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Abstract. We study dynamics of a qubit coupled to a single photon mode in a high-quality resonator. For a weak coupling constant the qubit and photon form a long-lived bound state which may be called as a "photon-qubit atom". We consider the stability and chaotic dynamics of the photon-qubit atom under the action of short electromagnetic pulses. The photon oscillator considered in the classical approximation. We plot the Poincaré surface to indicate the appearance of the chaotic behaviour. It is shown that depending on the qubit and field parameters the system may be demonstrating the chaotic behaviour.

1. Introduction

In recent years there have been advances in creating high-quality resonators with built-in Josephson junctions, the lower levels of which act as qubits. Interest in such systems is due to the possibility of creating devices for quantum computing, where artificial atoms play the role of qubits, and photons are used for qubit manipulation. Another group of applications is related to the fact that two-level atoms can be used to generate and detect photons [1].

When a qubit is coupled with a single-photon mode in a high-quality resonator, the exchange results in long-lived entangled photon-qubit states. The interaction of a single-photon mode with a qubit is described by the Rabi Hamiltonian [2]. In the rotating wave approximation the Schrödinger equation admits an exact solution, since there is an additional integral of motion. In this case, the system has a discrete spectrum of the ladder type [2]. The coupled photon-qubit system can be called as a "photon-qubit atom". If the system is excited by a time depended external field, the additional integral of motion is destroyed and a complex motion may be realized, which is similar to what occurs in systems with dynamical chaos [3].

We consider here the stability and chaotic dynamics of the photon-qubit atom under the action of short electromagnetic pulses which may be approximated by delta function series, called here as the Dirac comb [3]. This problem was motivated by recent experiments [4], in which a technique for manipulating individual qubits in resonators and waveguides using a sequence of short (~ns) microwave pulses of large amplitude has been developed.

2. The Hamiltonian of the photon-qubit atom in the field of the Dirac comb

Driven the photon-qubit atom describes by the Rabi Hamiltionian (we set the units when $\hbar = 1$):

$$H = \omega a^\dagger a + \frac{1}{2}\omega_q \sigma_z + g\sigma_x \left(a + a^\dagger\right) + g\left(\mu^*(t)\sigma_+ + \mu(t)\sigma_-\right), \tag{1}$$

where a^+, a are the creation and annihilation operators, ω, ω_q are the oscillation frequencies of the photon field and the qubit, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$, $\sigma_x, \sigma_y, \sigma_z$ are Pauli matrices, $\mu(t) = \mu_0 \delta_T(t)$, μ_0 — amplitude of an external driving force, $\delta_T(t) = \sum_{-\infty}^{\infty} \delta(t - nT)$ is the Dirac comb, T is the time between pulses of an external driving force, n is the number of the pulse. The first three terms in the Hamiltonian (1) described the dynamics of the photon mode, qubit, and the interaction between them with the coupling parameter g. The last term is an external driving force acting on the qubit.

If the coupling parameter g is small compared to the frequencies ω_q and ω, then only the resonance terms can be kept in the Hamiltonian and the rotating wave approximation can be used [2]. In this case the interaction of the photon mode with the qubit may be described by the Jaynes-Cummings Hamiltonian [2]:

$$H_0 = \omega a^\dagger a + \frac{1}{2}\omega_q \sigma_z + g(\sigma_- a + \sigma_+ a^\dagger). \tag{2}$$

For simplicity, we will limit ourselves to the case of exact resonance, $\omega_q = \omega$. The Hamiltonian (2) commutes with the operators:

$$N = a^\dagger a + \sigma_z + \frac{1}{2} \text{ and } C = g(\sigma_+ a + \sigma_- a^\dagger). \tag{3}$$

Exact diagonalisation of the Hamiltonian yields the eigenstates:

$$|+, n\rangle = \frac{1}{2}(|n\rangle |\uparrow\rangle + |n + 1\rangle |\downarrow\rangle), \tag{4}$$

$$|-, n\rangle = \frac{1}{2}(|n\rangle |\uparrow\rangle - |n + 1\rangle |\downarrow\rangle), \tag{5}$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the excited and ground states of the qubit, correspondingly. The energy of the system is written as:

$$E_{\pm n} = \frac{1}{2}\omega_q + (n + 1)\omega \pm g\sqrt{(n + 1)}. \tag{6}$$

Thus, the Hamiltonian (2) describes stationary states of the coupled photon-qubit atom. An interesting question arises about the excitation of such an atom by periodic external action. Below we will consider this question approximately, assuming that the photon mode in the resonator is populated by a large number of photons, and additional short pulses are inserted into the resonator through an external port, the similar way as it is done in the work [4].

