equation restrictions in (2) constitute the so called box constraints in the above problem and represent the nodal-power capabilities, flow capabilities and voltage regulation, respectively.

As mentioned above the obtained optimization problem (2) can be characterized as a non-convex due to the bilinear objective-constraints structure. However, the bi-linear characterization makes it possible to apply the celebrated nested optimization approach to the optimization problem we consider.

3. MATHEMATICAL FOUNDATIONS OF THE NESTED OPTIMIZATION

This section contains a collection of the necessary mathematical preliminaries. We study some results related to a specific nonlinear numerical optimization technique, namely, to the so called splitting methods. We refer to [Azhmyakov, 2019, Combettes and Pesquet, 2010, Eckstein and Svaiter, 2009] for some additional technical details and further ideas.

Assume $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is a Cartesian product of two real Hilbert spaces and examine the abstract optimization problem

$$\begin{aligned}
&\text{extremize } F(v_1, v_2) \\
&\text{subject to } (v_1, v_2) \in \mathcal{V}_1 \otimes \mathcal{V}_2 \subset H
\end{aligned} \tag{3}$$

where $\mathcal{V}_1 \subset \mathcal{H}_1, \; \mathcal{V}_2 \subset \mathcal{H}_2$. We next assume that $\mathcal{V}_1$ and $\mathcal{V}_2$ are bounded and convex. Moreover, we also suppose that the objective functional $F(\cdot, \cdot)$ in (3) possesses the following properties:

$$F(\cdot, v_2) : \mathcal{H}_1 \to [-\infty, \infty], \; F(v_1, \cdot) : \mathcal{H}_2 \to [-\infty, \infty]$$

are proper convex or concave (i.e., one can be convex while the other is concave) functionals for $v_1 \in \mathcal{H}_1,\ v_2 \in \mathcal{H}_2$, respectively. Note that a convex functional $F(\cdot, v_2)$ is called "proper" if $F(\cdot, v_2) \neq -\infty$ (over $\mathcal{H}_1$) and its effective domain is a non-empty set.

Let

$$v := (v_1, v_2),\ \mathcal{V} := \mathcal{V}_1 \otimes \mathcal{V}_2.$$

Note that the above concept can also be applied to $J(v_1, \cdot)$. We next omit the word "proper", because only such convex functionals will be considered in this paper.

The abstract optimization problem (3) belongs to the family of "nested convex programming" problems. It constitutes a generic theoretical (and numerical) framework for many practically oriented optimization problems (see e.g., Azhmyakov [2019], I. Ekeland [1976]). In addition to the above formal conditions, we next suppose that the full objective $F(\cdot)$ (as a function of v) is a bounded functional on

$$\mathcal{V} + \epsilon\mathcal{B} \subset \text{int}\{\text{dom}\{F(\cdot,\cdot)\}\}.$$

Here $\epsilon > 0$ and $\mathcal{B}$ is the open unit ball of $\mathcal{H}$. The "separate" functionals $F(v_1, \cdot)$ and $F(\cdot, v_2)$ can be convex or concave. Note that the "partial" convexity (or concavity) of $F(\cdot, v_2)$ and $F(v_1, \cdot)$ does not imply the "global" convexity (concavity) property of the complete objective $F(\cdot)$ in (3). Note that a bilinear objective

$$F(v) = \langle v_1, v_2 \rangle_{\mathcal{H}},$$

where $\langle \cdot, \cdot \rangle_{\mathcal{H}}$ denotes a scalar product in $\mathcal{H}$, is a non-convex functional. In the case of a convex $F(\cdot, v_2)$ and concave $F(\cdot, v_2)$ we call (3) a convex-concave problem.

The boundedness of $F(\cdot)$ on $\mathcal{V} + \epsilon\mathcal{B}$ implies the continuity property on the same set (see Azhmyakov [2019], I. Ekeland [1976]). Consequently, it is also Lipschitz continuous on the set $\mathcal{V}$ (see Rockafellar [1970], Thm. 10.4). The last fact involve the formal existence of an optimal solution

$$v^{opt} := (v_1^{opt}, v_2^{opt}) \in \mathcal{V}$$

to the abstract problem (3) is guaranteed by application of the Weierstrass Theorem (see e.g., F. H. Clarke [1998]).

Let us note that the abstract optimization problem (3) provides a necessary theoretic framework for a concrete optimization problem we are dealing with. The elements (objective functional, restrictions) of the applied optimization problem we next consider have the same properties as the corresponding elements of the abstract problem (3). Therefore, the abstract optimization problem (3) provides a universal theoretical framework for the concrete (applied) optimization problem.

Let us recall the following fundamental concept from Variational Analysis Azhmyakov [2019], Rockafellar [1970].

Definition 1. A sequence

$$\{v^k\} \subset \mathcal{H},\ k = 0, 1, ...$$

is called a minimizing (or maximizing) sequence for problem (3) if

$$\lim_{k \to \infty} F(v^k) = \min_{v \in \mathcal{V}} F(v) \text{ (or } \max_{v \in \mathcal{V}} F(v)).$$

Let us now consider the "minimization" version of the basic optimization problem (3) and introduce the following function:

$$\begin{aligned} &\mathcal{F}(\hat{v}_2) := \min_{v_1 \in \mathcal{V}_1} F(v_1, \hat{v}_2), \\ &\hat{v}_2 \in \mathcal{V}_2. \end{aligned} \tag{4}$$

Here $\hat{v}_2 \in \mathcal{V}_2$ is a fixed element. Consider now the auxiliary minimizing problem

$$\begin{aligned} &\text{minimize } \mathcal{F}(\hat{v}_2) \\ &\text{subject to } \hat{v}_2 \in \mathcal{V}_2. \end{aligned} \tag{5}$$

Since (4) and (5) constitute the conventional convex (or concave) programs, the auxiliary function $\mathcal{F}(\cdot)$ in (4) is well-defined and the existence of an optimal solution$v_1^0,\ v_2^o \in \mathcal{V}_2$ to the above problems is guaranteed (see Rockafellar [1970]). Evidently, v_1^o depends on a concrete selection $\hat{v}_2 \in \mathcal{V}_2$ in (4). The next fundamental result provides a theoretic basis for the nested optimization approach. Moreover, this result involves an implementable computational scheme we next will develop and use.

Theorem 1. The value

$$\theta := \min_{\hat{v}_2 \in \mathcal{V}_2} \mathcal{F}(\hat{v}_2)$$

in (5) is the overall minimal value for the initially given problem (3). A solution set $\text{argmin}_{v \in \mathcal{V}} F(v)$ of (3) is given as follows

$$\text{argmin}_{v \in \mathcal{V}} F(v) = F^{-1}(\theta).$$

In the case $(v_1^{opt}, v_2^{opt}) \in \text{argmin}_{v \in \mathcal{V}} F(v)$ we also have

$$(v_1^{opt}, v_2^o),\ (v_1^o, v_2^{opt}) \in \text{argmin}_{v \in \mathcal{V}} F(v).$$

A formal proof of this fundamental theorem can be found in Azhmyakov [2019]. The main numerical difficulty of this "nested optimization" approach consists in a constructive determination of an expression $v_1^o(\hat{v}_2)$. For a convex (or concave, or convex-concave) case Theorem 1 constitutes a constructive solution approach.

4. COMPUTATIONAL OPTIMIZATION OF THE POWER FLOW IN MT-HVDC GRIDS

4.1 The Nested Optimization Based Solution Approach

We now define a vector of nodal voltages $\hat{v}_k$ and the next (augmented) optimization problem as follows:

$$\begin{aligned} &\text{Optimization Problem (2)} \\ &\text{with the additional constraint} \\ &p_k - h_k(1 - v_k) = \\ &\sum_{m=0}^{n-1} g_{km} \hat{v}_k v_m \ \forall k \in \mathcal{N} \end{aligned} \tag{6}$$

The only difference between problem (2) and problem (6) is the additional equation we consider. This formal difference involves a rich geometric characterisation of the advanced problem (6). Since $\hat{v}_k$ is a constant, the additional equation in problem (6) defines an affine subspace and the last fact implies the convexity property. In addition, the partial objectives in (6) (in the sense of Section 3) are linear functions. This property guarantees the global optimum and the numerical convergence assuming a suitable numerical solution procedure will be applied into the nested optimization approach we developed in Section 3.