3. The semiclassical approximation for photons and dynamics of coupled system
If the conditions of the semi-classical approximation are satisfied for the exciting field (a lot of photons in the mode are received from the generator), then a simplification is possible: the field oscillator can be considered within the framework of classical mechanics. To do that, we first rewrite the Hamiltonian single mode photonic oscillator $\hat{H}_{ph} = \omega\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)$ in terms of the quadratures, $x = \frac{1}{\sqrt{2\omega}}(\hat{a} + \hat{a}^\dagger)$, and $p = i\sqrt{\frac{\omega}{2}}(\hat{a}^\dagger - \hat{a})$, in the form:

$$\hat{H}_c = \frac{p^2}{2} + \frac{\omega^2 x^2}{2}. \tag{7}$$

Notice, that the interaction of qubit with the field then reads as

$$\hat{H}_{int} = \sqrt{2\omega}\sigma_x x. \tag{8}$$

To describe our hybrid system: a classical oscillator interacting with a quantum system, we use the method proposed by Frenkel [5]. We construct the Lagrange function that would describe both the classical and quantum systems:

$$L = \frac{\dot{x}^2}{2} - \frac{\omega^2 x^2}{2} + \langle \Psi | \left(i \frac{\partial}{\partial t} - H_{int} \right) | \Psi \rangle, \quad | \Psi \rangle = \begin{pmatrix} u(t) \\ v(t) \end{pmatrix}. \quad (9)$$

The wave function $| \Psi \rangle$ is a two-component spinor that describes the dynamics of a qubit, each component of which is some function of time. The wave functions satisfy the normalization condition: $\langle \Psi | \Psi \rangle = 1$, $|u|^2 + |v|^2 = 1$. Further, we need the matrix elements

$$\langle \Psi | \sigma_z | \Psi \rangle = |u|^2 - |v|^2, \quad \langle \Psi | \sigma_x | \Psi \rangle = u^* v + u v^*. \quad (10)$$

By using of the equations (8), (9) and (10), we can write Lagrangian in the form

$$L = \frac{\dot{x}^2}{2} - \frac{\omega^2 x^2}{2} + i(u^* \dot{u} + \dot{u} u^*) - \frac{\omega_q}{2}(|u|^2 - |v|^2) - g_e(u^* v + u v^*) - f_{ext}(t)(u^* v + u v^*), \quad (11)$$

where $g_e = \sqrt{2\omega} g$. The external impulse is defined as $f_{ext}(t) = f_0 \delta_T(t)$, where $f_0 = g\mu_0$. When impulse is applied to the qubit, there is rotating of the qubit states taking place.

The equations of motion may be obtained from $\delta \int_{t_0}^{t_1} L dt = 0$:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0, \quad \frac{\partial L}{\partial u^*} = 0, \quad \frac{\partial L}{\partial v^*} = 0. \quad (12)$$

By substituting the Lagrange function (11) into the (12) we can write the equations of motion explicitly

$$\begin{cases} \dot{x} = p, \\ \dot{p} = -\omega^2 x - g_e(u^* v + u v^*), \\ i\dot{u} = \frac{\omega_q}{2} u + g_e v x + f_{ext}(t) v, \\ i\dot{v} = -\frac{\omega_q}{2} v + g_e u x + f_{ext}(t) u. \end{cases} \quad (13)$$

Taking in account the specifics of the Dirac comb perturbation, it is not difficult to obtain the map for the introduced variables for a period. Indeed, from the equation (13) it is followed that after the kick, the variables are related as

$$x_n^+ = x_n^-, \quad p_n^+ = p_n^-, \quad u_n^+ = \cos(\phi) u_n^- - i \sin(\phi) v_n^-, \quad v_n^+ = i \sin(\phi) u_n^- + \cos(\phi) v_n^-, \quad (14)$$

where the indices $\pm$ mean the time before and after the kick at the moment $t_n = nT$. To simplify the matching condition we have use the Euler angles parametrization for the components of the spinor u and v:

$$u = \cos \left(\frac{\theta}{2} \right) \exp \left[i \frac{\phi}{2} \right], v = \sin \left(\frac{\theta}{2} \right) \exp \left[-i \frac{\phi}{2} \right].$$