The nested convex optimization we discussed makes it possible to solve problem (6) by the generic interconnected iterative steps (see Section 3). The Lagrange function for the resulting optimization problem (6) can be written as follows:

$$\begin{aligned}
L(p,h,v,\mu) := {} & \mu_0 \sum_{k=0}^{n-1}\sum_{m=0}^{n-1} g_{km} v_k v_m + \\
& \mu_1[p_k - h_k(1-v_k) - \sum_{m=0}^{n-1} g_{km} v_k v_m] + \\
& \mu_2[p_{k(\min)} - p_k - h_k(1-v_k)] + \\
& \mu_3[p_k - h_k(1-v_k) - p_{k(\max)}] + \\
& \mu_4[-f_{km(\max)} - \frac{v_k - v_m}{r_{km}}] + \\
& \mu_5[\frac{v_k - v_m}{r_{km}} - f_{km(\max)}] + \\
& \mu_6[v_{\min} - v_k] + \mu_7[v_k - v_{\max}]
\end{aligned}$$

where $\mu := (\mu_0, ..., \mu 7)^T$ is a vector of the Lagrange multipliers. We next assume that the optimization problem (6) is Lagrange regular, i.e. $\mu_0 > 0$ (see Azhmyakov [2019], F. H. Clarke [1998] for the necessary mathematical details). The nested optimization expressed by partial optimization problems (4) - (5) can now be applied to the augmented optimization model (6) as well as to the following (unrestricted) Lagrange minimization problem

$$\begin{aligned}
&\text{minimize } L(p,h,v,\mu)\\
&\text{subject to the complete space}
\end{aligned} \tag{7}$$

Note that the Lagrange function from (7) also has a bilinear structure and the above auxiliary Lagrange minimization problem can also be solved by the nested algorithm discussed in Section 3.

In this paper, we apply the nested type solution procedure (4) - (5) directly to the optimization problem (6).

The main computational rule, namely, Algorithm 1 is presented in the form of pseudo-codes. The partial (nested) optimization problems are treated here using the conventional interior point algorithm. We start with an estimate of the value

$$\hat{v}_k = 1\text{pu}.$$

Note that the proposed initial point constitutes a common one for the usual practical optimization experience in power systems applications. Moreover, the value of loss P_L presents here a low change in comparison to the previous step measured using a prescribed tolerance. Problem (6) was solved by using the cvxpy in Python (see Agrawal et al. [2018])

Let us now discuss shortly the convergence properties of the presented algorithm.

Theorem 2. Assume that the formal technical conditions from Section 2 are satisfied. Then application of the nested optimization approach to the main optimization problem (6) implies the linear-convex partial optimization problems of the type (4) - (5).

The proof of the presented consistency result (Theorem 2) associated with the numerical scheme expressed by

Algorithm 1 Nested convex optimization for the OPF in MT-HVDC grids

Require: $g_{km}, p_{k(min)}, p_{k(max)}, f_{km(max)}, v_{k(min)}, v_{k(max)}$

$\hat{v}_k \leftarrow 1, \; \forall k \in \mathcal{N}$
$\hat{P}_L \leftarrow \infty$
$\epsilon \leftarrow \infty$
while $\epsilon >$ tolerance **do**
  $(v_k, p_k, P_L) \leftarrow$ Solve Model 2
  $\hat{v}_k \leftarrow v_k, \; \forall k \in \mathcal{N}$
  $\epsilon \leftarrow \left\| P_L - \tilde{P}_L \right\|$
  $\hat{P}_L \leftarrow P_L$
Print results

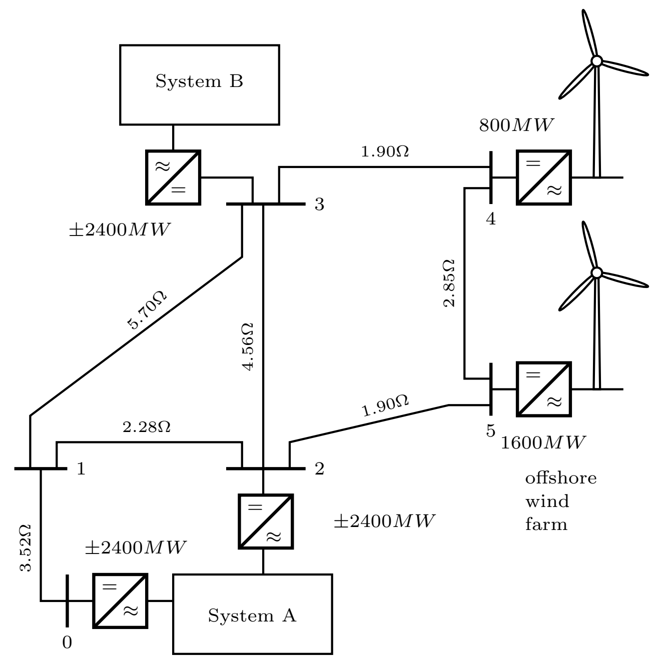

Fig. 2. Reduced version of the CIGRE MT-HVDC grid

Algorithm 1 constitutes in fact a simple consequence of the bilinear structure of the main optimization problem (6) (and the bilinear-type Lagrange function $L(p,h,v,\mu)$ associated with (6)).

4.2 Numerical Aspects

The proposed numerical scheme, namely, Algorithm 1 was finally applied to a simplified version of the CIGRE MT-HVDC test system (see [Vrana et al., 2013] for more technical details). The system under consideration consists on two large offshore wind farms, capable of generate up to 800MW and 1600MW. They are integrated to two transmission systems (A and B) through a meshed grid as depicted in Figure 2.

The physical systems parameters, namely, the parameters of resistance r_{km} are included in Ω and the maximal and minimal values of power in each terminal are given in MW. Moreover, every transmission line has a capability of 1100 MW. The system is represented in per-unit under a base of 400kV/10GW. System B require to receive at least 2000 MW.

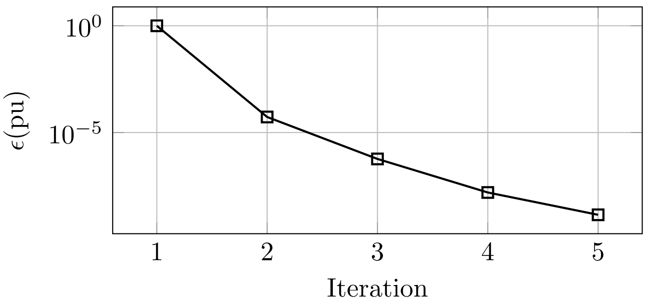

Fig. 3. Convergence of the proposed nested optimization algorithm

Table 1. Nodal variables for optimal operation of the CIGRE MT-HVDC system.

Node	v_k(pu)	p_k(MW)	$p_{k\min}$(MW)	$p_{k\max}$ (MW)
0	0.9918	-153.69	-2400.0	2400.0
1	0.9952	0.00	0.0	0.0
2	0.9923	-1846.31	-2400.0	2400.0
3	1.0081	1015.98	-2400.0	2400.0
4	1.0070	0.00	0.0	800.0
5	1.0053	1012.93	0.0	1600.0

Table 2. Branch flows for optimal operation of the CIGRE MT-HVDC system.

Branch	Flow(MW)	Maximum Flow (MW)
01	-155	1100
12	206	1100
13	-361	1100
23	-555	1100
25	-1100	1100
34	92	1100

The main Algorithm 1 was implemented using the standard Python packages. The main optimization problem (6) was solved using cvxpy routine. Each execution for problem (6) in averaging required 13 iterations. The numerical algorithm presents a convergence (by implementation) in few method iterations (as finally shown in Figure 3). The entire algorithm required 0.202 seconds of the processor time and the indicated computational effectiveness is a suitable one for practical applications.

Finally, let us note that the obtained calculated losses are equal to 29.17 MW. Some additional parameters and computational results are summarized in Tables 1 and 2. The proposed computational algorithm was also tested for various technically motivated scenarios and the practical convergence was similar for the all examined case studies.

5. CONCLUSIONS

This paper proposes a novel numerical optimization approach to a practically motivated power systems optimization problem. We consider the nested minimization approach in the concrete context of the power flow optimization in multi-terminal high-voltage direct-current electrical grids. Application of the nested optimization makes it possible to develop a constructive computational algorithm and solve the above engineering problem in a self-closed form of an iterative numerical scheme.

The resulting nested type solution scheme demonstrates a computational efficient and moreover, possesses the numerical consistence property. Let us also note that the realised calculations are completed in a suitable processor time. It is necessary to stress that the proposed numerical optimization algorithm deals with the originally given (non-linearized) optimization problem with a non-convex variational structure. It can be combined with the diverse numerical optimization schemes that, for example, with the gradient based approach proposed in Azhmyakov and Juarez [2017]. Finally note that the proposed methodology can also be applied to the various alternative classes of the power systems involved optimization problems [2].

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