Now, to find the mapping $(x_n^-, p_n^-, u_n^-, v_n^-) \mapsto (x_{n+1}^-, p_{n+1}^-, u_{n+1}^-, v_{n+1}^-)$ it is necessary after the transformation (14) to solve the equations (13) setting $f_{ext} = 0$ on the period T of the driving field. By repeating the calculations with period T, we obtain Poincaré surface that gives the simplest signature of the onset of chaos.

4. Numerical simulations and results

Bellow as starting conditions, we choose the initial point $x = 1$ and $p = 1$ in the phase space, which correspond to the energy of the oscillator $E = 1$, when $\omega = 1$, and we suppose that the qubit is initially in the ground state $u = 0$ and $v = 1$.

First of all, we want to investigate the case when there is very weak interaction between the photon field and the qubit. To do that, we set the system parameters as: $f_0 = 0, g_e = 0.0002, \omega_q = \omega, R = 0.33$, where f_0 is the external driving field amplitude and the fractional resonance, the ratio of the frequency of the photon field to the frequency of the external driving field, defined as $R = \dfrac{\omega T}{2\pi} = \dfrac{1}{k}$, where $k = 2, 3, 4...$ The results of calculations are displayed on the figure 1.
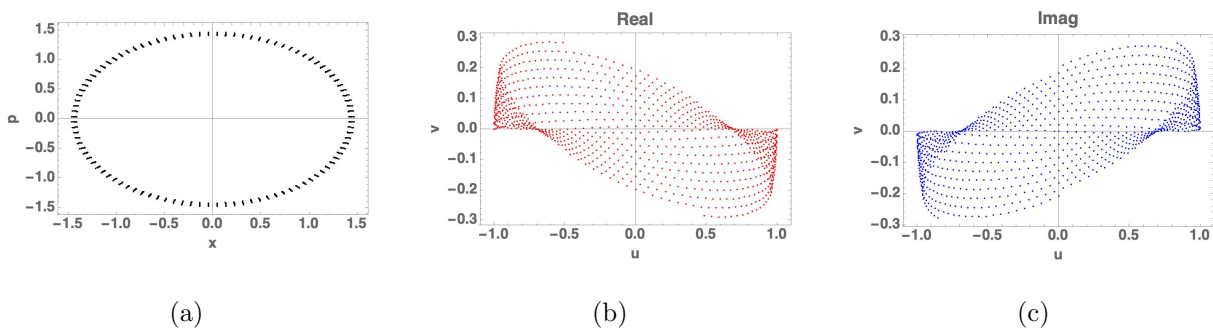

(a)

(b)

(c)

Figure 1: Poincaré surface (p, x), for $u = 0$ (a), and (v, u), real (b) and imaginary (c) parts, respectively, for $u = 0$. The system parameters are $f_0 = 0$, $g_e = 0.0002$, $R = 0.33$.

As have already mentioned in section 2, in the case of weak coupling, there are two integrals of motion in the system: the energy and the total number of particles, they limit the trajectories in the four-dimensional phase space by the intersection of these planes. Therefore, we see on the (p, x) plane, 1a, the almost invariant curve. Then, we found that with increasing the driving amplitude or the coupling parameter the system still shows regular motion.

Now we turn on the action of an external driving field, pumping energy into the qubit. The coupling parameter is assumed to be small.
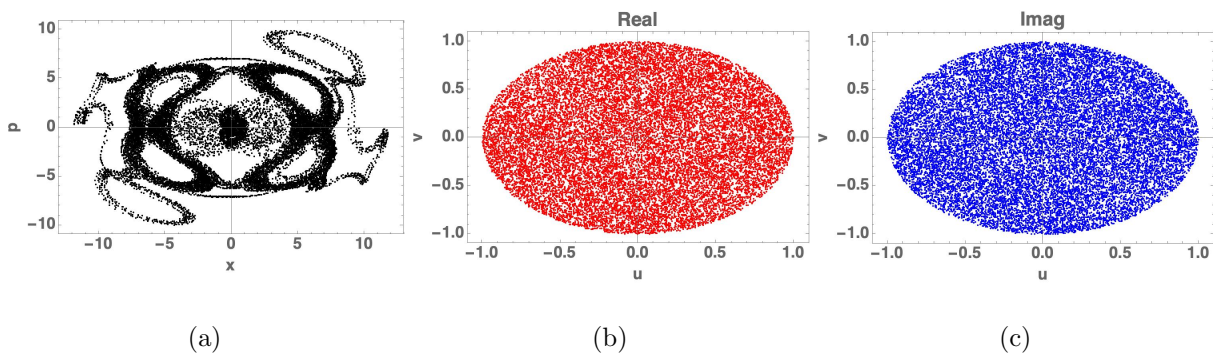

(a)

(b)

(c)

Figure 2: Poincaré surface (p, x), for $u = 0$ (a), and (v, u), real and imaginary parts respectively, for $u = 0$ (b), (c). The system parameters are $f_0 = 20$, $g_e = 0.41$, $R = 1/2$.

Figure 2 demonstrates the effect of destroying of the invariant curve shown in figure 1a and the formation of a characteristic rotating structure on the Poincare surface due to the ratio

between the qubit frequencies and the external force. Since stochastization is clearly observed for the chosen parameters in the qubit system (see figures in line from 2b to 2c), without loss of understanding, we can omit further display of the qubit's map.
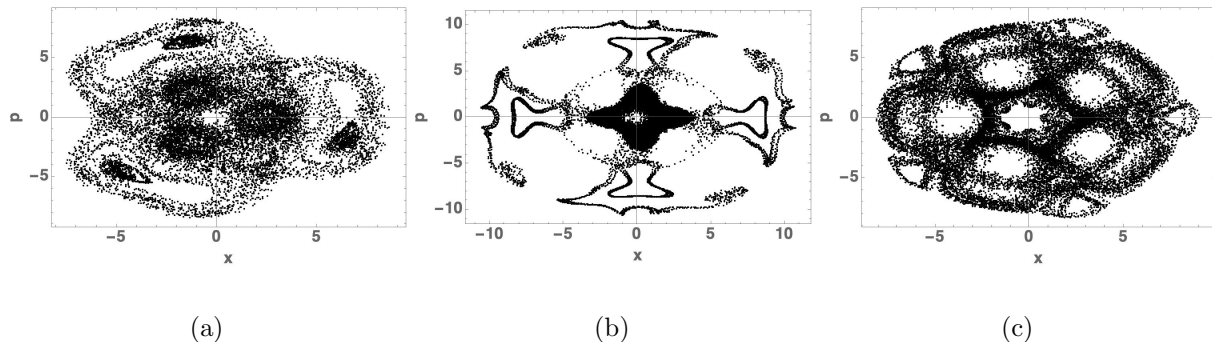

(a)   (b)   (c)

Figure 3: Poincaré surface (p, x), for $u = 0$. The system parameters are $f_0 = 20$, $g_e = 0.41$. (a) $R = 1/3$; (b) $R = 1/4$; (c) $R = 1/5$.

Figure 2a and figure 3 show that chaotic motion occurs, but there are so-called islands of stability. If such initial conditions are chosen that fall into these regions, then the dynamics of the system acquires regular behavior, but if the initial coordinates are on the 'petals', then the movement may be chaotic. This situation is similar to the "Delta-kicked harmonic oscillator" (see [3], p. 162) where the web-mapping was described.

5. Conclusions

In summary, our studies show a nontrivial dynamics of a hybrid quantum-classical systems - the photon-qubit atom, driven by sequences of the short electromagnetic pulses. We have found the map describing the stroboscopic motion in the phase space of the coupled systems. Using this map we have simulated the Poincaré surface which clarified the signature of the chaos and characteristic rotating structure in the phase space. It is shown that depending on the qubit and field parameters the regular motion of the system may be unstable and the system may demonstrating the chaotic behavior. The results can be further useful for studying a fully quantum model of a driving qubit coupled to a photon field.

6. Acknowledgments

7. References
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